2: Simultaneous Linear Inequalities

1. Linear Inequalities (recap)

We learnt in Sec 2 that there are several rules to solving inequalities and learnt how to reduce linear inequalities to one of these forms dependant on the inequality: $x > k$, $x < k$, $x \geq k$, $x \leq k$, where k is a real number. This is a solution to an inequality.

2. Properties of Inequalities (recap)

Let a, b, c be real numbers, where $\square \in \{<, >, \leq, \geq\}$ and the inequality sign is the same throughout.

Transitive Property: If $a \square b$ and $b \square c$, then $a \square c$

For example: When $a \leq b$ and $b \leq c$, then $a \leq c$

Addition Property: If $a \square b$, then $a + c \square b + c$

For example: When $a > b$, then $a + c > b + c$

Subtraction Property: If $a \square b$, then $a - c \square b - c$

For example: When $a \geq b$, then $a - c \geq b - c$

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Multiplication Property:

[Positive] If $a < b$, and c is **positive**, then $ac < ab$

[Negative] If $a < b$, and c is **negative**, then $ac > ab$

Division Property:

[Positive] If $a < b$, and c is **positive**, then $\frac{a}{c} < \frac{b}{c}$

[Negative] If $a < b$, and c is **negative**, then $\frac{a}{c} > \frac{b}{c}$

To put it simply:

If we multiply/divide both sides of an inequality with a **positive** number, the inequality sign **doesn't change**. However, if we multiply/divide both sides of an inequality with a **negative number**, the inequality sign will **flip/reverse direction**.

3. Solving Simultaneous Inequalities

For more detailed information on the step-by-step process of solving linear inequalities and expressing the solution on a number line, please refer to the *Sec 2 G3 Math Revision Notes*.

To solve linear inequalities simultaneously, we find the solutions to each inequality separately first. Thereafter, we consider the common solutions of the two, or the overlap or intersection of the solutions, as the overarching solution of the simultaneous inequalities.
Basically, we are finding an inequality in x that satisfies both linear inequalities.

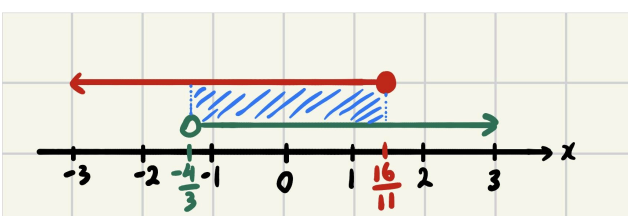
Example:
Solve the compound inequality, $\frac{x-8}{4} < \frac{5x+2}{2} \leq 9 - 3x$, with the help of a number line.

Break into two inequalities and solve individually:

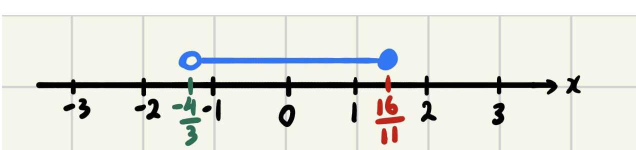
$$\frac{x-8}{4} < \frac{5x+2}{2}$$
$$4(\frac{x-8}{4}) < 4(\frac{5x+2}{2})$$
$$x - 8 < 2(5x + 2)$$
$$x - 8 < 10x + 4$$
$$x - 8 - x - 4 < 10x + 4 - x - 4$$
$$-12 < 9x$$
$$9x > -12$$
$$x > \frac{-12}{9}$$
$$x > -\frac{4}{3}$$

$$\frac{5x+2}{2} \leq 9 - 3x$$
$$2(\frac{5x+2}{2}) \leq 2(9 - 3x)$$
$$5x + 2 \leq 18 - 6x$$
$$5x + 2 + 6x - 2 \leq 18 - 6x + 6x - 2$$
$$11x \leq 16$$
$$x \leq \frac{16}{11}$$

Representing $x > -\frac{4}{3}$ and $x \leq \frac{16}{11}$ on a number line:



By overlapping the two, we get:



Therefore, the solution is $-\frac{4}{3} < x \leq \frac{16}{11}$.

4. Extra Practices

A) Solve $\frac{2x-7}{3} \leq 5 - x$ *and* $4(x + 1) > 7x + 9$. **Hence,** determine whether $x = \frac{22}{5}$ is a solution.

Solution:

$$\frac{2x-7}{3} \leq 5 - x$$
$$3(\frac{2x-7}{3}) \leq 3(5 - x)$$
$$2x - 7 \leq 15 - 3x$$
$$2x - 7 + 3x + 7 \leq 15 - 3x + 3x + 7$$
$$5x \leq 22$$
$$x \leq \frac{22}{5}$$

$$4(x + 1) > 7x + 9$$
$$4x + 4 > 7x + 9$$
$$4x + 4 - 4 - 7x > 7x + 9 - 4 - 7x$$
$$-3x > 5$$
$$\frac{-3x}{-3} < \frac{5}{-3}$$
$$x < -\frac{5}{3}$$

∴ The solution is $x < -\frac{5}{3}$. Hence, $x = \frac{22}{5}$ will not be a valid solution.

B) Solve $5 - 2x < \frac{x+4}{3}$ *and* $3(2x - 1) \geq 7x + 5$.

Solution:

$$5 - 2x < \frac{x+4}{3}$$
$$3(5 - 2x) < 3(\frac{x+4}{3})$$
$$15 - 6x < x + 4$$
$$15 - 6x + 6x - 4 < x + 4 + 6x - 4$$
$$11 < 7x$$
$$7x > 11$$
$$x > \frac{11}{7}$$

$$3(2x - 1) \geq 7x + 5$$
$$6x - 3 \geq 7x + 5$$
$$6x - 3 - 6x - 5 \geq 7x + 5 - 6x - 5$$
$$-8 \geq x$$
$$x \leq -8$$

∴ There is no solution to these simultaneous linear inequalities.

C) Find the range of values that satisfy $\frac{3n-2}{(n-5)^2} > 0$. Explain why.

Solution:

$\frac{3n-2}{(n-5)^2} > 0$ $(n - 5)^2 \frac{3n-2}{(n-5)^2} > 0(n - 5)^2$ $3n - 2 > 0$ $3n - 2 + 2 > 0 + 2$ $3n > 2$ $n > \frac{2}{3}$	However, when $n = 5$, the denominator of $\frac{3n-2}{(n-5)^2}$ will be 0, leaving the expression as undefined or unsolvable. Therefore, $n = 5$ cannot satisfy the inequality and is rejected. ∴ The range of values will be $n > \frac{2}{3}$, $n \neq 5$.