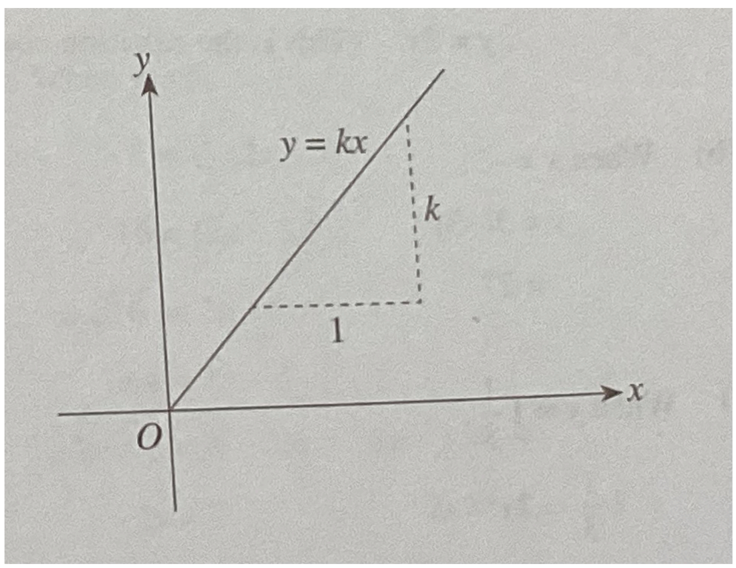
NOTES:

# 7: Direct and Inverse Proportions

**Direct Proportion:**
1, A quantity $y$ is directly proportional (or varies directly) to another quantity $x$ if the ratio of $y : x$ remains the same for all values. The relationship is often written as $y \propto x$.

2. The general equation to represent $y \propto x$ is $\frac{y}{x} = k$ or $y = kx$, where $k$ is known as the constant of variation which can be determined by a known ordered pair, e.g. $(x_1, y_1)$, that satisfies $y = kx$. Note that $k \neq 0$.

3. The graphical representation of $y \propto x$ (i.e. $y = kx$), which is a linear relationship, is a straight line with gradient $k$ that passes through the **origin**.




**Inverse Proportion:**
4. A quantity $y$ is inversely proportional (or varies inversely) to another quantity $x$ if $y$ is directly proportional to $\frac{1}{x}$. The relationship is often written as $y \propto \frac{1}{x}$.

5. The general equation to represent $y \propto \frac{1}{x}$ is $xy = k$ or $y = \frac{k}{x}$, where $k$ is a constant to be determined and $k \neq 0$.

**Solving Word Problems:**

**- Questions that need to solve/state one pair of $(x_1, y_1)$, and the relation of $y \propto x$ or $y \propto \frac{1}{x}$:**

Given that $y$ varies directly as $(2x - 3)^2$ and the difference in the values of $y$ when $x = 2$ and $x = 3$ is $4$.
(a) express $y$ in terms of $x$,
(b) find the values of $x$ when $y = 8$.

Solution:

(a) Since $y$ is directly proportional to $(2x - 3)^2$ i.e. $y \propto (2x - 3)^2$, then $y = k(2x - 3)^2$, where $k$ is a constant.

$(6 - 3)^2k - (4 - 3)^2k = 4$ (Substitute $x = 2$, $x = 3$ into $y = k(2x - 3)^2$ to find the corresponding values of $y$. The two values of $y$ have a difference of $4$.)

$9k - k = 4$

$8k = 4$

$k = \frac{1}{2}$ (Divide both sides by $8$.)

$\therefore y = \frac{1}{2}(2x - 3)^2$ (This is the equation connecting $x$ and $y$.)

(b) When $y = 8$,

$8 = \frac{1}{2}(2x - 3)^2$

$16 = (2x - 3)^2$

$\pm\sqrt{16} = 2x - 3$

$\pm 4 = 2x - 3$

$2x - 3 = 4$ or $2x - 3 = -4$

$2x = 7$ $\quad$ $2x = -1$

$x = 3\frac{1}{2}$ $\quad$ $x = -\frac{1}{2}$

$\therefore x = -\frac{1}{2}$ or $3\frac{1}{2}$

**questions may also provide two relationships, i.e. $y = \frac{kx^2}{z}$, where $y \propto x^2$ and $y \propto \frac{1}{z}$.*

**- Questions that give you a rate:**

A building project takes $12$ men $54$ days to complete.
(a) How long will it take $9$ men to finish the same project?
(b) How many men are needed to complete this project in $36$ days?

Solution:

(a) The number of men is inversely proportional to the number of days to complete the project.
$12$ men take $54$ days to complete the project.
$1$ man takes $(54 \times 12)$ days to complete the project.
$9$ men take $\frac{54\times12}{9} = 72$ days to complete the project.
$\therefore$ It will take $72$ days.

(b) It takes $(54 \times 12)$ days for $1$ man to complete the project.
It takes $1$ day for $(54 \times 12 \times 1)$ men to complete the project.
It takes $36$ days for $\frac{54\times12}{36} = 18$ men to complete the project.
$\therefore$ $18$ men are required.

**questions may be complex and may need more manipulation.*

**- Questions which gives you a change in one variable, i.e. $x$:**

The period, $T$, of a pendulum is inversely proportional to the square root of its gravitational acceleration. When its gravitational acceleration is $g$ m/$s^2$, the period is 2.5$s$
Given that the gravitational acceleration, $g$, is increased by $800\%$ on planet $W$, find
(a) an expression, in terms of $g$, for the gravitational acceleration on planet $W$,
(b) the period on planet $W$.

Solution:

(a) $T = \frac{k}{\sqrt{g}}$, where $k$ is a constant.
When $T = 2.5$,
$2.5 = \frac{k}{\sqrt{g}}$
When $g$ is increased by $800\%$,
$\therefore$ value of $g_{new}$ = $(100\% + 800\%) \times g$
= $9\,g$

(b) Method 1: find $k$

$2.5 = \frac{k}{\sqrt{g}}$

$k = 2.5\sqrt{g}$

$\therefore$ value of $T_{new} = \frac{2.5\sqrt{g}}{\sqrt{9g}}$, when substituting $k$ with $2.5\sqrt{g}$ and $g$ with $g_{new}$.

$= \frac{2.5\sqrt{g}}{3\sqrt{g}}$

$= \frac{5}{6}\ seconds$

Method 2: substitute $T$

$T = \frac{k}{\sqrt{g}}$, when $T = 2.5$

$\therefore$ value of $T_{new} = \frac{k}{\sqrt{9g}}$, when substituting $g$ with $g_{new}$.

$= \frac{k}{3\sqrt{g}}$

$= \frac{1}{3} \times \frac{k}{\sqrt{g}}$

$= \frac{1}{3}T$

$= \frac{1}{3}(2.5)$ (Substitute $T$ with $2.5$)

$= \frac{5}{6}\ seconds$



*Version 3*