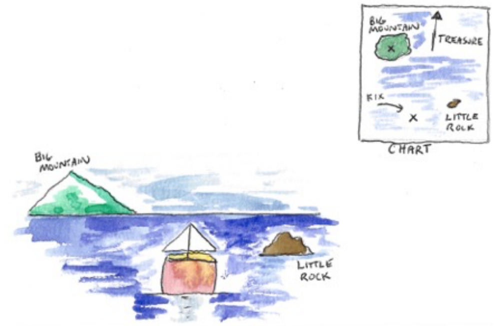
Types of Navigation

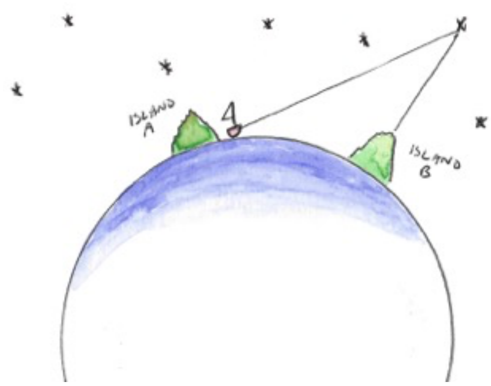


There are many ways to navigate a vessel. In this course we'll focus on the style of celestial navigation that modern yacht sailors and commercial mariners use. But with the goal of eventually trying to understand modern celestial navigation, it's helpful to review several different types of navigation.

The earliest method of navigation is piloting – simply looking at land, rocks, and ocean features and matching them to known objects, either through oral tradition, memory, or on nautical charts.

Next on our list is understanding wayfinding: the process of orienting and traveling from place to place, using processes that allow one to orient themselves on the Earth.

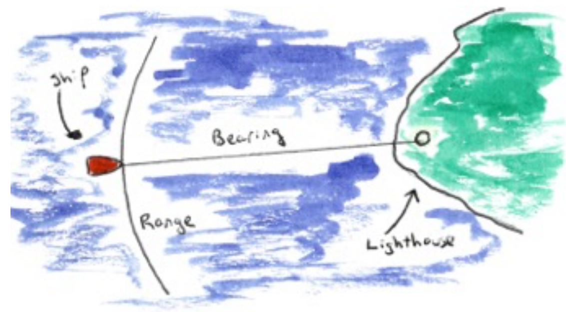
Early Polynesian navigators used natural cues such as the pattern of swells, the presence of birds, the changing color of the horizon or sky or water, and other natural clues to find their way. Coupling this with an oral and written account of the location of certain islands in the Ocean, the Polynesian navigators were able to execute amazing journeys.



For example, all stars have an annual pattern of rising, transiting, and setting which are helpful for orienting oneself on the ocean – this knowledge can be memorized and passed down from navigator to navigator and can be used to create "stick charts" to help understand the local area.

Another type of navigation is Dead Reckoning. This technique relies on periodically recording a vessel's course and speed and projecting this data forward in time. Still used today, this ability to estimate one's future position based on past data is useful, but subject to flaws: it doesn't usually account for oceanic and tidal currents or the effects of winds.

Celestial navigation developed first by using the sun, stars, planets, and moon as steering aids to keep a vessel on course. It grew out of wayfinding and dead reckoning techniques to provide periodic "resets" of known or estimated position on a voyage. However, the increased position knowledge required greater amounts of equipment and knowledge. Thus, the development of the quadrant, the sextant, the chronometer, tables of information, and guides to assist in calculations. We'll spend more time thinking about celestial navigation throughout this course.

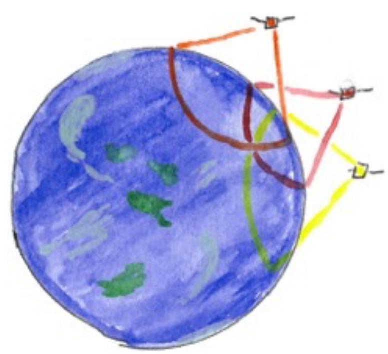


Electronic navigation includes techniques such as decoding time signals from a known location using the relationship between the speed of light, distance, and time. It also includes systems such as the Long-Range Radio Navigation (LORAN) system, which relies on hyperbolic radio signals from multiple locations to result in a position "fix" at sea.

Another modern navigation technique is radio detection and ranging (RADAR) navigation. This technique involves sending radio signals outward from a ship and bouncing them off objects. The return signal, subject to some signal processing and calculation, results in a bearing and range to objects, facilitating navigation.

Finally, satellite navigation combines several previous techniques: it relies on the known position of orbiting satellites, and equipment which can process and decode a time signal from the satellite – with 4 or more satellite measurements, one can plot their position very accurately.

Navigation also relies on using the right tool for the job. For example, when navigating out of a known harbor, piloting, visual, and radar techniques might be most helpful. In the open ocean, satellite navigation and celestial navigation are best suited to the job.



In our study of celestial navigation, the takeaway point for the modern student is that they are studying merely one method of navigation, which should be used in combination with any other navigational tool to facilitate a voyage.

References:

University of Hawaii at Manoa Explanation of Wayfinding and Navigation https://manoa.hawaii.edu/exploringourfluidearth/physical/navigation-and-transportation/wayfinding-and-navigation

Bowditch, 2024 edition, Article 101. https://msi.nga.mil/Publications/APN

The Celestial Navigation 5-Step Process

At this stage in the course, it is helpful to see an overview of the 5 key steps which allow you to use celestial objects to determine a line of position at sea.

We have the tools to calculate a spot on Earth directly beneath a celestial object. We can use a sextant to directly determine our distance from that position, just like taking a radar range from a known object. However, the scales involved are huge, which means we need a technique to make the process more human scale.

The method we use in this course is the Altitude-Intercept Method.

<u>The Altitude-Intercept Method compares a calculated and an observed height of a celestial object and plots the difference on a realistic-scale chart, resulting in one line of position.</u>

Here are the five specific steps that we use in the course:

1. The first step in our process is to use a marine sextant to observe a celestial body. We note the height of the object above the horizon in terms of degrees and minutes. Then, we apply several corrections to that measurement to increase its accuracy.

2. Next, we use the Nautical Almanac to determine where on Earth the celestial body we observed was directly overhead. This **geographic position** forms one point of a spherical triangle that we later solve.

3. The third step in the process is to build the remaining points and sides of the spherical triangle by using the north or south pole, and a carefully chosen **assumed position** somewhere near our dead reckoning location.

4. Using tables or a math formula, we can solve for the unknown parts of the spherical triangle and determine a computed height for the celestial object we observed in step 1, if we were standing exactly at the position that we selected in step 3.

5. Lastly, we compare the computed and observed measurements to determine a difference. We plot the assumed position and the difference from it to help create a celestial **line of position**.

The rest of this part of the course deals with Step 1 of the process. Onward!

The Marine Sextant

The main tool used for celestial navigation, in addition to tables and a timepiece, is the marine sextant. Historical tools used for measuring the angle between a celestial object and the horizon include quadrants, staffs, kamals, and other methods. These are all very interesting for historical purposes, but we'll focus on the modern marine sextant in this course.

The general principle of operation is that the light from a celestial body will arrive and bounce off a series of mirrors, which allow the user to bring the body and the horizon together, then reading the measured angle from the device.

Here are the main parts of the sextant:

- Handle – this is how you hold the sextant in your right hand.

- Index Mirror – this is where the light first arrives.

- Index Shades – these are shades to help dim the light as it arrives.

- Horizon Mirror – this mirror comes in two main varieties, but the function of each is to allow the user to see the horizon and bounce the light of the celestial body from the index mirror further onward.

- Horizon Shades – these are shades to help dim the horizon light.

- Telescope – this is where the light arrives after bouncing off the index and horizon mirror. The telescope has various zoom and focusing capabilities.

- Index Arm – this arm moves via two methods and allows you to move the index mirror in relation to the horizon mirror to allow the objects to come into coincidence.

- Micrometer Drum – this is the method of moving the index arm very slowly.

- Index Release – this is the method of moving the index arm very quickly.

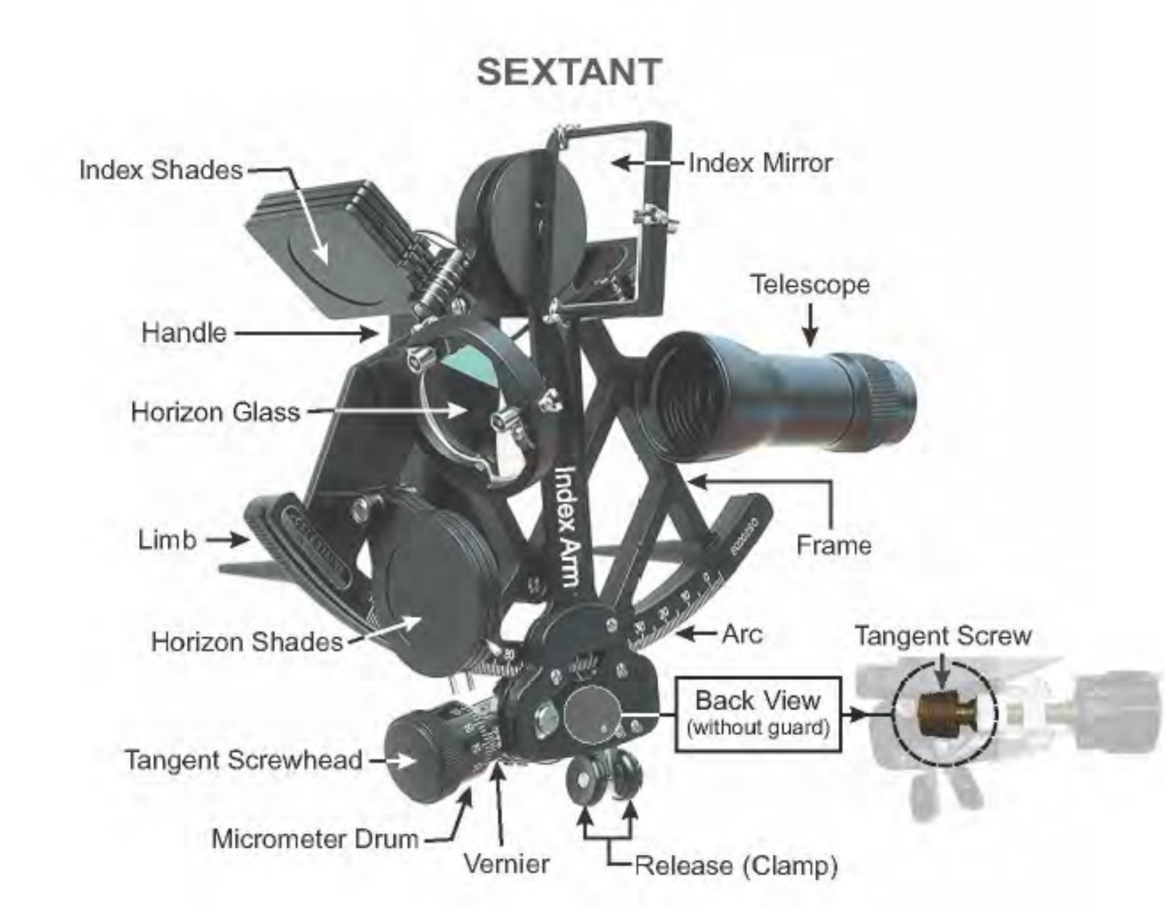
- Arc – this is a graduated scale allowing you to read the ultimate angle of the measurement in terms of degrees.

- Vernier Scale – each sextant will be different, but there is generally a method of reading the ultimate angle of measurement in terms of minutes and seconds or tenths of minutes.

Reference:

Bowditch, 2024 edition, Article 1500 - 1503. https://msi.nga.mil/Publications/APN

Figure Reproduced from Bowditch.



Sextant Cleaning and Tuning

Generally, the sextant will take care of itself, especially if left safely in its case in average temperatures and humidities and areas of low vibration – in other words not in a shipboard environment! But sextants are made for the open sea. If the sextant needs cleaning, here are a few tips:

- Washing with fresh water is a good idea if the sextant has received salt spray.
- Keeping a small silica pouch in the sextant box will help remove moisture from the stored sextant.
- Use lens paper instead of rags on the mirrors – the mirrors are coated with a special coating that you don't want to wear away prematurely.
- You can lubricate the screws and teeth of the sextant with the oil provided in the sextant box.

The sextant, like all human-made objects, is also subject to error and occasionally needs to be tuned. Here are sources of error in the sextant and the means of correcting it:

- Three sources of error that are not adjustable include prismatic error (if the shades and mirrors are not parallel), graduation error (the metal components are improperly manufactured), and centering error (the index arm is not centered). Some sextants arrive from the manufacturer with a card indicating the overall **instrument error** which can be used for calculations.

- Perpendicularity occurs when the index mirror is not perpendicular to the frame. This can be measured with a technique using similarly sized objects placed on the frame and observed through the mirrors. Error can be corrected using the index mirror screw.

- Side error occurs when the horizon mirror is not perpendicular to the frame. This can be measured by observing a known object and tilting the sextant. Error can be corrected using the horizon mirror screw.

- **Index error** occurs when the horizon mirror and the index mirror are not parallel to each other. This common error should be determined each time the sextant is used and mathematically removed from observations. When the index error is unsuitably large (e.g. generally larger than 3 minutes of arc), it can be removed using the second horizon mirror screw.

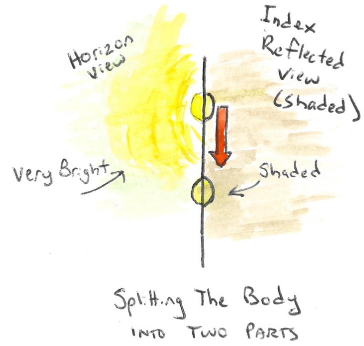
- Collimation error occurs when the telescope is misaligned with the frame of the sextant. This is more challenging to determine and often results from rough handling of the sextant, but proper seating of the telescope should be checked periodically.

Reference:

Bowditch, 2024 edition, Article 1510-1511. https://msi.nga.mil/Publications/APN

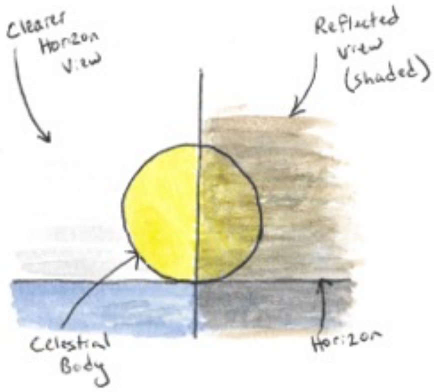
Making Sextant Observations

The best objects to observe are generally between 30° and 70° above the horizon. They should be spaced out about 30° - 45° degrees horizontally to give the best fix.



First – observe the index error, which was described earlier and can be mathematically removed from any sight that you take.

When it comes to taking a sight, the general process is to observe the object through the telescope with the sextant set to zero degrees. Then, keeping the object in view, slowly lower the angle of the sextant until it is parallel to the horizon, while moving the index arm forward, either with the index release or the micrometer drum.



When close to the horizon, swing the sextant back and forth so that the object traces an arc around the horizon. Adjust the sextant until the object just touches the horizon at its lowest point and then note the time and sextant reading.

An alternative method of observation involves using the sextant upside down – and bringing the horizon up to the object. This is helpful when the object is faint or in partly cloudy skies. Once you have the object and horizon close to each other, you can turn the sextant into its normal configuration, sweep the horizon until the object is visible, and continue the sight.

Reading the sextant can be challenging but it is helpful to remember to think in two parts: degrees, and then minutes.

The degree measurement is read off the index arc. Whichever whole number of degrees has been surpassed by the index arm is your value. It is easy to make an error of 1° or 5° or 10°, so take your time and carefully note the reading.

The vernier scale on the micrometer drum will give you minutes and tenths of a minute. Each sextant is unique, so learn your device, but in general look for the whole value of minutes that has been surpassed by the micrometer drum and then use the vernier scale to read the tenths of minutes.

Although the majority of time spent on any celestial sight involves the reduction processes which follow, developing skill as a sextant operator is crucial and will avoid the disappointment of useless results at the conclusion of your subsequent math work.

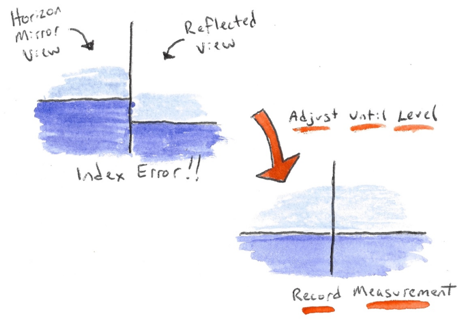
Reference:

Bowditch, 2024 edition, Article 1507 - 1508. https://msi.nga.mil/Publications/APN

Index Corrections

Previously, we've discussed some of the errors that can strike at a sextant, but the most common error you will routinely deal with is index error.

Index error happens when the two mirrors are not exactly parallel. However, it is usually easy to mathematically remove from the sight rather than trying to adjust the mirrors on the sextant. You should check the index error every time you take a sight.



The key concept to remember is that index error can be either <u>on the arc</u> or <u>off the arc</u>, meaning that the index error is a little above zero or below zero. If the error is on the arc, you must take it off (subtract). If the index error is off the arc, you must add it back on.

To measure for index error, set the sextant to zero and observe the horizon. The horizon should look straight and level across the entire view. If one part of the horizon appears stepped or broken above or below the other, adjust the micrometer drum of the sextant until the horizon is level.

Then, read the sextant and pay close attention to the minutes of arc. In situation 1 (below), it might read 1.0'. In situation 2 (below), it might read 59.0'.

<u>Situation 1:</u>

If you performed the above procedure and the sextant read 01.0 minutes, you have some index error. The error is <u>on the arc</u> since it is higher than zero, and you must remove it.

Once you make your sextant measurement for any celestial body, you'll just need to subtract 1.0' of arc from the measurement to account for the index error. This is called making an index correction.

<u>Situation 2:</u>

If, after checking for index error, the sextant read 59.0 minutes, you also have some index error. This time the error is <u>off the arc</u> since it is less than zero (given that the micrometer drum moves continuously. You must add it back on.

So, once you make a celestial sight, to apply index correction in this case, you would add 1.0' of arc to the measurement to account for the index error.

Reference:

Bowditch, 2024 edition, Article 1511. https://msi.nga.mil/Publications/APN

Index Corrections – Practice Problems

Definitions:

Hs – this is the height you measure with the sextant: Height per Sextant.

Ha – this is the apparent altitude once the measurement has been corrected for index error and Dip (which we will learn about later): Height Apparent or Apparent Altitude.

Example Problem 1.1. You observe an object at a height of 40° 05.0'. This is your Hs – height per sextant. You previously determined that the sextant index error was 2.0' on the arc. To determine the apparent altitude (Ha), you must add or subtract the index error from the Hs. Since the index error is on the arc you must subtract it. What is the Ha?

- Height per sextant (Hs) = 40° 05.0' (observed directly from the sextant)
- Index correction = -2.0' (on the arc needs to be subtracted)
- Height apparent (Ha) = 40° 03.0'

Problem 1.1 Answer. 40° 03.0'

Example Problem 1.2. You observe an object at a height (Hs) of 40° 05.0'. The index error is 2.0' off the arc. What is the Ha?

- Height per sextant (Hs) = 40° 05.0' (observed directly from the sextant)
- Index correction = +2.0' (off the arc needs to be added)
- Height apparent (Ha) = 40° 07.0'

Problem 1.2 Answer: 40° 07.0'

Problem 1.3. You observe an object at a height (Hs) of 44° 13.5’. The index error is 2.5’ off the arc. What is the Ha?

Problem 1.4. You observe an object at a height (Hs) of 40° 59.5’. The index error is 2.0’ off the arc. What is the Ha?

Problem 1.5. You observe an object at a height (Hs) of 66° 01.0’. The index error is 1.5’ on the arc. What is the Ha?

Problem 1.6. You observe an object at a height (Hs) of 40° 00.5’. The index error is 1.0’ on the arc. What is the Ha?

Problem 1.3. Answer: 44° 16.0’
Problem 1.4. Answer: 41° 01.5’
Problem 1.5. Answer: 65° 59.5’
Problem 1.6. Answer: 39° 59.5’

The Nautical Almanac (Part 1) – Standard Corrections

The nautical almanac is a key document for us in our study of celestial navigation and learning its intricacies is important.

One of your key tasks in step 1 of the celestial navigation process is applying **standard corrections**, and the Nautical Almanac can help you with that.

These standard corrections include index correction, which we learned about previously, as well as height of eye corrections and altitude corrections, which we'll learn about next.

For now, let's examine the front matter of the Nautical Almanac and get familiar with how the book is laid out.

Here are some key features:

- The 1981 nautical almanac is the standard training book, and we'll use it for this course.
- It is a joint publication of the USA's Naval Observatory, as well as the UK's Navy Office.
- The table of contents is unremarkable, and you'll soon memorize where things live in the almanac.
- The front of the almanac contains correction tables, annual events such as calendar, holidays, phases of the moon, days of the week, eclipses, visibility of the planets.
- The middle section of the almanac contains the daily pages, which we'll explore more later.
- The back of the almanac contains instructions, procedures, tables, and additional methods of correction.

For the next several topics, we'll focus on how to make standard corrections to sextant observations using the nautical almanac's tables for height of eye and altitude corrections. Remember that you've already learned how to apply index corrections, which is also a key standard correction you'll apply nearly ever time you take a celestial sight.

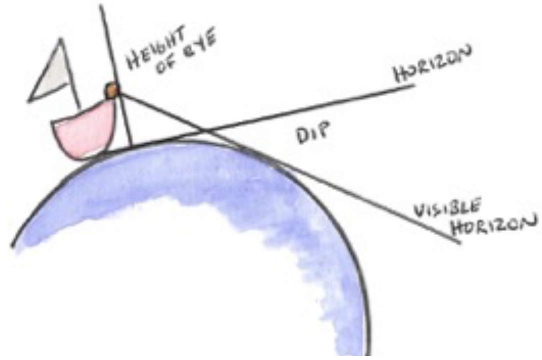
References:

Free website version: https://www.thenauticalalmanac.com/

Government version: https://aa.usno.navy.mil/publications/na

Training Almanac:
https://www.dco.uscg.mil/Portals/9/NMC/pdfs/examinations/10_1981_nautical_almanac.pdf

Height of Eye Corrections (Dip)

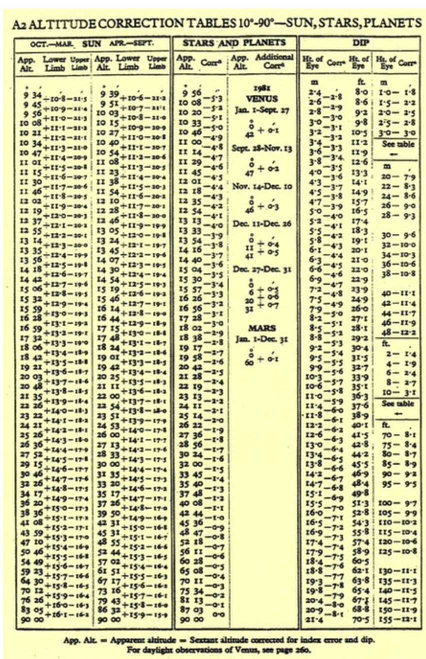


Despite appearances, when you make sextant observations on a boat, you are NOT at sea level. Your eye is above sea level by your height plus the distance from the deck to the sea. Although it is usually a small amount, it is important - all almanac figures and calculations are based on an observer at sea level.

In terms of geometry, there is a difference between the visible horizon that you can see, and the sensible horizon. What you observe through the sextant is always too big, and therefore a negative correction (subtraction) is required to these sights.

Of note, if you are using an artificial horizon to practice sextant sights, this correction does not apply.

You can find the correction in the front tables of the Nautical Almanac → it is called <u>DIP</u>.


A2 ALTITUDE CORRECTION TABLES 10°–90°—SUN, STARS, PLANETS

App. Alt. = Apparent altitude = Sextant altitude corrected for index error and dip.
For daylight observations of Venus, see page 260.

→

DIP

Ht. of Eye	Corrⁿ	Ht. of Eye	Ht. of Eye	Corrⁿ
m		ft.	m	′
2·4		8·0	1·0 —	1·8
	−2·8			
2·6		8·6	1·5 —	2·2
	−2·9			
2·8		9·2	2·0 —	2·5
	−3·0			
3·0		9·8	2·5 —	2·8
	−3·1			
3·2		10·5	3·0 —	3·0
	−3·2			
3·4		11·2	See table	
	−3·3		←	
3·6		11·9		
	−3·4			
3·8		12·6	m	′
	−3·5			
4·0		13·3	20 —	7·9
	−3·6			
4·3		14·1	22 —	8·3
	−3·7			
4·5		14·9	24 —	8·6
	−3·8			
4·7		15·7	26 —	9·0
	−3·9			
5·0		16·5	28 —	9·3
	−4·0			
5·2		17·4		
	−4·1			

References:

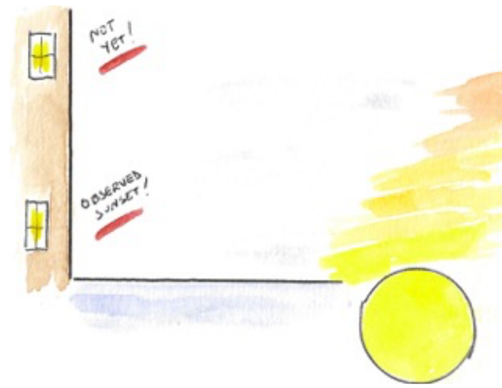
Bowditch, 2024 edition, Article 1604. https://msi.nga.mil/Publications/APN

Free website version: https://www.thenauticalalmanac.com/

Government version: https://aa.usno.navy.mil/publications/na

Height of Eye Corrections (Dip) – Practice Problems

These practice problems will illustrate how to apply height of eye (dip) corrections to sextant sights. This correction is applied nearly every time you take a sextant sight.



Example Problem 1.7. You observe an object at a sextant height (Hs) of 40° 05.0'. There is no index error, so there is no index correction needed. Your height of eye above the surface of the sea is 10.0 feet. Referring to the dip tables, you note that for a height of 10 feet, the dip correction is -3.1. Dip corrections are almost always subtracted. What is the Apparent Altitude (Ha)?

- Height per sextant (Hs) = 40° 05.0' (observed directly from the sextant)
- Index correction = no correction in this problem.
- Height of eye correction (dip) = -3.1' (from the dip tables for a height of eye of 10 ft)
- Height apparent (Ha) = 40° 01.9'

Problem 1.7 Answer: 40° 01.9'

Example Problem 1.8. You observe an object at a Sextant Height (Hs) of 55° 30.0'. There is no index error. Your height of eye is 20 feet. What is the Apparent Altitude (Ha)?

- Height per sextant (Hs) = 55° 30.0' (observed directly from the sextant)
- Index correction = no correction in this problem.
- Height of eye correction (dip) = -4.3' (from the dip tables for a height of eye of 20 ft)
- Height apparent (Ha) = 55° 25.7'

Problem 1.8 Answer: 55° 25.7'

Problem 1.9. You observe an object at a Sextant Height (Hs) of 25° 45'. There is no index error. Your height of eye is 1.5 meters. What is the Apparent Altitude (Ha)?

Problem 1.10. You observe an object at a Sextant Height (Hs) of 33° 25'. There is no index error. Your height of eye is 12 feet. What is the Apparent Altitude (Ha)?

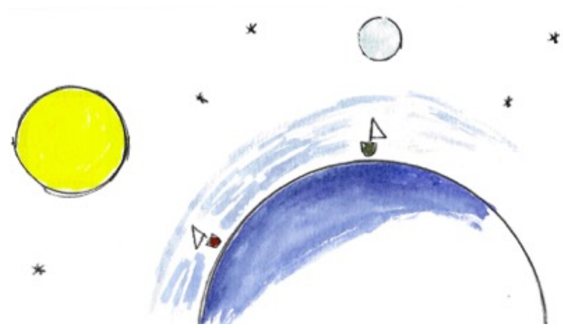
Problem 1.11. You observe an object at a Sextant Height (Hs) of 46° 01.5'. There is no index error. Your height of eye is 2.5 meters. What is the Apparent Altitude (Ha)?

Problem 1.12. You observe an object at Hs 35° 13.2'. The index error is 2.0' on the arc. The height of eye is 9 feet. What is the Ha?

Problem 1.13. You observe an object at Hs 66° 00.5'. The index error is 2.2' off the arc. The height of eye is 3.7 meters. What is the Ha?

Problem 1.9. Answer: 25° 42.8’
Problem 1.10. Answer: 33° 21.6’
Problem 1.11. Answer: 45° 58.7’
Problem 1.12. Answer: 35° 08.3’
Problem 1.13. Answer: 65° 59.3’

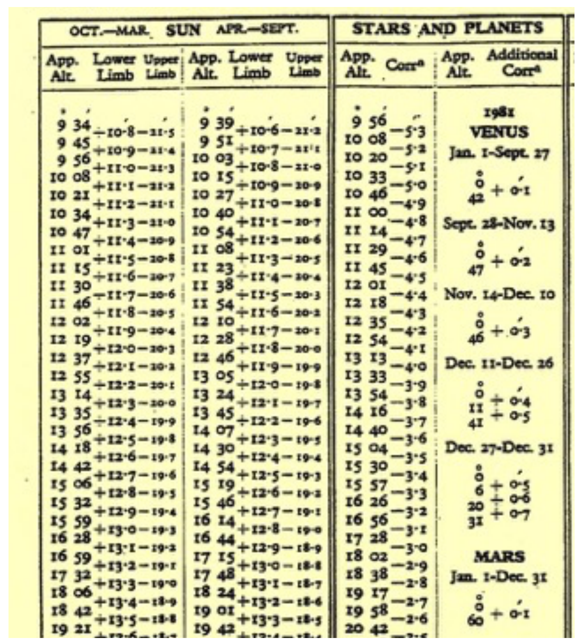
Altitude Corrections (Normal Conditions)



Once we have corrected a sextant sight for index error and dip, we have obtained the **Apparent Altitude**. The next step is to correct this figure for variables including the diameter of the object, the refraction of the atmosphere, parallax issues for various bodies, the temperature and pressure, and the proximity to Earth.

Although that sounds complicated, it is generally all wrapped up into one or two additional corrections. First, some notes for observations in "Normal Conditions":

- Refraction: the bending of light in the atmosphere due to the differing properties of air between the observer and the object. For example, consider observing something directly near the horizon (lots of atmosphere to look through) versus something directly overhead (less atmosphere to look through).



OCT.—MAR. SUN APR.—SEPT.						STARS AND PLANETS			
App. Alt.	Lower Limb	Upper Limb	App. Alt.	Lower Limb	Upper Limb	App. Alt.	Corr^n	App. Alt.	Additional Corr^n
9 34	+10.8	−21.5	9 39	+10.6	−21.2	9 56	−5.3		1981
9 45	+10.9	−21.4	9 51	+10.7	−21.1	10 08	−5.2		VENUS
9 56	+11.0	−21.3	10 03	+10.8	−21.0	10 20	−5.1		Jan. 1-Sept. 27
10 08	+11.1	−21.2	10 15	+10.9	−20.9	10 33	−5.0	0°	+0.1'
10 21	+11.2	−21.1	10 27	+11.0	−20.8	10 46	−4.9	42	
10 34	+11.3	−21.0	10 40	+11.1	−20.7	11 00	−4.8		Sept. 28-Nov. 13
10 47	+11.4	−20.9	10 54	+11.2	−20.6	11 14	−4.7	0°	+0.2'
11 01	+11.5	−20.8	11 08	+11.3	−20.5	11 29	−4.6	47	
11 15	+11.6	−20.7	11 23	+11.4	−20.4	11 45	−4.5		Nov. 14-Dec. 10
11 30	+11.7	−20.6	11 38	+11.5	−20.3	12 01	−4.4	0°	+0.3'
11 46	+11.8	−20.5	11 54	+11.6	−20.2	12 18	−4.3	46	
12 02	+11.9	−20.4	12 10	+11.7	−20.1	12 35	−4.2		Dec. 11-Dec. 26
12 19	+12.0	−20.3	12 28	+11.8	−20.0	12 54	−4.1	0°	+0.4'
12 37	+12.1	−20.2	12 46	+11.9	−19.9	13 13	−4.0	11	+0.5
12 55	+12.2	−20.1	13 05	+12.0	−19.8	13 33	−3.9	41	
13 14	+12.3	−20.0	13 24	+12.1	−19.7	13 54	−3.8		Dec. 27-Dec. 31
13 35	+12.4	−19.9	13 45	+12.2	−19.6	14 16	−3.7	0°	+0.5'
13 56	+12.5	−19.8	14 07	+12.3	−19.5	14 40	−3.6	6	+0.6
14 18	+12.6	−19.7	14 30	+12.4	−19.4	15 04	−3.5	20	+0.7
14 42	+12.7	−19.6	14 54	+12.5	−19.3	15 30	−3.4	31	
15 06	+12.8	−19.5	15 19	+12.6	−19.2	15 57	−3.3		
15 32	+12.9	−19.4	15 46	+12.7	−19.1	16 26	−3.2		
15 59	+13.0	−19.3	16 14	+12.8	−19.0	16 56	−3.1		
16 28	+13.1	−19.2	16 44	+12.9	−18.9	17 28	−3.0		
16 59	+13.2	−19.1	17 15	+13.0	−18.8	18 02	−2.9		MARS
17 32	+13.3	−19.0	17 48	+13.1	−18.7	18 38	−2.8		Jan. 1-Dec. 31
18 06	+13.4	−18.9	18 24	+13.2	−18.6	19 17	−2.7	0°	+0.1'
18 42	+13.5	−18.8	19 01	+13.3	−18.5	19 58	−2.6	60	
19 21			19 42			20 42			

- Semi-diameter: all celestial calculations are based on the exact center of an object. However, if the object is large, it is easier for us to observe the objects lower limb or upper limb (think about the sun or the moon). Semi-diameter correction accounts for the difference between the limb/edge of the object and the center.

- The Sun's proximity to Earth changes with the seasons, and this can affect celestial navigation calculations.

Making apparent altitude corrections in normal conditions is relatively straightforward using the tables at the front of the Nautical Almanac – just be sure to pay attention to the season, the object, the limb (sun and moon), and the sign of the correction from the tables.

Apply this correction to the Ha to determine the **Height observed**, knows as Ho.

References:

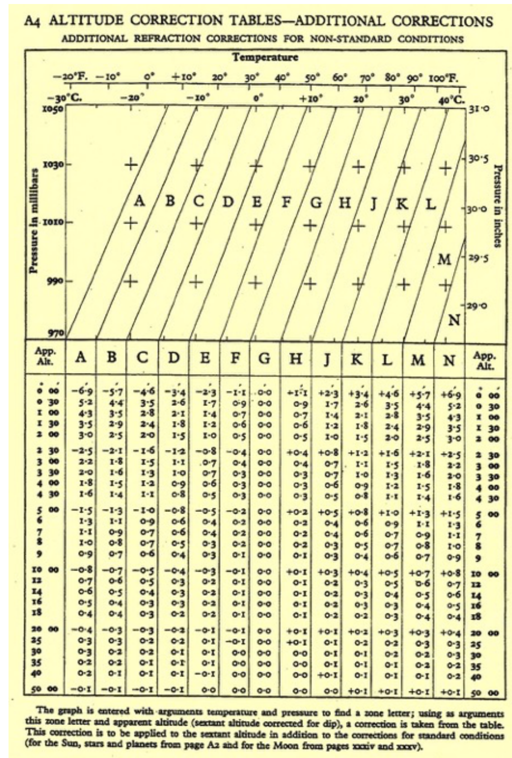
Bowditch, 2024 edition, Article 1605, 1608. https://msi.nga.mil/Publications/APN

Free website version: https://www.thenauticalalmanac.com/

Government version: https://aa.usno.navy.mil/publications/na

Altitude Corrections (Abnormal Conditions)

Sometimes, atmospheric conditions are so abnormal that additional corrections are required. These corrections are usually small and can almost always be ignored in practical navigation.


A4 ALTITUDE CORRECTION TABLES—ADDITIONAL CORRECTIONS

ADDITIONAL REFRACTION CORRECTIONS FOR NON-STANDARD CONDITIONS

App. Alt.	A	B	C	D	E	F	G	H	J	K	L	M	N	App. Alt.
° ′	′	′	′	′	′	′	′	′	′	′	′	′	′	° ′
0 00	-6.9	-5.7	-4.6	-3.4	-2.3	-1.1	0.0	+1.1	+2.3	+3.4	+4.6	+5.7	+6.9	0 00
0 30	5.2	4.4	3.5	2.6	1.7	0.9	0.0	0.9	1.7	2.6	3.5	4.4	5.2	0 30
1 00	4.3	3.5	2.8	2.1	1.4	0.7	0.0	0.7	1.4	2.1	2.8	3.5	4.3	1 00
1 30	3.5	2.9	2.4	1.8	1.2	0.6	0.0	0.6	1.2	1.8	2.4	2.9	3.5	1 30
2 00	3.0	2.5	2.0	1.5	1.0	0.5	0.0	0.5	1.0	1.5	2.0	2.5	3.0	2 00
2 30	-2.5	-2.1	-1.6	-1.2	-0.8	-0.4	0.0	+0.4	+0.8	+1.2	+1.6	+2.1	+2.5	2 30
3 00	2.2	1.8	1.5	1.1	0.7	0.4	0.0	0.4	0.7	1.1	1.5	1.8	2.2	3 00
3 30	2.0	1.6	1.3	1.0	0.7	0.3	0.0	0.3	0.7	1.0	1.3	1.6	2.0	3 30
4 00	1.8	1.5	1.2	0.9	0.6	0.3	0.0	0.3	0.6	0.9	1.2	1.5	1.8	4 00
4 30	1.6	1.4	1.1	0.8	0.5	0.3	0.0	0.3	0.5	0.8	1.1	1.4	1.6	4 30
5 00	-1.5	-1.3	-1.0	-0.8	-0.5	-0.2	0.0	+0.2	+0.5	+0.8	+1.0	+1.3	+1.5	5 00
6	1.3	1.1	0.9	0.6	0.4	0.2	0.0	0.2	0.4	0.6	0.9	1.1	1.3	6
7	1.1	0.9	0.7	0.6	0.4	0.2	0.0	0.2	0.4	0.6	0.7	0.9	1.1	7
8	1.0	0.8	0.7	0.5	0.3	0.2	0.0	0.2	0.3	0.5	0.7	0.8	1.0	8
9	0.9	0.7	0.6	0.4	0.3	0.1	0.0	0.1	0.3	0.4	0.6	0.7	0.9	9
10 00	-0.8	-0.7	-0.5	-0.4	-0.3	-0.1	0.0	+0.1	+0.3	+0.4	+0.5	+0.7	+0.8	10 00
12	0.7	0.6	0.5	0.3	0.2	0.1	0.0	0.1	0.2	0.3	0.5	0.6	0.7	12
14	0.6	0.5	0.4	0.3	0.2	0.1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	14
16	0.5	0.4	0.3	0.3	0.2	0.1	0.0	0.1	0.2	0.3	0.3	0.4	0.5	16
18	0.4	0.4	0.3	0.2	0.2	0.1	0.0	0.1	0.2	0.2	0.3	0.4	0.4	18
20 00	-0.4	-0.3	-0.3	-0.2	-0.1	-0.1	0.0	+0.1	+0.1	+0.2	+0.3	+0.3	+0.4	20 00
25	0.3	0.3	0.2	0.2	0.1	-0.1	0.0	+0.1	0.1	0.2	0.2	0.3	0.3	25
30	0.3	0.2	0.2	0.1	0.1	0.0	0.0	0.0	0.1	0.1	0.2	0.2	0.3	30
35	0.2	0.2	0.1	0.1	0.1	0.0	0.0	0.0	0.1	0.1	0.1	0.2	0.2	35
40	0.2	0.1	0.1	0.1	-0.1	0.0	0.0	0.0	+0.1	0.1	0.1	0.1	0.2	40
50 00	-0.1	-0.1	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	+0.1	+0.1	+0.1	+0.1	50 00

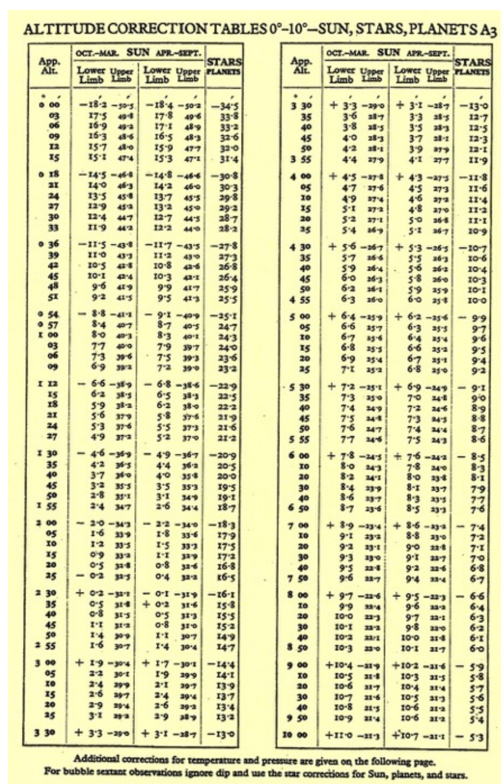
The graph is entered with arguments temperature and pressure to find a zone letter; using as arguments this zone letter and apparent altitude (sextant altitude corrected for dip), a correction is taken from the table. This correction is to be applied to the sextant altitude in addition to the corrections for standard conditions (for the Sun, stars and planets from page A2 and for the Moon from pages xxxiv and xxxv).

- Temperature affects the atmosphere by changing its density, resulting in variable refraction. Usually, standard almanac corrections are sufficient, but in cases of extremely high or low temperature, an additional correction is needed.

- Pressure can also affect the refraction of the atmosphere. The correction is typically extremely small, but it does exist.

- The phase of a celestial object can also be significant enough to warrant a correction. We will discuss the moon later, but Mars and Venus are also close enough to the Earth to demonstrate phases. These corrections are typically small and seasonal.

- Extreme refraction occurs at altitudes of less than 10 degrees, so the Nautical Almanac includes special tables for these "low" sights.


ALTITUDE CORRECTION TABLES 0°–10°—SUN, STARS, PLANETS A3

App. Alt.	Oct.–Mar. Sun Lower Limb	Oct.–Mar. Sun Upper Limb	Apr.–Sept. Sun Lower Limb	Apr.–Sept. Sun Upper Limb	Stars Planets
° ′	′	′	′	′	′
0 00	-18.2	-50.5	-18.4	-50.2	-34.5
03	17.5	49.8	17.8	49.6	33.8
06	16.9	49.2	17.1	48.9	33.2
09	16.3	48.6	16.5	48.3	32.6
12	15.7	48.0	15.9	47.7	32.0
15	15.1	47.4	15.3	47.1	31.4
0 18	-14.5	-46.8	-14.8	-46.6	-30.8
21	14.0	46.3	14.2	46.0	30.3
24	13.5	45.8	13.7	45.5	29.8
27	12.9	45.2	13.2	45.0	29.2
30	12.4	44.7	12.7	44.5	28.7
33	11.9	44.2	12.2	44.0	28.2
0 36	-11.5	-43.8	-11.7	-43.5	-27.8
39	11.0	43.3	11.2	43.0	27.3
42	10.5	42.8	10.8	42.6	26.8
45	10.1	42.4	10.3	42.1	26.4
48	9.6	41.9	9.9	41.7	25.9
51	9.2	41.5	9.5	41.3	25.5
0 54	- 8.8	-41.1	- 9.1	-40.9	-25.1
0 57	8.4	40.7	8.7	40.5	24.7
1 00	8.0	40.3	8.3	40.1	24.3
03	7.7	40.0	7.9	39.7	24.0
06	7.3	39.6	7.5	39.3	23.6
09	6.9	39.2	7.2	39.0	23.2
1 12	- 6.6	-38.9	- 6.8	-38.6	-22.9
15	6.2	38.5	6.5	38.3	22.5
18	5.9	38.2	6.2	38.0	22.2
21	5.6	37.9	5.8	37.6	21.9
24	5.3	37.6	5.5	37.3	21.6
27	4.9	37.2	5.2	37.0	21.2
1 30	- 4.6	-36.9	- 4.9	-36.7	-20.9
35	4.2	36.5	4.4	36.2	20.5
40	3.7	36.0	4.0	35.8	20.0
45	3.2	35.5	3.5	35.3	19.5
50	2.8	35.1	3.1	34.9	19.1
1 55	2.4	34.7	2.6	34.4	18.7
2 00	- 2.0	-34.3	- 2.2	-34.0	-18.3
05	1.6	33.9	1.8	33.6	17.9
10	1.2	33.5	1.5	33.3	17.5
15	0.9	33.2	1.1	32.9	17.2
20	0.5	32.8	0.8	32.6	16.8
25	- 0.2	32.5	0.4	32.2	16.5
2 30	+ 0.2	-32.1	- 0.1	-31.9	-16.1
35	0.5	31.8	+ 0.2	31.6	15.8
40	0.8	31.5	0.5	31.3	15.5
45	1.1	31.2	0.8	31.0	15.2
50	1.4	30.9	1.1	30.7	14.9
2 55	1.6	30.7	1.4	30.4	14.7
3 00	+ 1.9	-30.4	+ 1.7	-30.1	-14.4
05	2.2	30.1	1.9	29.9	14.1
10	2.4	29.9	2.1	29.7	13.9
15	2.6	29.7	2.4	29.4	13.7
20	2.9	29.4	2.6	29.2	13.4
25	3.1	29.2	2.9	28.9	13.2
3 30	+ 3.3	-29.0	+ 3.1	-28.7	-13.0

App. Alt.	Oct.–Mar. Sun Lower Limb	Oct.–Mar. Sun Upper Limb	Apr.–Sept. Sun Lower Limb	Apr.–Sept. Sun Upper Limb	Stars Planets
° ′	′	′	′	′	′
3 30	+ 3.3	-29.0	+ 3.1	-28.7	-13.0
35	3.6	28.7	3.3	28.5	12.7
40	3.8	28.5	3.5	28.3	12.5
45	4.0	28.3	3.7	28.1	12.3
50	4.2	28.1	3.9	27.9	12.1
3 55	4.4	27.9	4.1	27.7	11.9
4 00	+ 4.5	-27.8	+ 4.3	-27.5	-11.8
05	4.7	27.6	4.5	27.3	11.6
10	4.9	27.4	4.6	27.2	11.4
15	5.1	27.2	4.8	27.0	11.2
20	5.2	27.1	5.0	26.8	11.1
25	5.4	26.9	5.1	26.7	10.9
4 30	+ 5.6	-26.7	+ 5.3	-26.5	-10.7
35	5.7	26.6	5.5	26.3	10.6
40	5.9	26.4	5.6	26.2	10.4
45	6.0	26.3	5.8	26.0	10.3
50	6.2	26.1	5.9	25.9	10.1
4 55	6.3	26.0	6.0	25.8	10.0
5 00	+ 6.4	-25.9	+ 6.2	-25.6	- 9.9
05	6.6	25.7	6.3	25.5	9.7
10	6.7	25.6	6.4	25.4	9.6
15	6.8	25.5	6.6	25.2	9.5
20	6.9	25.4	6.7	25.1	9.4
25	7.1	25.2	6.8	25.0	9.2
5 30	+ 7.2	-25.1	+ 6.9	-24.9	- 9.1
35	7.3	25.0	7.0	24.8	9.0
40	7.4	24.9	7.2	24.6	8.9
45	7.5	24.8	7.3	24.5	8.8
50	7.6	24.7	7.4	24.4	8.7
5 55	7.7	24.6	7.5	24.3	8.6
6 00	+ 7.8	-24.5	+ 7.6	-24.2	- 8.5
10	8.0	24.3	7.8	24.0	8.3
20	8.2	24.1	8.0	23.8	8.1
30	8.4	23.9	8.1	23.7	7.9
40	8.6	23.7	8.3	23.5	7.7
6 50	8.7	23.6	8.5	23.3	7.6
7 00	+ 8.9	-23.4	+ 8.6	-23.2	- 7.4
10	9.1	23.2	8.8	23.0	7.2
20	9.2	23.1	9.0	22.8	7.1
30	9.3	23.0	9.1	22.7	7.0
40	9.5	22.8	9.2	22.6	6.8
7 50	9.6	22.7	9.4	22.4	6.7
8 00	+ 9.7	-22.6	+ 9.5	-22.3	- 6.6
10	9.9	22.4	9.6	22.2	6.4
20	10.0	22.3	9.7	22.1	6.3
30	10.1	22.2	9.8	22.0	6.2
40	10.2	22.1	10.0	21.8	6.1
8 50	10.3	22.0	10.1	21.7	6.0
9 00	+10.4	-21.9	+10.2	-21.6	- 5.9
10	10.5	21.8	10.3	21.5	5.8
20	10.6	21.7	10.4	21.4	5.7
30	10.7	21.6	10.5	21.3	5.6
40	10.8	21.5	10.6	21.2	5.5
9 50	10.9	21.4	10.6	21.2	5.4
10 00	+11.0	-21.3	+10.7	-21.1	- 5.3

Additional corrections for temperature and pressure are given on the following page.
For bubble sextant observations ignore dip and use the star corrections for Sun, planets, and stars.

Making altitude corrections in abnormal conditions is rare, but relatively straightforward – simply make an additional correction to your previous calculations to determine your Ho.

References: Bowditch, 2024 edition, Article 1606, 1607, 1609. https://msi.nga.mil/Publications/APN

Free website version:
https://www.thenauticalalmanac.com/

Government version:
https://aa.usno.navy.mil/publications/na

Altitude Corrections (Normal Conditions) – Practice Problems

Example Problem 1.14. You have observed the sun's lower limb on October 11th. You previously determined the Apparent Altitude (Ha) was 43° 13.0'. What is the Observed Altitude (Ho)?

- Height apparent (Ha) = 43° 13.0' (Hs already corrected for index error and dip)
- Apparent altitude correction = +15.2' (lower limb, October 11th, per table)
- Height observed (Ho) = 43° 28.2'

Problem 1.14 Answer: 43° 28.2'

Example Problem 1.15. You have observed the Sun's lower limb on June 27th. You corrected your observation for index error and height of eye to obtain an Apparent Altitude (Ha) of 62° 45.0'. What is the Observed Altitude (Ho)?

- Height apparent (Ha) = 62° 45.0' (Hs already corrected for index error and dip)
- Apparent altitude correction = +15.5' (lower limb, June 27th, per table)
- Height observed (Ho) = 63° 00.5'

Problem 1.15. Answer: 63° 00.5'

Problem 1.16. On October 31, you observed the Sun's lower limb and have determined the Apparent Altitude (Ha) of the Sun to be 10° 12.0'. What is the Ho?

Problem 1.17. On November 2, you observed the Sun's lower limb and have determined the Apparent Altitude (Ha) of the Sun to be 13° 43.7'. What is the Ho?

Problem 1.18. You observe the Sun's lower limb on April 13th. The Apparent Altitude is 27° 12.5'. What is the Observed Altitude?

Problem 1.19. You observe the Sun's UPPER limb on 2 October. The Ha is 65° 30.0'. What is the Ho?

Problem 1.20. You have taken a sextant sight of the sun's lower limb on 12 December. The index error is 2.0' on the arc. The height of eye is 10 feet. The Hs is 45° 30.0'. What is the Observed Altitude (Ho)?

Problem 1.21. It is April 15th, and you have taken a sextant sight of the sun and measured 53° 59.5'. You observed the sun's lower limb. The height of eye is 2.5 meters, and the index error is 3.5 minutes off the arc. What is the Ho?

Problem 1.22. You observe the lower limb of the Sun at a sextant altitude of 54° 28.2' on 22 July. The index error is 1.5' off the arc. The height of eye is 56 feet. What is the observed altitude?

Problem 1.23. You observe the star Sirius at an apparent altitude of 30° 15.0'. What is the Observed Altitude?

Problem 1.24. You have observed the star Mirfak. The apparent altitude is 55° 12.9'. What is the Ho?

Problem 1.25. On December 15th, you take an observation of Venus. The Apparent Altitude is 18° 33.6'. You desire to make an apparent altitude correction, as well as a seasonal/phase correction for Venus using the Nautical Almanac. What is the Observed Altitude?

Problem 1.26. It is 5 January, and you observe Venus at an Apparent Altitude of 12° 15.9'. What is the Observed Altitude?

Problem 1.16. Answer: 10° 23.1’
Problem 1.17. Answer: 13° 56.1’
Problem 1.18. Answer: 27° 26.6’
Problem 1.19. Answer: 65° 13.5’
Problem 1.20. Answer: 45° 40.2’
Problem 1.21. Answer: 54° 15.5’
Problem 1.22. Answer: 54° 37.7’
Problem 1.23. Answer: 30° 13.3’
Problem 1.24. Answer: 55° 12.2’
Problem 1.25. Answer: 18° 31.2’
Problem 1.26. Answer: 12° 11.6’

Altitude Corrections (Abnormal Conditions) – Practice Problems

Example Problem 1.27. You have observed the Sun's lower limb on 2 October and determined the apparent altitude of the Sun to be 15° 30'. There are abnormal conditions, with pressure of 1020mb and temperature of 10F. Estimating the values in the Altitude Correction Tables are sufficient. What is the Observed Altitude?

- Height apparent (Ha) = 15° 30.0' (given)
- Apparent altitude correction = +12.8' (lower limb, October 2nd, per table)
- Intermediate step: 15° 42.8'
- Additional correction = -0.4' (pressure 1020 and temperature +10° F = Zone B)
- Height observed (Ho) = 15° 42.4'

Problem 1.27. Answer: 15° 42.4'

For the below problems, accuracy to the nearest 0.5' is sufficient, and estimating values in the Altitude Correction Table for Non-Standard Conditions is acceptable.

Problem 1.28. On 5 April you observed the Sun's lower limb and determined the Apparent Altitude to be 22° 13.9'. The pressure is 972mb and the temperature is 70F. What is the Observed Altitude?

Problem 1.29. It is the 20th of October, and you take a low-altitude sight of the sun's lower limb. You determined the Apparent Altitude (Ha) to be 06° 14.5'. Using the low-altitude sight correction tables, determine the Observed Altitude (Ho).

Problem 1.30. You have taken a low altitude sight of Jupiter on September 5th. The apparent altitude is 08° 11.0'. What is the Observed Altitude?

Problem 1.31. On 3 January you observe Venus at a Sextant Altitude (Hs) of 42° 12.6'. The index error is 1.0 on the arc and the height of eye is 2 meters. The atmospheric pressure is 1030mb and the temperature is 20F. What is the observed altitude (Ho)?

Problem 1.32. You observe the sun's lower limb on March 5th and determine the Hs to be 06° 13.0'. The height of eye is 12.5 meters, and the sextant has an index error of 01.1 minutes off the arc. The pressure is 1010mb and the temperature is 15F. What is the Ho?

Problem 1.28. Answer: 22° 27.8'
Problem 1.29. Answer: 06° 22.6'
Problem 1.30. Answer: 08° 04.6'
Problem 1.31. Answer: 42° 08.0'
Problem 1.32. Answer: 06° 15.0'

Altitude Corrections for the Moon

The moon is a very challenging object to work with because it is very close to Earth, it changes phases, and it alters the visibility of the horizon when making sights. However, it is in the sky during the day half the month, and is impossible to mistake for another object, making it quite reliable for position fixing.

Moon corrections are found at the back of the Nautical Almanac and account for several things:

- The moon has its own Dip table near the back of the Nautical Almanac; however, this is simply duplicated from the front of the Almanac for convenience.
- The first apparent altitude correction for the moon is at the upper part of the table, and accounts for refraction and semi-diameter, like the sun.
- The second altitude correction for the moon is at the lower part of the table and accounts for **horizontal parallax** – the proximity of the moon to Earth. It is important to note the table has values for each of the upper and lower limb. We will learn how to determine the horizontal parallax later.
- All corrections for the moon are added to the apparent altitude (Ha), except 30' is subtracted when using the upper limb of the moon.

The moon is challenging but enjoyable to observe and can result in excellent daytime fixes with the sun at the right time of the lunar cycle.

ALTITUDE CORRECTION TABLES 0°–35°—MOON

App. Alt.	0°–4° Corr^n	5°–9° Corr^n	10°–14° Corr^n	15°–19° Corr^n	20°–24° Corr^n	25°–29° Corr^n	30°–34° Corr^n	App. Alt.
′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	′
00	0 33·8	5 58·2	10 62·1	15 62·8	20 62·2	25 60·8	30 58·9	00
10	35·9	58·5	62·2	62·8	62·1	60·8	58·8	10
20	37·8	58·7	62·2	62·8	62·1	60·7	58·8	20
30	39·6	58·9	62·3	62·8	62·1	60·7	58·7	30
40	41·2	59·1	62·3	62·8	62·0	60·6	58·6	40
50	42·6	59·3	62·4	62·7	62·0	60·6	58·5	50
00	1 44·0	6 59·5	11 62·4	16 62·7	21 62·0	26 60·5	31 58·5	00
10	45·2	59·7	62·4	62·7	61·9	60·4	58·4	10
20	46·3	59·9	62·5	62·7	61·9	60·4	58·3	20
30	47·3	60·0	62·5	62·7	61·9	60·3	58·2	30
40	48·3	60·2	62·5	62·7	61·8	60·3	58·2	40
50	49·2	60·3	62·6	62·7	61·8	60·2	58·1	50
00	2 50·0	7 60·5	12 62·6	17 62·7	22 61·7	27 60·1	32 58·0	00
10	50·8	60·6	62·6	62·6	61·7	60·1	57·9	10
20	51·4	60·7	62·6	62·6	61·6	60·0	57·8	20
30	52·1	60·9	62·7	62·6	61·6	59·9	57·8	30
40	52·7	61·0	62·7	62·6	61·5	59·9	57·7	40
50	53·3	61·1	62·7	62·6	61·5	59·8	57·6	50
00	3 53·8	8 61·2	13 62·7	18 62·5	23 61·5	28 59·7	33 57·5	00
10	54·3	61·3	62·7	62·5	61·4	59·7	57·4	10
20	54·8	61·4	62·7	62·5	61·4	59·6	57·4	20
30	55·2	61·5	62·8	62·5	61·3	59·6	57·3	30
40	55·6	61·6	62·8	62·4	61·3	59·5	57·2	40
50	56·0	61·6	62·8	62·4	61·2	59·4	57·1	50
00	4 56·4	9 61·7	14 62·8	19 62·4	24 61·2	29 59·3	34 57·0	00
10	56·7	61·8	62·8	62·3	61·1	59·3	56·9	10
20	57·1	61·9	62·8	62·3	61·1	59·2	56·9	20
30	57·4	61·9	62·8	62·3	61·0	59·1	56·8	30
40	57·7	62·0	62·8	62·2	60·9	59·1	56·7	40
50	57·9	62·1	62·8	62·2	60·9	59·0	56·6	50

H.P.	L	U	L	U	L	U	L	U	L	U	L	U	L	U	H.P.
′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′
54·0	0·3	0·9	0·3	0·9	0·4	1·0	0·5	1·1	0·6	1·2	0·7	1·3	0·9	1·5	54·0
54·3	0·7	1·1	0·7	1·2	0·7	1·2	0·8	1·3	0·9	1·4	1·1	1·5	1·2	1·7	54·3
54·6	1·1	1·4	1·1	1·4	1·1	1·4	1·2	1·5	1·3	1·6	1·4	1·7	1·5	1·8	54·6
54·9	1·4	1·6	1·5	1·6	1·5	1·6	1·6	1·7	1·6	1·8	1·8	1·9	1·9	2·0	54·9
55·2	1·8	1·8	1·8	1·8	1·9	1·9	1·9	1·9	2·0	2·0	2·1	2·1	2·2	2·2	55·2
55·5	2·2	2·0	2·2	2·0	2·3	2·1	2·3	2·1	2·4	2·2	2·4	2·3	2·5	2·4	55·5
55·8	2·6	2·2	2·6	2·2	2·6	2·3	2·7	2·3	2·7	2·4	2·8	2·4	2·9	2·5	55·8
56·1	3·0	2·4	3·0	2·5	3·0	2·5	3·0	2·5	3·1	2·6	3·1	2·6	3·2	2·7	56·1
56·4	3·4	2·7	3·4	2·7	3·4	2·7	3·4	2·7	3·4	2·8	3·5	2·8	3·5	2·9	56·4
56·7	3·7	2·9	3·7	2·9	3·8	2·9	3·8	2·9	3·8	3·0	3·8	3·0	3·9	3·0	56·7
57·0	4·1	3·1	4·1	3·1	4·1	3·1	4·1	3·1	4·2	3·1	4·2	3·2	4·2	3·2	57·0
57·3	4·5	3·3	4·5	3·3	4·5	3·3	4·5	3·3	4·5	3·3	4·5	3·4	4·6	3·4	57·3
57·6	4·9	3·5	4·9	3·5	4·9	3·5	4·9	3·5	4·9	3·5	4·9	3·5	4·9	3·6	57·6
57·9	5·3	3·8	5·3	3·8	5·2	3·8	5·2	3·7	5·2	3·7	5·2	3·7	5·2	3·7	57·9
58·2	5·6	4·0	5·6	4·0	5·6	4·0	5·6	4·0	5·6	3·9	5·6	3·9	5·6	3·9	58·2
58·5	6·0	4·2	6·0	4·2	6·0	4·2	6·0	4·2	6·0	4·1	5·9	4·1	5·9	4·1	58·5
58·8	6·4	4·4	6·4	4·4	6·4	4·4	6·3	4·4	6·3	4·3	6·3	4·3	6·2	4·2	58·8
59·1	6·8	4·6	6·8	4·6	6·7	4·6	6·7	4·6	6·7	4·5	6·6	4·5	6·6	4·4	59·1
59·4	7·2	4·8	7·1	4·8	7·1	4·8	7·1	4·8	7·0	4·7	7·0	4·7	6·9	4·6	59·4
59·7	7·5	5·1	7·5	5·0	7·5	5·0	7·5	5·0	7·4	4·9	7·3	4·8	7·2	4·7	59·7
60·0	7·9	5·3	7·9	5·3	7·9	5·2	7·8	5·2	7·8	5·1	7·7	5·0	7·6	4·9	60·0
60·3	8·3	5·5	8·3	5·5	8·2	5·4	8·2	5·4	8·1	5·3	8·0	5·2	7·9	5·1	60·3
60·6	8·7	5·7	8·7	5·7	8·6	5·7	8·6	5·6	8·5	5·5	8·4	5·4	8·2	5·3	60·6
60·9	9·1	5·9	9·0	5·9	9·0	5·9	8·9	5·8	8·8	5·7	8·7	5·6	8·6	5·4	60·9
61·2	9·5	6·2	9·4	6·1	9·4	6·1	9·3	6·0	9·2	5·9	9·1	5·8	8·9	5·6	61·2
61·5	9·8	6·4	9·8	6·3	9·7	6·3	9·7	6·2	9·5	6·1	9·4	5·9	9·2	5·8	61·5

DIP

Ht. of Eye	Corr^n	Ht. of Eye	Ht. of Eye	Corr^n	Ht. of Eye
m	′	ft.	m	′	ft.
2·4	−2·8	8·0	9·5	−5·5	31·5
2·6	−2·9	8·6	9·9	−5·6	32·7
2·8	−3·0	9·2	10·3	−5·7	33·9
3·0	−3·1	9·8	10·6	−5·8	35·1
3·2	−3·2	10·5	11·0	−5·9	36·3
3·4	−3·3	11·2	11·4	−6·0	37·6
3·6	−3·4	11·9	11·8	−6·1	38·9
3·8	−3·5	12·6	12·2	−6·2	40·1
4·0	−3·6	13·3	12·6	−6·3	41·5
4·3	−3·7	14·1	13·0	−6·4	42·8
4·5	−3·8	14·9	13·4	−6·5	44·2
4·7	−3·9	15·7	13·8	−6·6	45·5
5·0	−4·0	16·5	14·2	−6·7	46·9
5·2	−4·1	17·4	14·7	−6·8	48·4
5·5	−4·2	18·3	15·1	−6·9	49·8
5·8	−4·3	19·1	15·5	−7·0	51·3
6·1	−4·4	20·1	16·0	−7·1	52·8
6·3	−4·5	21·0	16·5	−7·2	54·3
6·6	−4·6	22·0	16·9	−7·3	55·8
6·9	−4·7	22·9	17·4	−7·4	57·4
7·2	−4·8	23·9	17·9	−7·5	58·9
7·5	−4·9	24·9	18·4	−7·6	60·5
7·9	−5·0	26·0	18·8	−7·7	62·1
8·2	−5·1	27·1	19·3	−7·8	63·8
8·5	−5·2	28·1	19·8	−7·9	65·4
8·8	−5·3	29·2	20·4	−8·0	67·1
9·2	−5·4	30·4	20·9	−8·1	68·8
9·5		31·5	21·4		70·5

MOON CORRECTION TABLE

The correction is in two parts; the first correction is taken from the upper part of the table with argument apparent altitude, and the second from the lower part, with argument H.P., in the same column as that from which the first correction was taken. Separate corrections are given in the lower part for lower (L) and upper (U) limbs. All corrections are to be **added** to apparent altitude, *but 30′ is to be subtracted from the altitude of the upper limb.*

For corrections for pressure and temperature see page A4.

For bubble sextant observations ignore dip, take the mean of upper and lower limb corrections and subtract 15′ from the altitude.

App. Alt. = Apparent altitude = Sextant altitude corrected for index error and dip.

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References:

Bowditch, 2024 edition, Article 1611.
https://msi.nga.mil/Publications/APN

Free website version:
https://www.thenauticalalmanac.com/

Government version:
https://aa.usno.navy.mil/publications/na

Altitude Corrections for the Moon – Practice Problems

Example Problem 1.33. You have observed the lower limb of the moon. The apparent altitude is 29° 10.0’. The horizontal parallax is 54.0’. What is the observed altitude?

- Height apparent (Ha) = 29° 10.0’ (already corrected for index error and dip)
- Apparent altitude correction #1 = +59.3’ (upper portion of table)
- Apparent altitude correction #2 = +0.7’ (HP = 54.0’, same column as correction 1)
- Height observed (Ho) = 30° 10.0’

Problem 1.33. Answer: 30° 10.0’

Problem 1.34. You have observed the lower limb of the moon. The apparent altitude is 37° 41.0'. The horizontal parallax is 58.5'. What is the observed altitude?

Problem 1.35. You take an observation of the UPPER limb of the moon. The apparent altitude is 61° 29.0'. The horizontal parallax is 55.5. What is the observed altitude?

Problem 1.36. At about 1100 GMT on 2 June, the lower limb of the moon is observed. The Hs is 18° 04.6'. The index error is 3.2' off the arc. The height of eye is 32 feet. The horizontal parallax is 59.7. What is the observed altitude?

Problem 1.33. Answer: 38° 41.9
Problem 1.34. Answer: 61° 39.9’
Problem 1.35. Answer: 19° 12.3’

Key Skill: From Hs to Ho

In this first section of the course, we've discussed several key topics:

- Types of navigation
- Marine sextant and cleaning/tuning of the sextant
- Making celestial observations with a sextant
- The Nautical Almanac
- Standard corrections including index correction, dip, and altitude corrections for all celestial bodies

The key skill for you at this point is being able to make a sextant measurement, and then correcting that measurement for standard corrections, ultimately arriving at Ho (height observed).

In the five-step celestial sight reduction process, we will now put this Ho aside. Later, we'll compare it to a calculated height for the celestial object, and the difference between the two will help us find a line of position.

Introduction to Geographic Position

It's important to remember there are different modes of navigation. For example, you wouldn't use celestial navigation in a harbor or near land…. instead, you would use piloting techniques by matching landmarks to charts and determining position that way.

Likewise, when you are approaching land from the open ocean, you can use signs such as birds, waves, clouds, smells, and changing water colors to help ease the transition from ocean navigation to piloting.

So celestial navigation is used when you are far offshore, where there are no terrestrial landmarks…only the lighthouses in the sky: the sun, moon, stars, and planets.

In this section, we'll talk about using those lighthouses in the sky and establish some shared vocabulary so we can best take advantage of our knowledge base.

Accuracy is also inherently less important in the open ocean. Of course, there are rocks and shoals, but you'll generally know where you start and where your destination lies.

As you work on your celestial navigation skills, remember that you will be in the open ocean, where an accuracy of 3 miles or so is an A+, and anything less than 10 miles is great.

When near land, or lighthouses, for example, you can use a compass to determine a line of position…you are somewhere on that line. If you had two lighthouses, then you could fix your exact position by using two lines of position.

The problem with celestial lines of position is that the earth is vast.

Each celestial object has a **geographic position** directly beneath it on the Earth. You use your sextant to determine your distance from this position, to obtain a circle of equal altitude. If you had three celestial objects, you could fix your position.

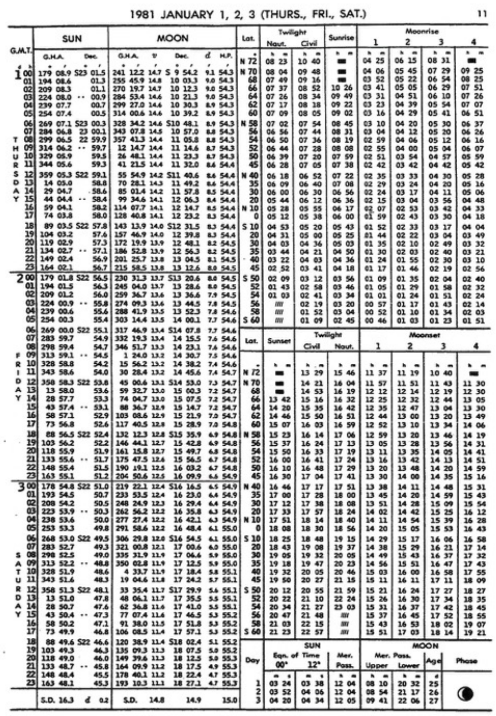
It's important that we can find that position for the geographic position at any time. It is described by its **declination** (latitude), and **Greenwich hour angle**, or GHA (longitude west of Greenwich).

The Nautical Almanac (Part 2) – Geographic Position

In the first part of this course, we took a quick look at the Nautical Almanac, focusing on standard corrections for Dip and Altitude Correction.

The Nautical Almanac can also provide you with the declination and Greenwich Hour Angle of any celestial body. This position is called the geographic position (GP) and can be found in the daily pages of the Almanac.

All time values in the Almanac are listed in **Greenwich mean time** (GMT), sometimes known as universal coordinated time (UTC).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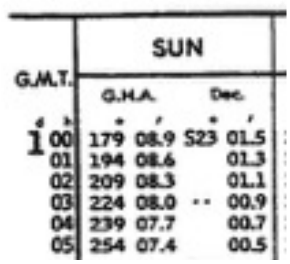
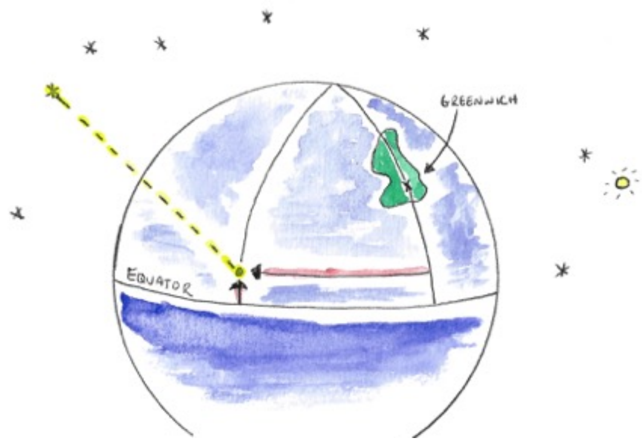


G.M.T.	SUN G.H.A.	SUN Dec.
1 00	179 08.9	S23 01.5
01	194 08.6	01.3
02	209 08.3	01.1
03	224 08.0	·· 00.9
04	239 07.7	00.7
05	254 07.4	00.5



References:

Free website version: https://www.thenauticalalmanac.com/

Government version: https://aa.usno.navy.mil/publications/na

Training Almanac:
https://www.dco.uscg.mil/Portals/9/NMC/pdfs/examinations/10_1981_nautical_almanac.pdf

Determining Declination

At any moment, there is a spot on Earth directly beneath any celestial object. Imagine that you were at that spot. The sun, for instance, would be directly overhead, at your **zenith.** This spot moves rapidly as the Earth rotates, but you can describe the position using terms of latitude and longitude.

The latitude of the spot on Earth directly beneath the celestial body is called declination. You can find this in the Nautical Almanac. Declination can be either north or south of the equator.

The declination of the Sun varies with the seasons, oscillating from the Tropic of Cancer to the Tropic of Capricorn and back. When the sun's declination reaches either of these latitudes, we call it the **solstice**, which is Latin for "sun stands still." This is appropriate because the sun stops moving towards a given tropic line, and reverses direction towards the other tropic line.

Similarly, when the Sun crosses the equator, we call this the **equinox** which is Latin for "equal nights" – this is the time when the length of day and night are equal all over Earth.

We will learn to determine declination very precisely later in the course, but here is a basic example problem for determining declination:

Example problem 2.0.1. What is the declination of the sun on 1 January at 0100 GMT?

Problem 2.0.1 Answer: S23° 01.3'

G.M.T.		SUN			
		G.H.A.		Dec.	
d	h	°	'	°	'
1	00	179	08.9	S23	01.5
	01	194	08.6		01.3
	02	209	08.3		01.1
	03	224	08.0	..	00.9
	04	239	07.7		00.7
	05	254	07.4		00.5

Problem 2.0.2 What is the declination of the sun on 1 January at 0200 GMT?

Problem 2.0.3 Answer: S 23° 01.1'

Reference:

Bowditch, 2024 edition, Article 1426. https://msi.nga.mil/Publications/APN

Determining Greenwich Hour Angle

Remember that every celestial body has a spot on Earth directly beneath it. The latitude of this spot is called declination, which we learned about in the last lesson.

The longitude of this spot is called **Greenwich hour angle,** or GHA. However, GHA is a little more complicated because it is not annotated with a hemisphere like longitude is.

Instead, Greenwich hour angle is measured westward from the Prime Meridian, which runs through Greenwich. It is only measured westward.

For example, a GHA of 15° would have a longitude of 15° W. A GHA of 170° would have a longitude of 170° W. A GHA of 190° would have a longitude of 170° E. Finally, a GHA of 345° would have a longitude of 15° E. This measurement westward from Greenwich is a very important distinction.

The reason this is called **hour angle** is due to the movement of celestial bodies. As the Earth rotates forever eastward, it appears that celestial bodies move forever westward – with increasing GHA.

Just like declination, Greenwich hour angle is found in the Nautical Almanac for any time. We will practice determining GHA very precisely later in the course, but here is a basic example problem showing how to determine Greenwich hour angle:

Example Problem 2.0.3. What is the GHA of the sun on 18 April at 1700 GMT?

Example problem answer: 75° 10.8’

Reference:

Bowditch, 2024 edition, Article 1426. https://msi.nga.mil/Publications/APN

	G.M.T.	SUN G.H.A.	Dec.	
	d h	° ′	°	′
16	00	180 01.3	N10	00.7
	01	195 01.5		01.6
	02	210 01.6		02.5
	03	225 01.8	··	03.4
	04	240 01.9		04.3
	05	255 02.1		05.2
	06	270 02.2	N10	06.1
	07	285 02.4		07.0
T	08	300 02.5		07.8
H	09	315 02.7	··	08.7
U	10	330 02.8		09.6
R	11	345 03.0		10.5
S	12	0 03.1	N10	11.4
D	13	15 03.3		12.3
A	14	30 03.4		13.2
Y	15	45 03.6	··	14.0
	16	60 03.7		14.9
	17	75 03.9		15.8
	18	90 04.0	N10	16.7
	19	105 04.2		17.6
	20	120 04.3		18.5
	21	135 04.5	··	19.3
	22	150 04.6		20.2
	23	165 04.8		21.1
17	00	180 04.9	N10	22.0
	01	195 05.1		22.9
	02	210 05.2		23.7
	03	225 05.4	··	24.6
	04	240 05.5		25.5
	05	255 05.6		26.4
	06	270 05.8	N10	27.3
	07	285 05.9		28.1
	08	300 06.1		29.0
F	09	315 06.2	··	29.9
R	10	330 06.4		30.8
I	11	345 06.5		31.7
D	12	0 06.7	N10	32.5
A	13	15 06.8		33.4
Y	14	30 07.0		34.3
	15	45 07.1	··	35.2
	16	60 07.2		36.0
	17	75 07.4		36.9
	18	90 07.5	N10	37.8
	19	105 07.7		38.7
	20	120 07.8		39.6
	21	135 08.0	··	40.4
	22	150 08.1		41.3
	23	165 08.2		42.2
18	00	180 08.4	N10	43.1
	01	195 08.5		43.9
	02	210 08.7		44.8
	03	225 08.8	··	45.7
	04	240 09.0		46.5
	05	255 09.1		47.4
	06	270 09.2	N10	48.3
	07	285 09.4		49.2
S	08	300 09.5		50.0
A	09	315 09.7	··	50.9
T	10	330 09.8		51.8
U	11	345 10.0		52.7
R	12	0 10.1	N10	53.5
D	13	15 10.2		54.4
A	14	30 10.4		55.3
Y	15	45 10.5	··	56.1
	16	60 10.7		57.0
	17	75 10.8		57.9
	18	90 10.9	N10	58.7
	19	105 11.1	10	59.6
	20	120 11.2	11	00.5
	21	135 11.4	··	01.3
	22	150 11.5		02.2
	23	165 11.6		03.1
		S.D. 16.0	d	0.9

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Interpolation

One key skill when using tables such as the Nautical Almanac is **interpolation**. This is simply choosing (wisely) a value from a table which falls between tabulated values.

For example, if a table provided data for cookie sales on certain days during the week, you might be able to interpolate a likely value for the data that is missing. As we all know, we tend to eat more cookies as the work week progresses….

Cookies Sold	Chocolate Chip	Peanut Butter
Monday	10	6
Wednesday	20	12
Friday	30	18

In this case, it would be reasonable to assume that on Tuesday, you sold 15 chocolate chip and 9 peanut butter cookies.

This process of using tables to infer non-tabulated values is very important in celestial navigation, where we use tables for declination, GHA, and other figures.

In some cases, you can mentally interpolate values. In others, you may need to use ratios to solve a problem.

For example, in the situation above, it is relatively easy to determine the amount of chocolate chip cookies sold on Tuesday. But mathematically, the process looks like this:

- The difference between tabulated values for Monday and Wednesday is 10 (from 10 to 20).
- The value we are looking for is exactly between the tabulated values, so we need to choose a value exactly halfway between 10 and 20, which is 15.

If the table were different, such as Table 1 below, we may need to solve things using ratios.

Table 1		
Time	Category 1	Category 2
1100	26	75
1200	28	78
1300	30	81
1400	32	84

Example Problem 2.1.5. Given table 1, what is the value of Category 2 at 1320?

- The tabulated values which bracket our desired time are 1300 (81) and 1400 (84).
- Our desired value is 1320, which is 1/3 of the way from 1300 to 1400. (20 minutes divided by 60 minutes).
- Therefore, we need a value 1/3 of the way from 81 to 84. In this case, (84-81)/3 is 1. So, we add 1 to the value for 1300 and obtain our final answer of 82.

Problem 2.1.5 Answer: 82

Reference:

Wikipedia article on Linear Interpolation: https://en.wikipedia.org/wiki/Linear_interpolation

Problem 2.1.1. Given table 1, what is the value of category 1 at 1200?

Problem 2.1.2. Given table 1, what is the value of category 1 at 1230?

Problem 2.1.3. Given table 1, what is the value of Category 1 at 1245?

Problem 2.1.4. Given table 1, what is the value of Category 1 at 1300?

Problem 2.1.5. Solved as an example in text.

Problem 2.1.6. What is the value of Category 2 at 1330?

Table 1		
Time	Category 1	Category 2
1100	26	75
1200	28	78
1300	30	81
1400	32	84

Problem 2.1.1. Answer: 28
Problem 2.1.2. Answer: 29
Problem 2.1.3. Answer: 29.5
Problem 2.1.4. Answer: 30
Problem 2.1.6. Answer: 82.5

Fundamentals of Time Zones

All times noted in the Nautical Almanac are given as **Greenwich mean time** (GMT), so a key skill for navigators is the ability to convert time between time zones.

The local time on a mariner's watch is known as the **local mean time** (LMT), otherwise known as a time zone.

Since there are 360° in the sphere of Earth, and there are 24 hours per day, each time zone is therefore 15° wide (360/15).

Zone descriptors (ZD) for times west of Greenwich are usually noted as +, which indicates that you must add a certain number of hours to your local time to obtain GMT. Zone descriptors east of Greenwich are noted as – values.

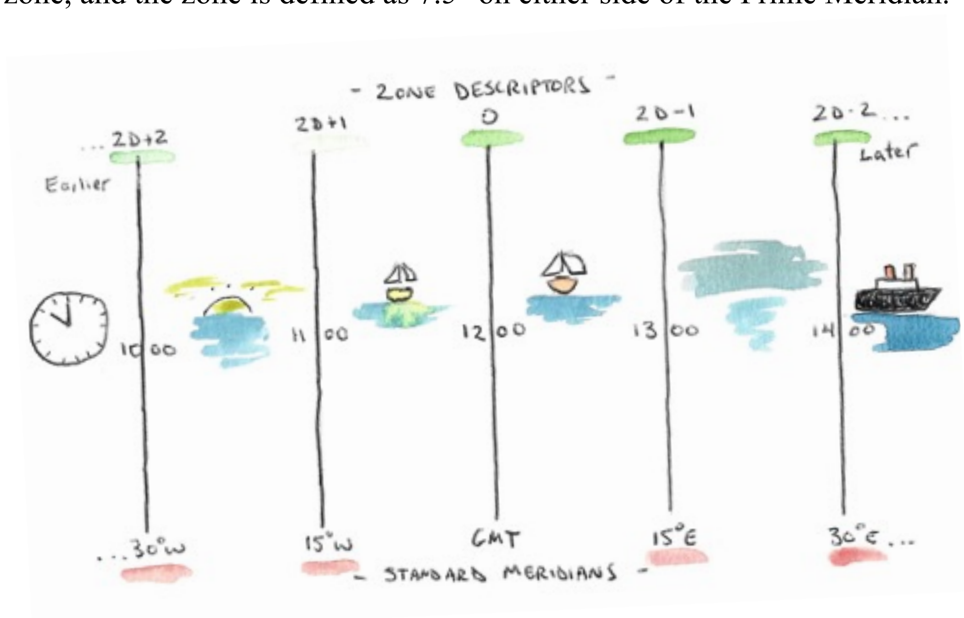
For example, if you are 2 hours west of Greenwich, you would be observing zone descriptor (ZD +2). If your local watch read a time of 1400, you would need to add 2 hours to obtain Greenwich time of 1600 GMT.

Likewise, if you are 5 hours east of Greenwich, you observe zone descriptor (ZD -5), and if your local watch read a time of 2100, you would need to subtract 5 hours to obtain 1600 GMT.

The opposite is also true: when converting from GMT to local, you would note the GMT and subtract the hours west of Greenwich or add the hours east of Greenwich.

These time zones can be associated with longitudes. If time zones are 15 degrees wide, everyone inside that time zone would use the same local time on their watches.

The zero point of longitude on Earth, or the Prime Meridian, marks the center point of the Greenwich mean time zone, and the zone is defined as 7.5° on either side of the Prime Meridian. This notation continues around the earth, with time zone centers at 15°, 30°, 45°, etc.



It is also important to note that landlubbers vary their time zones according to political will, while mariners are likewise able to observe any time zone they choose, but they generally follow the time notation described above.

Since the Earth rotates towards the east, it is also sometimes helpful to remember that things happen “first” to the east. For example, local noon happens first in Italy (ZD -2). Later, local noon happens in England (GMT). Later still, local noon happens in Iceland (ZD +3).

Example Problem 2.1.7. It is 1300 GMT. Your longitude is 15° W, and you are observing zone descriptor (+1). What is your local time?

Problem 2.1.7. Answer: 1200 local.

Reference:

Bowditch, 2024 edition, Article 1700-1714. https://msi.nga.mil/Publications/APN

Problem 2.1.8. It is 1430 GMT. Your longitude is 60° W, and you are observing zone descriptor (ZD +4). What is the local time?

Problem 2.1.9. It is 1130 GMT. Your longitude is 30° E, and you are observing zone descriptor (ZD -2). What is the local time?

Problem 2.1.10. It is 2300 GMT. Your longitude is 45° E, and you are observing zone descriptor (ZD -3). What is the local time?

Problem 2.1.11. It is 1400 local time. You are observing zone descriptor (ZD +1). What is the GMT?

Problem 2.1.12. It is 1530 local time. You are observing zone descriptor (ZD -2). What is the GMT?

Problem 2.1.8. Answer: 1030 local
Problem 2.1.9. Answer: 1330 local
Problem 2.1.10. Answer: 0200 local the following day
Problem 2.1.11. Answer: 1500 GMT
Problem 2.1.12. Answer: 1330 GMT

Conversion of Arc to Time

One principle of living on a rotating sphere is that clock time and the degrees and minutes of arc are related.

For example, we've already learned that the Earth rotates 360° in 24 hours, or 15° per hour. Importantly, this is in relation to the Sun. Other objects in the sky appear to move at different rates as the Earth rotates beneath them.

The Nautical Almanac has two important tools for converting arc to time and vice versa.

The first is a table which coverts arc to time in terms of degrees, minutes, and tenths of a minute.

This table is typically used for converting certain amounts of arc into degrees and minutes of longitude.

CONVERSION OF ARC TO TIME

0°–59°		60°–119°		120°–179°		180°–239°		240°–299°		300°–359°			0'·00	0'·25	0'·50	0'·75
°	h m	°	h m	°	h m	°	h m	°	h m	°	h m	'	m s	m s	m s	m s
0	0 00	60	4 00	120	8 00	180	12 00	240	16 00	300	20 00	0	0 00	0 01	0 02	0 03
1	0 04	61	4 04	121	8 04	181	12 04	241	16 04	301	20 04	1	0 04	0 05	0 06	0 07
2	0 08	62	4 08	122	8 08	182	12 08	242	16 08	302	20 08	2	0 08	0 09	0 10	0 11
3	0 12	63	4 12	123	8 12	183	12 12	243	16 12	303	20 12	3	0 12	0 13	0 14	0 15
4	0 16	64	4 16	124	8 16	184	12 16	244	16 16	304	20 16	4	0 16	0 17	0 18	0 19
5	0 20	65	4 20	125	8 20	185	12 20	245	16 20	305	20 20	5	0 20	0 21	0 22	0 23
6	0 24	66	4 24	126	8 24	186	12 24	246	16 24	306	20 24	6	0 24	0 25	0 26	0 27
7	0 28	67	4 28	127	8 28	187	12 28	247	16 28	307	20 28	7	0 28	0 29	0 30	0 31
8	0 32	68	4 32	128	8 32	188	12 32	248	16 32	308	20 32	8	0 32	0 33	0 34	0 35
9	0 36	69	4 36	129	8 36	189	12 36	249	16 36	309	20 36	9	0 36	0 37	0 38	0 39
10	0 40	70	4 40	130	8 40	190	12 40	250	16 40	310	20 40	10	0 40	0 41	0 42	0 43
11	0 44	71	4 44	131	8 44	191	12 44	251	16 44	311	20 44	11	0 44	0 45	0 46	0 47
12	0 48	72	4 48	132	8 48	192	12 48	252	16 48	312	20 48	12	0 48	0 49	0 50	0 51
13	0 52	73	4 52	133	8 52	193	12 52	253	16 52	313	20 52	13	0 52	0 53	0 54	0 55
14	0 56	74	4 56	134	8 56	194	12 56	254	16 56	314	20 56	14	0 56	0 57	0 58	0 59
15	1 00	75	5 00	135	9 00	195	13 00	255	17 00	315	21 00	15	1 00	1 01	1 02	1 03
16	1 04	76	5 04	136	9 04	196	13 04	256	17 04	316	21 04	16	1 04	1 05	1 06	1 07
17	1 08	77	5 08	137	9 08	197	13 08	257	17 08	317	21 08	17	1 08	1 09	1 10	1 11
18	1 12	78	5 12	138	9 12	198	13 12	258	17 12	318	21 12	18	1 12	1 13	1 14	1 15
19	1 16	79	5 16	139	9 16	199	13 16	259	17 16	319	21 16	19	1 16	1 17	1 18	1 19
20	1 20	80	5 20	140	9 20	200	13 20	260	17 20	320	21 20	20	1 20	1 21	1 22	1 23
21	1 24	81	5 24	141	9 24	201	13 24	261	17 24	321	21 24	21	1 24	1 25	1 26	1 27
22	1 28	82	5 28	142	9 28	202	13 28	262	17 28	322	21 28	22	1 28	1 29	1 30	1 31
23	1 32	83	5 32	143	9 32	203	13 32	263	17 32	323	21 32	23	1 32	1 33	1 34	1 35
24	1 36	84	5 36	144	9 36	204	13 36	264	17 36	324	21 36	24	1 36	1 37	1 38	1 39
25	1 40	85	5 40	145	9 40	205	13 40	265	17 40	325	21 40	25	1 40	1 41	1 42	1 43
26	1 44	86	5 44	146	9 44	206	13 44	266	17 44	326	21 44	26	1 44	1 45	1 46	1 47
27	1 48	87	5 48	147	9 48	207	13 48	267	17 48	327	21 48	27	1 48	1 49	1 50	1 51
28	1 52	88	5 52	148	9 52	208	13 52	268	17 52	328	21 52	28	1 52	1 53	1 54	1 55
29	1 56	89	5 56	149	9 56	209	13 56	269	17 56	329	21 56	29	1 56	1 57	1 58	1 59
30	2 00	90	6 00	150	10 00	210	14 00	270	18 00	330	22 00	30	2 00	2 01	2 02	2 03
31	2 04	91	6 04	151	10 04	211	14 04	271	18 04	331	22 04	31	2 04	2 05	2 06	2 07
32	2 08	92	6 08	152	10 08	212	14 08	272	18 08	332	22 08	32	2 08	2 09	2 10	2 11
33	2 12	93	6 12	153	10 12	213	14 12	273	18 12	333	22 12	33	2 12	2 13	2 14	2 15
34	2 16	94	6 16	154	10 16	214	14 16	274	18 16	334	22 16	34	2 16	2 17	2 18	2 19
35	2 20	95	6 20	155	10 20	215	14 20	275	18 20	335	22 20	35	2 20	2 21	2 22	2 23
36	2 24	96	6 24	156	10 24	216	14 24	276	18 24	336	22 24	36	2 24	2 25	2 26	2 27
37	2 28	97	6 28	157	10 28	217	14 28	277	18 28	337	22 28	37	2 28	2 29	2 30	2 31
38	2 32	98	6 32	158	10 32	218	14 32	278	18 32	338	22 32	38	2 32	2 33	2 34	2 35
39	2 36	99	6 36	159	10 36	219	14 36	279	18 36	339	22 36	39	2 36	2 37	2 38	2 39
40	2 40	100	6 40	160	10 40	220	14 40	280	18 40	340	22 40	40	2 40	2 41	2 42	2 43
41	2 44	101	6 44	161	10 44	221	14 44	281	18 44	341	22 44	41	2 44	2 45	2 46	2 47
42	2 48	102	6 48	162	10 48	222	14 48	282	18 48	342	22 48	42	2 48	2 49	2 50	2 51
43	2 52	103	6 52	163	10 52	223	14 52	283	18 52	343	22 52	43	2 52	2 53	2 54	2 55
44	2 56	104	6 56	164	10 56	224	14 56	284	18 56	344	22 56	44	2 56	2 57	2 58	2 59
45	3 00	105	7 00	165	11 00	225	15 00	285	19 00	345	23 00	45	3 00	3 01	3 02	3 03
46	3 04	106	7 04	166	11 04	226	15 04	286	19 04	346	23 04	46	3 04	3 05	3 06	3 07
47	3 08	107	7 08	167	11 08	227	15 08	287	19 08	347	23 08	47	3 08	3 09	3 10	3 11
48	3 12	108	7 12	168	11 12	228	15 12	288	19 12	348	23 12	48	3 12	3 13	3 14	3 15
49	3 16	109	7 16	169	11 16	229	15 16	289	19 16	349	23 16	49	3 16	3 17	3 18	3 19
50	3 20	110	7 20	170	11 20	230	15 20	290	19 20	350	23 20	50	3 20	3 21	3 22	3 23
51	3 24	111	7 24	171	11 24	231	15 24	291	19 24	351	23 24	51	3 24	3 25	3 26	3 27
52	3 28	112	7 28	172	11 28	232	15 28	292	19 28	352	23 28	52	3 28	3 29	3 30	3 31
53	3 32	113	7 32	173	11 32	233	15 32	293	19 32	353	23 32	53	3 32	3 33	3 34	3 35
54	3 36	114	7 36	174	11 36	234	15 36	294	19 36	354	23 36	54	3 36	3 37	3 38	3 39
55	3 40	115	7 40	175	11 40	235	15 40	295	19 40	355	23 40	55	3 40	3 41	3 42	3 43
56	3 44	116	7 44	176	11 44	236	15 44	296	19 44	356	23 44	56	3 44	3 45	3 46	3 47
57	3 48	117	7 48	177	11 48	237	15 48	297	19 48	357	23 48	57	3 48	3 49	3 50	3 51
58	3 52	118	7 52	178	11 52	238	15 52	298	19 52	358	23 52	58	3 52	3 53	3 54	3 55
59	3 56	119	7 56	179	11 56	239	15 56	299	19 56	359	23 56	59	3 56	3 57	3 58	3 59

The above table is for converting expressions in arc to their equivalent in time; its main use in this Almanac is for the conversion of longitude for application to L.M.T. (*added if west, subtracted if east*) to give G.M.T. or vice versa, particularly in the case of sunrise, sunset, etc.

The second resource in the Nautical Almanac are the increments and corrections pages. These tables are used for converting values of time in terms of minutes and seconds into angular values of arc.

Since certain objects move at different apparent rates in the sky, there are values for the Sun/Planets, Stars, and the Moon.

INCREMENTS AND CORRECTIONS

0m

0m	SUN PLANETS	ARIES	MOON	v or d	Corrn	v or d	Corrn	v or d	Corrn
s	° '	° '	° '	'	'	'	'	'	'
00	0 00·0	0 00·0	0 00·0	0·0	0·0	6·0	0·1	12·0	0·1
01	0 00·3	0 00·3	0 00·2	0·1	0·0	6·1	0·1	12·1	0·1
02	0 00·5	0 00·5	0 00·5	0·2	0·0	6·2	0·1	12·2	0·1
03	0 00·8	0 00·8	0 00·7	0·3	0·0	6·3	0·1	12·3	0·1
04	0 01·0	0 01·0	0 01·0	0·4	0·0	6·4	0·1	12·4	0·1
05	0 01·3	0 01·3	0 01·2	0·5	0·0	6·5	0·1	12·5	0·1
06	0 01·5	0 01·5	0 01·4	0·6	0·0	6·6	0·1	12·6	0·1
07	0 01·8	0 01·8	0 01·7	0·7	0·0	6·7	0·1	12·7	0·1
08	0 02·0	0 02·0	0 01·9	0·8	0·0	6·8	0·1	12·8	0·1
09	0 02·3	0 02·3	0 02·1	0·9	0·0	6·9	0·1	12·9	0·1

1m

1m	SUN PLANETS	ARIES	MOON	v or d	Corrn	v or d	Corrn	v or d	Corrn
s	° '	° '	° '	'	'	'	'	'	'
00	0 15·0	0 15·0	0 14·3	0·0	0·0	6·0	0·2	12·0	0·3
01	0 15·3	0 15·3	0 14·6	0·1	0·0	6·1	0·2	12·1	0·3
02	0 15·5	0 15·5	0 14·8	0·2	0·0	6·2	0·2	12·2	0·3
03	0 15·8	0 15·8	0 15·0	0·3	0·0	6·3	0·2	12·3	0·3
04	0 16·0	0 16·0	0 15·3	0·4	0·0	6·4	0·2	12·4	0·3
05	0 16·3	0 16·3	0 15·5	0·5	0·0	6·5	0·2	12·5	0·3
06	0 16·5	0 16·5	0 15·7	0·6	0·0	6·6	0·2	12·6	0·3
07	0 16·8	0 16·8	0 16·0	0·7	0·0	6·7	0·2	12·7	0·3
08	0 17·0	0 17·0	0 16·2	0·8	0·0	6·8	0·2	12·8	0·3
09	0 17·3	0 17·3	0 16·5	0·9	0·0	6·9	0·2	12·9	0·3

Example Problem 2.1.18. What is the equivalent of 35 degrees, 30.5 minutes of arc in time units?

- Use the Conversion of Arc to Time table and note the time value for 35 whole degrees of arc (2 hours and 20 minutes).
- On the right side of the table, find the value for 30.5 minutes of arc and note the time value (2 minutes and 2 seconds).
- Add the values together for the total conversion of arc to time (2 hours, 22 minutes, and 2 seconds).

Problem 2.1.18. Answer: 2 hours, 22 minutes, 2 seconds.

Example Problem 2.1.25. What is the equivalent of 7 minutes and 59 seconds of time in arc units for the sun?

- Use the increments and corrections tables for 7 minutes.
- Using the "Sun" column, pull the arc value for a time of 7 minutes and 59 seconds (1° 59.8').

Problem 2.1.25. Answer: 1° 59.8'

m 7	SUN PLANETS	ARIES	MOON	v or d	Corr^n	v or d	Corr^n	v or d	Corr^n
s	° ′	° ′	° ′	′	′	′	′	′	′
00	1 45·0	1 45·3	1 40·2	0·0	0·0	6·0	0·8	12·0	1·5
01	1 45·3	1 45·5	1 40·5	0·1	0·0	6·1	0·8	12·1	1·5
02	1 45·5	1 45·8	1 40·7	0·2	0·0	6·2	0·8	12·2	1·5
03	1 45·8	1 46·0	1 40·9	0·3	0·0	6·3	0·8	12·3	1·5
04	1 46·0	1 46·3	1 41·2	0·4	0·1	6·4	0·8	12·4	1·6
05	1 46·3	1 46·5	1 41·4	0·5	0·1	6·5	0·8	12·5	1·6
06	1 46·5	1 46·8	1 41·6	0·6	0·1	6·6	0·8	12·6	1·6
07	1 46·8	1 47·0	1 41·9	0·7	0·1	6·7	0·8	12·7	1·6
08	1 47·0	1 47·3	1 42·1	0·8	0·1	6·8	0·9	12·8	1·6
09	1 47·3	1 47·5	1 42·4	0·9	0·1	6·9	0·9	12·9	1·6
10	1 47·5	1 47·8	1 42·6	1·0	0·1	7·0	0·9	13·0	1·6
11	1 47·8	1 48·0	1 42·8	1·1	0·1	7·1	0·9	13·1	1·6
12	1 48·0	1 48·3	1 43·1	1·2	0·2	7·2	0·9	13·2	1·7
13	1 48·3	1 48·5	1 43·3	1·3	0·2	7·3	0·9	13·3	1·7
14	1 48·5	1 48·8	1 43·6	1·4	0·2	7·4	0·9	13·4	1·7
15	1 48·8	1 49·0	1 43·8	1·5	0·2	7·5	0·9	13·5	1·7
16	1 49·0	1 49·3	1 44·0	1·6	0·2	7·6	1·0	13·6	1·7
17	1 49·3	1 49·5	1 44·3	1·7	0·2	7·7	1·0	13·7	1·7
18	1 49·5	1 49·8	1 44·5	1·8	0·2	7·8	1·0	13·8	1·7
19	1 49·8	1 50·1	1 44·8	1·9	0·2	7·9	1·0	13·9	1·7
20	1 50·0	1 50·3	1 45·0	2·0	0·3	8·0	1·0	14·0	1·8
21	1 50·3	1 50·6	1 45·2	2·1	0·3	8·1	1·0	14·1	1·8
22	1 50·5	1 50·8	1 45·5	2·2	0·3	8·2	1·0	14·2	1·8
23	1 50·8	1 51·1	1 45·7	2·3	0·3	8·3	1·0	14·3	1·8
24	1 51·0	1 51·3	1 45·9	2·4	0·3	8·4	1·1	14·4	1·8
25	1 51·3	1 51·6	1 46·2	2·5	0·3	8·5	1·1	14·5	1·8
26	1 51·5	1 51·8	1 46·4	2·6	0·3	8·6	1·1	14·6	1·8
27	1 51·8	1 52·1	1 46·7	2·7	0·3	8·7	1·1	14·7	1·8
28	1 52·0	1 52·3	1 46·9	2·8	0·4	8·8	1·1	14·8	1·9
29	1 52·3	1 52·6	1 47·1	2·9	0·4	8·9	1·1	14·9	1·9
30	1 52·5	1 52·8	1 47·4	3·0	0·4	9·0	1·1	15·0	1·9
31	1 52·8	1 53·1	1 47·6	3·1	0·4	9·1	1·1	15·1	1·9
32	1 53·0	1 53·3	1 47·9	3·2	0·4	9·2	1·2	15·2	1·9
33	1 53·3	1 53·6	1 48·1	3·3	0·4	9·3	1·2	15·3	1·9
34	1 53·5	1 53·8	1 48·3	3·4	0·4	9·4	1·2	15·4	1·9
35	1 53·8	1 54·1	1 48·6	3·5	0·4	9·5	1·2	15·5	1·9
36	1 54·0	1 54·3	1 48·8	3·6	0·5	9·6	1·2	15·6	2·0
37	1 54·3	1 54·6	1 49·0	3·7	0·5	9·7	1·2	15·7	2·0
38	1 54·5	1 54·8	1 49·3	3·8	0·5	9·8	1·2	15·8	2·0
39	1 54·8	1 55·1	1 49·5	3·9	0·5	9·9	1·2	15·9	2·0
40	1 55·0	1 55·3	1 49·8	4·0	0·5	10·0	1·3	16·0	2·0
41	1 55·3	1 55·6	1 50·0	4·1	0·5	10·1	1·3	16·1	2·0
42	1 55·5	1 55·8	1 50·2	4·2	0·5	10·2	1·3	16·2	2·0
43	1 55·8	1 56·1	1 50·5	4·3	0·5	10·3	1·3	16·3	2·0
44	1 56·0	1 56·3	1 50·7	4·4	0·6	10·4	1·3	16·4	2·1
45	1 56·3	1 56·6	1 51·0	4·5	0·6	10·5	1·3	16·5	2·1
46	1 56·5	1 56·8	1 51·2	4·6	0·6	10·6	1·3	16·6	2·1
47	1 56·8	1 57·1	1 51·4	4·7	0·6	10·7	1·3	16·7	2·1
48	1 57·0	1 57·3	1 51·7	4·8	0·6	10·8	1·4	16·8	2·1
49	1 57·3	1 57·6	1 51·9	4·9	0·6	10·9	1·4	16·9	2·1
50	1 57·5	1 57·8	1 52·1	5·0	0·6	11·0	1·4	17·0	2·1
51	1 57·8	1 58·1	1 52·4	5·1	0·6	11·1	1·4	17·1	2·1
52	1 58·0	1 58·3	1 52·6	5·2	0·7	11·2	1·4	17·2	2·2
53	1 58·3	1 58·6	1 52·9	5·3	0·7	11·3	1·4	17·3	2·2
54	1 58·5	1 58·8	1 53·1	5·4	0·7	11·4	1·4	17·4	2·2
55	1 58·8	1 59·1	1 53·3	5·5	0·7	11·5	1·4	17·5	2·2
56	1 59·0	1 59·3	1 53·6	5·6	0·7	11·6	1·5	17·6	2·2
57	1 59·3	1 59·6	1 53·8	5·7	0·7	11·7	1·5	17·7	2·2
58	1 59·5	1 59·8	1 54·1	5·8	0·7	11·8	1·5	17·8	2·2
59	1 59·8	2 00·1	1 54·3	5·9	0·7	11·9	1·5	17·9	2·2
60	2 00·0	2 00·3	1 54·5	6·0	0·8	12·0	1·5	18·0	2·3

Reference:

Bowditch, 2024 edition, Article 1705.
https://msi.nga.mil/Publications/APN

Problem 2.1.13. What is the equivalent of 4 degrees of arc in time units?

Problem 2.1.13. What is the equivalent of 87 degrees of arc in time units?

Problem 2.1.15. What is the equivalent of 3 minutes of arc in time units?

Problem 2.1.16. What is the equivalent of 15 minutes of arc in time units?

Problem 2.1.17. What is the equivalent of 8.5 minutes of arc in time units?

Problem 2.1.18. What is the equivalent of 35 degrees, 30.5 minutes of arc in time units?

Problem 2.1.19. What is the equivalent of 40° 46.25’ of arc in time units?

Problem 2.1.20. What is the equivalent of 0 minutes and 4 seconds of time in arc units for the Sun?

Problem 2.1.21. What is the equivalent of 0 minutes and 25 seconds of time in arc units for the Sun?

Problem 2.1.22. What is the equivalent of 0 minutes and 40 seconds of time in arc units for the Moon?

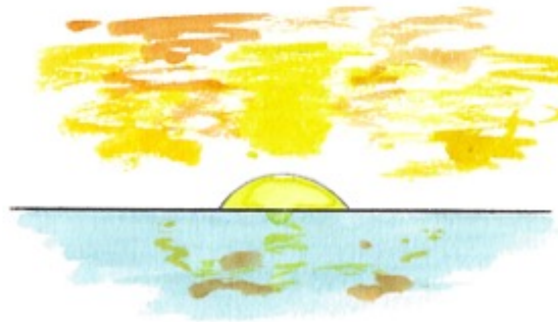
Problem 2.1.23. What is the equivalent of 1 minute and 30 seconds of time in arc units for Aries (stars)?

Problem 2.1.24. What is the equivalent of 7 minutes and 59 seconds in arc units for Jupiter?

Problem 2.1.25. What is the equivalent of 7 minutes and 59 seconds in arc units for the Sun?

Problem 2.1.13. Answer: 16 minutes
Problem 2.1.13. Answer: 5 hours and 48 minutes
Problem 2.1.15. Answer: 12 seconds
Problem 2.1.16. Answer:1 minute
Problem 2.1.17. Answer: 34 seconds
Problem 2.1.18. Answer: 2 hours, 22 minutes, 2 seconds.
Problem 2.1.19. Answer: 2 hours, 43 minutes, 5 seconds
Problem 2.1.20. Answer: 1’
Problem 2.1.21. Answer: 6.3’
Problem 2.1.22. Answer: 9.5’
Problem 2.1.23. Answer: 22.6’
Problem 2.1.24. Answer: 1° 57.3’
Problem 2.1.25. Answer. 1° 59.8’

Mathematics of Time and Arc



Adding and subtracting time or arc measurements can be tricky, but it is a routine task in celestial navigation. Here are some tips on how to be more accurate with these calculations:

- When adding time, separate the hours and minutes of each value. Add them separately. If the minutes value exceeds 60, add an extra hour and subtract 60 minutes. Likewise, if the hour value exceeds 24, simply add a day and subtract 24 hours.

- When subtracting time, the same suggestions apply for addition. However, when subtracting, you cannot use negative values for minutes or hours…. instead, you must think ahead and occasionally convert 1 hour into 60 additional minutes, or 1 day into 24 additional hours.

- When adding or subtracting degrees and minutes of arc, it is likewise helpful to break the degrees and minutes apart and borrow as necessary.

Example Problem 2.1.31. What is the result of subtracting 11:51 from 02:14?

- 02:14 – 11:51 is tough to do mentally.
- 02:14 is the same as 25:74 (add 24 hours for 1 day, and borrow 1 hour for 60 minutes)
- 25:74 – 11:51 = 14:23 (the previous day).

Problem 2.1.31. Answer: 14:23 the previous day.

Problem 2.1.26. What is the result of adding 12:51 and 1:12?

Problem 2.1.27. What is the result of adding 13:06 and 4:55?

Problem 2.1.28. What is the result of adding 23:50 and 01:12?

Problem 2.1.29. What is the result of subtracting 01:03 from 10:44?

Problem 2.1.30. What is the result of subtracting 02:47 from 22:12?

Problem 2.1.32. What is the result of adding 14° 55.2’ and 2° 06.9’?

Problem 2.1.33. What is the result of subtracting 22° 56.8’ from 47° 12.2’?

Problem 2.1.26. Answer: 14:03
Problem 2.1.27. Answer: 18:01
Problem 2.1.28. Answer: 01:02 the following day
Problem 2.1.29. Answer: 09:41
Problem 2.1.30. Answer: 19:25
Problem 2.1.32. Answer: 17° 02.1’
Problem 2.1.33. Answer: 24° 15.4’

V and D Corrections

Using the tables in the Nautical Almanac involves lots of nuance, but the tables also provide many tools to make your life easier. Two of these tools include v and d corrections.

All celestial objects appear to move through the sky at certain rates. The Nautical Almanac tables calculate the average (mean) movement for a certain period. However, sometimes that rate accelerates or decelerates based on the peculiarities of orbits. The v and d corrections account for these small changes.

D	13	13 51.0		47.8	48 06.1	11.7	17 35.5	5.5	55.1
A	14	28 50.7		47.6	62 36.8	11.6	17 41.0	5.5	55.1
Y	15	43 50.4	··	47.3	77 07.4	11.6	17 46.5	5.3	55.2
	16	58 50.2		47.1	91 38.0	11.5	17 51.8	5.3	55.2
	17	73 49.9		46.8	106 08.5	11.4	17 57.1	5.3	55.2
	18	88 49.6	S22	46.6	120 38.9	11.4	S18 02.4	5.1	55.2
	19	103 49.3		46.3	135 09.3	11.3	18 07.5	5.0	55.2
	20	118 49.0		46.0	149 39.6	11.3	18 12.5	5.0	55.3
	21	133 48.7	··	45.8	164 09.9	11.2	18 17.5	4.9	55.3
	22	148 48.4		45.5	178 40.1	11.2	18 22.4	4.7	55.3
	23	163 48.1		45.3	193 10.3	11.1	18 27.1	4.7	55.3
		S.D. 16.3	d	0.2	S.D.	14.8	14.9		15.0

52	20 22	21 10	22 24	15 26	16 30	17 34	18 35
54	20 34	21 27	23 03	15 31	16 37	17 42	18 45
56	20 47	21 48	////	15 37	16 45	17 52	18 55
58	21 03	22 15	////	15 43	16 53	18 02	19 07
S 60	21 23	22 57	////	15 51	17 03	18 14	19 21

Day	SUN Eqn. of Time 00ʰ	SUN Eqn. of Time 12ʰ	SUN Mer. Pass.	MOON Mer. Pass. Upper	MOON Mer. Pass. Lower	MOON Age	MOON Phase
	m s	m s	h m	h m	h m	d	
1	03 24	03 38	12 04	08 10	20 32	25	
2	03 52	04 06	12 04	08 54	21 17	26	
3	04 20	04 34	12 05	09 41	22 06	27	

Found at the bottom of the daily pages in the almanac, the v correction adjusts for the actual rate of change of orbit for GHA.

Likewise, the d correction adjusts for slight adjustments in the rate of declination change from hour to hour.

Once found, the v and d corrections are applied using the increments and corrections pages in the back of the nautical almanac for the time in question.

Later, we'll learn to use the v and d corrections in detail, but at this stage it would be helpful to practice locating the corrections and applying them for a given time.

Example Problem 2.1.35. Given a time of 1 minute and a v value of 9.8, what is the correction?

Problem 2.1.35. Answer: 0.2'.

Reference:

Bowditch, 2024 edition, Appendix D14.
https://msi.nga.mil/Publications/APN

ND CORRECTIONS 1ᵐ

1ᵐ	SUN PLANETS	ARIES	MOON	v or d	Corrⁿ	v or d	Corrⁿ	v or d	Corrⁿ
s	° ′	° ′	° ′	′	′	′	′	′	′
00	0 15·0	0 15·0	0 14·3	0·0	0·0	6·0	0·2	12·0	0·3
01	0 15·3	0 15·3	0 14·6	0·1	0·0	6·1	0·2	12·1	0·3
02	0 15·5	0 15·5	0 14·8	0·2	0·0	6·2	0·2	12·2	0·3
03	0 15·8	0 15·8	0 15·0	0·3	0·0	6·3	0·2	12·3	0·3
04	0 16·0	0 16·0	0 15·3	0·4	0·0	6·4	0·2	12·4	0·3
05	0 16·3	0 16·3	0 15·5	0·5	0·0	6·5	0·2	12·5	0·3
06	0 16·5	0 16·5	0 15·7	0·6	0·0	6·6	0·2	12·6	0·3
07	0 16·8	0 16·8	0 16·0	0·7	0·0	6·7	0·2	12·7	0·3
08	0 17·0	0 17·0	0 16·2	0·8	0·0	6·8	0·2	12·8	0·3
09	0 17·3	0 17·3	0 16·5	0·9	0·0	6·9	0·2	12·9	0·3
10	0 17·5	0 17·5	0 16·7	1·0	0·0	7·0	0·2	13·0	0·3
11	0 17·8	0 17·8	0 16·9	1·1	0·0	7·1	0·2	13·1	0·3
12	0 18·0	0 18·0	0 17·2	1·2	0·0	7·2	0·2	13·2	0·3
13	0 18·3	0 18·3	0 17·4	1·3	0·0	7·3	0·2	13·3	0·3
14	0 18·5	0 18·6	0 17·7	1·4	0·0	7·4	0·2	13·4	0·3
15	0 18·8	0 18·8	0 17·9	1·5	0·0	7·5	0·2	13·5	0·3
16	0 19·0	0 19·1	0 18·1	1·6	0·0	7·6	0·2	13·6	0·3
17	0 19·3	0 19·3	0 18·4	1·7	0·0	7·7	0·2	13·7	0·3
18	0 19·5	0 19·6	0 18·6	1·8	0·0	7·8	0·2	13·8	0·3
19	0 19·8	0 19·8	0 18·9	1·9	0·0	7·9	0·2	13·9	0·3
20	0 20·0	0 20·1	0 19·1	2·0	0·1	8·0	0·2	14·0	0·4
21	0 20·3	0 20·3	0 19·3	2·1	0·1	8·1	0·2	14·1	0·4
22	0 20·5	0 20·6	0 19·6	2·2	0·1	8·2	0·2	14·2	0·4
23	0 20·8	0 20·8	0 19·8	2·3	0·1	8·3	0·2	14·3	0·4
24	0 21·0	0 21·1	0 20·0	2·4	0·1	8·4	0·2	14·4	0·4
25	0 21·3	0 21·3	0 20·3	2·5	0·1	8·5	0·2	14·5	0·4
26	0 21·5	0 21·6	0 20·5	2·6	0·1	8·6	0·2	14·6	0·4
27	0 21·8	0 21·8	0 20·8	2·7	0·1	8·7	0·2	14·7	0·4
28	0 22·0	0 22·1	0 21·0	2·8	0·1	8·8	0·2	14·8	0·4
29	0 22·3	0 22·3	0 21·2	2·9	0·1	8·9	0·2	14·9	0·4
30	0 22·5	0 22·6	0 21·5	3·0	0·1	9·0	0·2	15·0	0·4
31	0 22·8	0 22·8	0 21·7	3·1	0·1	9·1	0·2	15·1	0·4
32	0 23·0	0 23·1	0 22·0	3·2	0·1	9·2	0·2	15·2	0·4
33	0 23·3	0 23·3	0 22·2	3·3	0·1	9·3	0·2	15·3	0·4
34	0 23·5	0 23·6	0 22·4	3·4	0·1	9·4	0·2	15·4	0·4
35	0 23·8	0 23·8	0 22·7	3·5	0·1	9·5	0·2	15·5	0·4
36	0 24·0	0 24·1	0 22·9	3·6	0·1	9·6	0·2	15·6	0·4
37	0 24·3	0 24·3	0 23·1	3·7	0·1	9·7	0·2	15·7	0·4
38	0 24·5	0 24·6	0 23·4	3·8	0·1	9·8	0·2	15·8	0·4
39	0 24·8	0 24·8	0 23·6	3·9	0·1	9·9	0·2	15·9	0·4
40	0 25·0	0 25·1	0 23·9	4·0	0·1	10·0	0·3	16·0	0·4
41	0 25·3	0 25·3	0 24·1	4·1	0·1	10·1	0·3	16·1	0·4
42	0 25·5	0 25·6	0 24·3	4·2	0·1	10·2	0·3	16·2	0·4
43	0 25·8	0 25·8	0 24·6	4·3	0·1	10·3	0·3	16·3	0·4
44	0 26·0	0 26·1	0 24·8	4·4	0·1	10·4	0·3	16·4	0·4

Problem 2.1.34. Given a time of 1 minute and a d value of 2.2, what is the correction?

Problem 2.1.35. Given a time of 1 minute and a v value of 9.8, what is the correction?

Problem 2.1.36. Given a time of 30 minutes and a d value of 1.0, what is the correction?

Problem 2.1.37. Given a time of 30 minutes and a v value of 4.0, what is the correction?

Problem 2.1.38. Given a time of 40 minutes and a d value of 3.1, what is the correction?

Problem 2.1.39. Given a time of 40 minutes and a d value of 16.2, what is the correction?

Problem 2.1.34. Answer: 0.1’
Problem 2.1.35. Answer: 0.2’.
Problem 2.1.36. Answer: 0.5’
Problem 2.1.37. Answer: 2.0’
Problem 2.1.38. Answer: 2.1’
Problem 2.1.39. Answer: 10.9’

Finding the Geographic Position of the Sun at Any Time

Earlier we learned where to find declination and GHA of the sun in the Nautical Almanac, and we practiced pulling values of each for whole GMT hours.

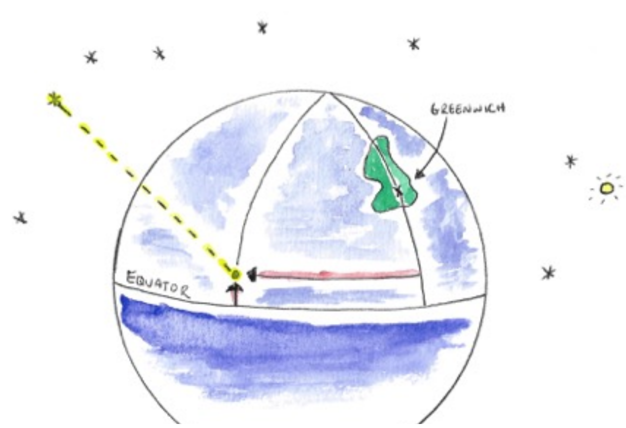
However, we rarely take sights at exact hours of GMT – here are the steps to determining GHA and declination for the sun at any time:

- Enter the daily page with the whole hour of time. For example, if the sight was 14:32:12 GMT, pull the GHA and declination values for 1400 GMT.
- Inspect and note the trend in declination – is it increasing or decreasing? GHA always increases with time.
- Record the d value at the bottom of the daily pages for declination.
- Enter the increments and corrections pages for the minutes and seconds that are left over (in our example, 32 minutes and 12 seconds).
- Pull the value for GHA from the Sun/Planets column and add it to the whole hours of GHA obtained earlier.
- Use the same increments and corrections page to obtain a d correction using the d value pulled earlier. Add or subtract this to the whole value of declination, based on the trend of the declination (increasing or decreasing).

At this point, you have obtained the geographic position of the sun for the exact moment in question in terms of GHA and declination – a significant achievement!

This process combines the skills we learned earlier by using the daily pages, the increments and corrections pages, and the fundamentals of declination and GHA.

It is a critical step in the overall celestial sight reduction process – use the practice problems included in the course to drill the skill thoroughly before moving on.



Reference:

Bowditch, 2024 edition, Article 1804.https://msi.nga.mil/Publications/APN

Declination Basics:

Problem 2.2.1. What is the declination of the sun on 1 January at 0100 GMT?

Problem 2.2.2. What is the declination of the sun on 1 January at 0200 GMT?

Problem 2.2.3. Without using the increments and corrections pages, mentally interpolate the value for the declination of the sun on 1 January at 0130 GMT.

Problem 2.2.4 Without using the increments and corrections pages, mentally interpolate the value for the declination of the sun on 1 January at 01:55:46 GMT.

Problem 2.2.5. On January 1, is the sun's declination moving towards or away from the equator?

Problem 2.2.6. Use the 1981 Nautical Almanac to find the date in which the sun's declination changes from North to South.

Problem 2.2.7. Use the 1981 Nautical Almanac to find the date in which the sun's declination reaches its maximum northerly declination.

GHA Basics:

Problem 2.2.8. What is the GHA of the sun on 1 January at 0300 GMT?

Problem 2.2.9. What is the GHA of the sun on 1 January at 0400 GMT?

Problem 2.2.10. To the nearest whole degree, what is the hourly rate of change for the GHA of the sun on 1 January?

Declination Advanced:

Problem 2.2.11. What is the declination of the sun on 4 February at 1800 GMT?

Problem 2.2.12. What is the d value for the sun on 4 February?

Problem 2.2.13. Using a d value of 0.7 and the increments and corrections pages for a time of 11 minutes, determine the d correction.

Problem 2.2.14. On 4 February is the declination of the sun increasing or decreasing?

Problem 2.2.15. What is the declination of the sun on 4 February at 18:11 GMT?

Problem 2.2.16. What is the declination of the sun on 4 February at 18:36 GMT?

Problem 2.2.17. What is the declination of the sun on 4 February at 18:44:12 GMT?

Problem 2.2.18. What is the declination of the sun on 24 July at 1900 GMT?

Problem 2.2.19. What is the declination of the sun on 24 July at 19:26:26 GMT?

Problem 2.2.20. What is the declination of the sun on 8 November at 0900 GMT?

Problem 2.2.21. Is the declination of the sun increasing or decreasing on 8 November?

Problem 2.2.22. What is the declination of the sun on 8 November at 09:41:58 GMT?

Problem 2.2.23. It is 6 November. Your longitude is 32° W, and you are observing zone descriptor (+2). What is the declination of the sun at 1800 local time?

GHA Advanced:

Problem 2.2.24. What is the GHA of the sun on 18 April at 1700 GMT?

Problem 2.2.25. Using the increments and corrections pages, what is the correction for the sun for a value of 1 minute and zero seconds?

Problem 2.2.26. What is the GHA of the sun on 18 April at 17:01:00 GMT?

Problem 2.2.27. Using the increments and corrections pages, what is the correction for the sun for a value of 1 minute and 50 seconds?

Problem 2.2.28. What is the GHA of the sun on 18 April at 17:01:50 GMT?

Problem 2.2.29. What is the GHA of the sun on 18 April at 17:09:26 GMT?

Problem 2.2.30. What is the GHA of the sun on 21 May at 02:31:02 GMT?

Problem 2.2.31. What is the GHA of the sun on 3 October at 18:49:55 GMT?

Geographic Position – Summative:

Problem 2.2.32. It is 26 February. You make an observation of the sun at 10:29:12 local time. Your dead reckoning (DR) longitude is 45° W, and you are observing zone descriptor (ZD +3). What is the geographic position (GP) of the sun?

Problem 2.2.33. It is 19 April. You observe the sun at 18:02:03 local time. Your DR longitude is 15° E and you are observing (ZD -1). What is the geographic position (GP) of the sun?

Problem 2.2.34. On 14 September you observe the sun at 09:23:49 local time. Your longitude is 135° W and you are observing (ZD +9). What is the GHA and declination of the sun?

Problem 2.2.35. It is 02:22:22 GMT on 13 September. What is the GP of the sun?

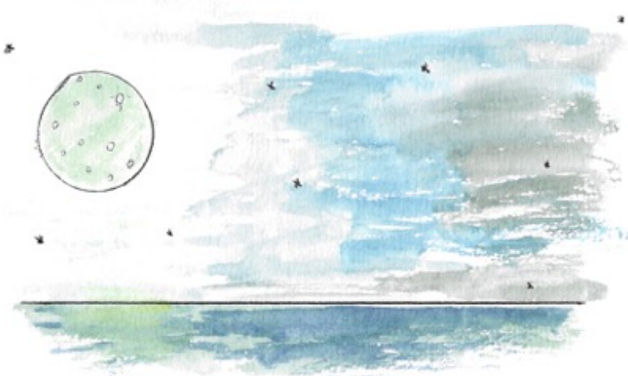
Problem 2.2.1. Answer: S 23° 01.3'
Problem 2.2.2. Answer: S 23° 01.1'
Problem 2.2.3. Answer: S 23° 01.2'
Problem 2.2.4 Answer: S 23° 01.1' (rounding to the nearest tenth is acceptable)
Problem 2.2.5. Answer: Towards the equator/decreasing southerly
Problem 2.2.6. Answer: 20 March
Problem 2.2.7. Answer: 21 June
Problem 2.2.8. Answer: 224° 08.0'
Problem 2.2.9. Answer: 239° 07.7'
Problem 2.2.10. Answer: 15 degrees per hour, standard for the Sun.

Problem 2.2.11. Answer: S 16° 05.2'
Problem 2.2.12. Answer: d = 0.7
Problem 2.2.13. Answer: Correction = 0.1'
Problem 2.2.14. Answer: Decreasing
Problem 2.2.15. Answer: S 16° 05.1'
Problem 2.2.16. Answer: S 16° 04.8'
Problem 2.2.17. Answer: S 16° 04.7'
Problem 2.2.18. Answer: N 19° 46.1'
Problem 2.2.19. Answer: N 19° 45.9'
Problem 2.2.20. Answer: S 16° 35.4'
Problem 2.2.21. Answer: Increasing
Problem 2.2.22. Answer: S 16° 35.9'
Problem 2.2.23. Answer: S 16° 08.3'

Problem 2.2.24. Answer: 75° 10.8'
Problem 2.2.25. Answer: 0° 15.0'
Problem 2.2.26. Answer: 75° 25.8'
Problem 2.2.27. Answer: 0° 27.5'
Problem 2.2.28. Answer: 75° 38.3'
Problem 2.2.29. Answer: 77° 32.3'
Problem 2.2.30. Answer: 218° 38.3'
Problem 2.2.31. Answer: 105° 14.6'

Problem 2.2.32. Answer: Dec S 8° 37.2', GHA 19° 04.2'
Problem 2.2.33. Answer: Dec N 11° 18.6', GHA 75° 44.9'
Problem 2.2.34. Answer: Dec N 3° 14.3', GHA 97° 05.6'
Problem 2.2.35. Answer: Dec N 3° 52.7', GHA 216° 34.9'

Geographic Position of Other Bodies



Finding the geographic position of the moon, planets, and stars is relatively straightforward once you know the process for the sun. Here are some general notes about GHA and declination for other bodies:

- Declination for planets is listed in each planet column. The process is the same as the sun.
- GHA for planets is listed in each planet column. The process is the same as the sun with the addition of the v correction found at the bottom of each column in the almanac.
- Declination for stars are listed next to each of the 57 navigational stars. The declination changes extremely slowly for stars, so no d correction for increments is needed.
- GHA is unique for stars – instead of listing the GHA for every navigational star, the almanac instead lists the GHA of Aries and provides a correction factor for each star known as sidereal hour angle (SHA). To obtain the GHA of a star, add the SHA to the GHA of Aries (hourly and increments) for the desired time. The **first point of Aries** is a designated point in the sky which corresponds to the location of the constellation Aries on the vernal equinox. You can consider the first point of Aries as the "Greenwich of the Sky."
- Declination for the moon changes extremely rapidly due to the moon's proximity to Earth. The declination is listed in the almanac like the sun; however, a d value is listed for each hour, rather than the whole day.
- GHA for the moon is calculated like the sun, with the exception that a v correction is listed hourly. As noted in part 1 of the course, the moon's horizontal parallax is also listed hourly.

Reference:

Bowditch, 2024 edition, Article 1805-1807. https://msi.nga.mil/Publications/APN

Geographic Position of Planets and Stars (Basic):

Problem 2.2.36. It is 1 March at 1200 GMT. What is the GP of Venus?

Problem 2.2.37. It is 1 March at 1800 GMT. What is the GP of Saturn?

Problem 2.2.38. It is 1 March at 2300 GMT. What is the GP of Jupiter?

Problem 2.2.39. It is 1 March at 0000 GMT. What is the GHA of Aries?

Problem 2.2.40. It is 1 March. What is the sideral hour angle (SHA) of Vega?

Problem 2.2.41. It is 1 March. What is the SHA and declination of Spica?

Problem 2.2.42. It is 1 March at 0000 GMT. Using the GHA of Aries and the SHA of Vega, determine the geographic position of Vega.

Problem 2.2.43. It is 1 March at 1800 GMT. What is the GP of Spica?

Geographic Position of Planets, Stars, and Moon (Advanced):

Problem 2.2.44. You observe the star Arcturus on 4 August at 04:13:12 GMT. What is the GP?

Problem 2.2.45. You observe the star Rigel on 10 October at 16:44:02 GMT. What is the GP?

Problem 2.2.46. You observe the planet Saturn on 10 March at 18:12:00 GMT. What is the GP?

Problem 2.2.47. You observe the planet Mars on 26 September at 02:55:40 GMT. What is the GP?

Problem 2.2.48. You observe the moon on 15 February at 06:33:12 GMT. What is the GP?

Problem 2.2.49. You observe the moon on 5 December at 23:40:10 GMT. What is the GP?

Problem 2.2.36. Answer: Dec S 12° 07.9', GHA 005° 09.3'
Problem 2.2.37. Answer: Dec S 0° 53.8', GHA 240° 47.4'
Problem 2.2.38. Answer: Dec S 1° 54.4' GHA 316° 19.2'
Problem 2.2.39. Answer: 158° 42.6'
Problem 2.2.40. Answer: 80° 55.8'
Problem 2.2.41. Answer: Dec S 11° 03.8', SHA 158° 57.0'
Problem 2.2.42. Answer: Dec N 38° 45.7', GHA 239° 38.4'
Problem 2.2.43. Answer: Dec S 11° 03.8', GHA 228° 24.0'

Problem 2.2.44. Answer: Dec N 19° 17.0', GHA 162° 14.5'
Problem 2.2.45. Answer: Dec S 8° 13.2', GHA 191° 47.5'
Problem 2.2.46. Answer: Dec S 0° 38.0', GHA 253° 13.7'
Problem 2.2.47. Answer: Dec N 17° 28.8', GHA 270° 52.9'
Problem 2.2.48. Answer: Dec N 20° 21.4', GHA 140° 52.8'
Problem 2.2.49. Answer: Dec S 4° 24.3', GHA 67° 53.7'

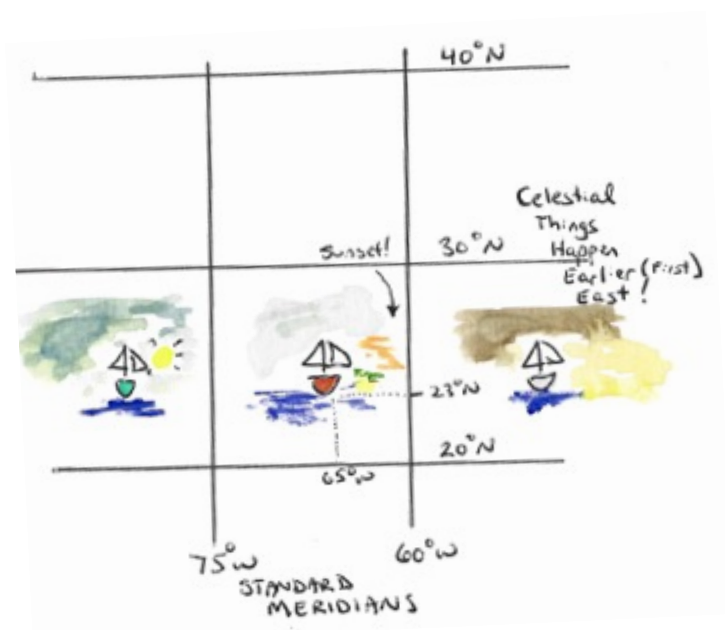
Time of Sunrise and Sunset

Although not strictly part of the celestial sight reduction process, the skill of determining the time of sunrise and sunset is helpful to introduce at this point in the course.

The sun is generally observed during mid-morning and mid-afternoon local times for reasons that we will learn about later. However most celestial observations of stars, planets, and the moon occur near twilight when a horizon is visible. So, determining the time of sunrise, sunset, and twilight is a useful skill.

There are several ways to determine any time of phenomena (sunrise, etc.). In this course we will focus on the shortest method, which relies on using Table 1 in the rear of the Nautical Almanac. Here are the steps:

- Look in the daily pages of the Nautical Almanac for the day in question and retrieve the time of phenomena for the nearest whole latitude less than the desired value.
- Note the interval between the latitudes which bracket the desired value, (e.g. 10°, 5°, or 2°).
- Enter the left side of Table 1 with the interval between bracketing latitudes and the difference between your desired value and the next less latitude.
- Scroll right to the column closest to the difference in consecutive latitudes that you retrieved earlier.
- Obtain the correction and add or subtract it, as necessary, to the tabulated latitude value lower than your desired latitude.
- Once the latitude has been corrected, note the difference in longitude from your position to the standard meridian which you are observing.
- Convert arc to time using the procedures we learned about in previous lessons to obtain a longitude correction and your final answer.



This process works for sunrise, sunset, and twilight. Exact times of moonrise are seldom needed, but Table 2 on the same page in the Almanac will help you interpolate the longitude for the moon.

Accuracy for times of sunrise, sunset, and twilight to the nearest 3 minutes is generally sufficient. In the following practice problems, if you obtain an answer within 5 minutes of the given solution, consider that a success.

TABLES FOR INTERPOLATING SUNRISE, MOONRISE, ETC.

TABLE I—FOR LATITUDE

Tabular Interval			Difference between the times for consecutive latitudes															
10°	5°	2°	5m	10m	15m	20m	25m	30m	35m	40m	45m	50m	55m	60m	1h 05m	1h 10m	1h 15m	1h 20m
° ′	° ′	° ′	m	m	m	m	m	m	m	m	m	m	m	m	h m	h m	h m	h m
0 30	0 15	0 06	0	0	1	1	1	1	1	2	2	2	2	2	0 02	0 02	0 02	0 02
1 00	0 30	0 12	0	1	1	2	2	3	3	3	4	4	4	5	05	05	05	05
1 30	0 45	0 18	1	1	2	3	3	4	4	5	5	6	7	7	07	07	07	07
2 00	1 00	0 24	1	2	3	4	5	5	6	7	7	8	9	10	10	10	10	10
2 30	1 15	0 30	1	2	4	5	6	7	8	9	9	10	11	12	12	13	13	13
3 00	1 30	0 36	1	3	4	6	7	8	9	10	11	12	13	14	0 15	0 15	0 16	0 16
3 30	1 45	0 42	2	3	5	7	8	10	11	12	13	14	16	17	18	18	19	19
4 00	2 00	0 48	2	4	6	8	9	11	13	14	15	16	18	19	20	21	22	22
4 30	2 15	0 54	2	4	7	9	11	13	15	16	18	19	21	22	23	24	25	26
5 00	2 30	1 00	2	5	7	10	12	14	16	18	20	22	23	25	26	27	28	29
5 30	2 45	1 06	3	5	8	11	13	16	18	20	22	24	26	28	0 29	0 30	0 31	0 32
6 00	3 00	1 12	3	6	9	12	14	17	20	22	24	26	29	31	32	33	34	36
6 30	3 15	1 18	3	6	10	13	16	19	22	24	26	29	31	34	36	37	38	40
7 00	3 30	1 24	3	7	10	14	17	20	23	26	29	31	34	37	39	41	42	44
7 30	3 45	1 30	4	7	11	15	18	22	25	28	31	34	37	40	43	44	46	48
8 00	4 00	1 36	4	8	12	16	20	23	27	30	34	37	41	44	0 47	0 48	0 51	0 53
8 30	4 15	1 42	4	8	13	17	21	25	29	33	36	40	44	48	0 51	0 53	0 56	0 58
9 00	4 30	1 48	4	9	13	18	22	27	31	35	39	43	47	52	0 55	0 58	1 01	1 04
9 30	4 45	1 54	5	9	14	19	24	28	33	38	42	47	51	56	1 00	1 04	1 08	1 12
10 00	5 00	2 00	5	10	15	20	25	30	35	40	45	50	55	60	1 05	1 10	1 15	1 20

Reference:

Bowditch, 2024 edition, Article 1808-1810. https://msi.nga.mil/Publications/APN

Example Problem 2.3.1. It is 4 August, and you are in position 25° N and 65° W. You are observing ZD (+4). What is the time of sunrise?

Latitude correction: Using the daily pages and the sunrise column, note the following:

- the interval between consecutive latitudes surrounding our position is 10° (e.g. 20-30)
- the time of sunrise for the next lower latitude is 0536 (for 20° N)
- the difference in time for consecutive latitudes is 16 minutes (e.g. 0536 at 20° N and 0520 at 30° N).

Using Table 1, enter the tabular interval of 10°, the difference between consecutive latitudes of 16 minutes (round to 15m), and the difference between tabulated and desired latitudes of 5°.

The correction retrieved is 7 minutes. Apply this correction to the next lower latitude (e.g. the value for 20° N of 0536). Therefore, the latitude corrected time is 0529.

Longitude correction: Determine the difference between the ship's longitude and the observed standard meridian. In this case the ship is observing (ZD +4), a standard meridian of 60° W. The longitude is 65° W.

Convert this difference in arc (5°) to time and apply that to the latitude corrected time to obtain a final answer of 0549 (0529 + 20 minutes west of standard meridian).

Problem 2.3.1 Answer: 0549

Problem 2.3.2. It is 4 August, and you are in position 25° N and 65° W. You are observing ZD (+4). What is the time of sunset?

Problem 2.3.3. What is the time of sunset on 12 June in position 44° 13'N and 138° 14'W? You are keeping ZD (+9).

Problem 2.3.4. What is the time of sunrise on 2 February in position 51° 44' N and 17° 00.0' E? You are observing ZD (-1).

Problem 2.3.5. You desire to observe evening stars on 17 June in position 32° 40' S and 47° 20' W. You are observing ZD (+3). What is the time of evening nautical twilight?

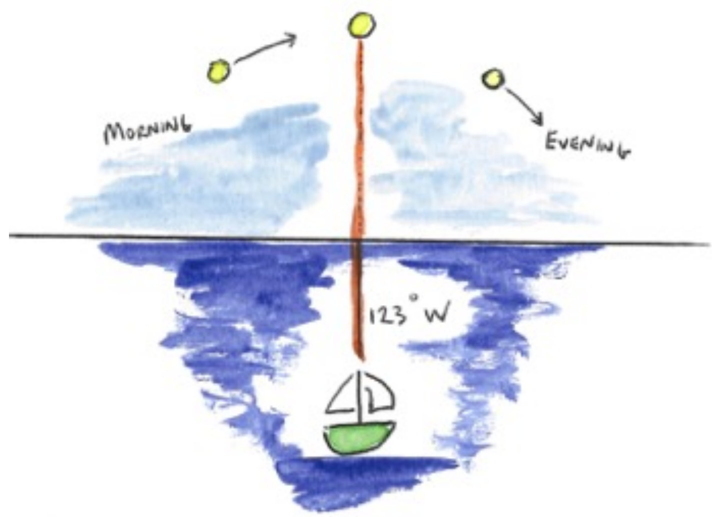
Problem 2.3.6. You are underway in position 22° 50' N and 163° 20' E and are observing ZD (-11). What is the time of sunrise on 21 December?

Answers within +/- 5 minutes are acceptable

Problem 2.3.2. Answer: 1903
Problem 2.3.3. Answer: 1956
Problem 2.3.4. Answer: 0730
Problem 2.3.5. Answer: 1807
Problem 2.3.6. Answer: 0630

Time of Meridian Passage

When the sun reaches its highest point in the sky for the day at any given location, it is passing the observer's **meridian** or line of longitude. The sun is due south or due north of the observer.

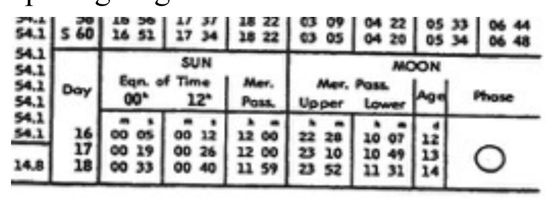


Although this "solar noon" is a local phenomenon that occurs near 1200 local time, the actual time of meridian passage is useful – it allows a celestial navigator to prepare for taking the **noon sight**, which is discussed in the next lesson.

Calculating the time of meridian passage is like calculating the time of sunrise or sunset, however no latitude correction is required – since the sun is on the observer's meridian, the time is the same at any latitude for a given longitude.

Example Problem 2.3.7. You are underway on April 18th and your DR position is 20° 00' N and 64° 30' W. You are observing ZD (+4). What is the local time of meridian passage for the sun?

- No latitude correction is required for meridian passage sights.
- Retrieve the standard time of meridian passage from the daily page in the Nautical Almanac. In this case, the time is 1159.



54.1	S 60	16 51	17 34	18 22	03 05	04 20	05 34	06 48

Day	SUN Eqn. of Time 00h	12h	Mer. Pass.	MOON Mer. Pass. Upper	Lower	Age	Phase
	m s	m s	h m	h m	h m	d	
16	00 05	00 12	12 00	22 28	10 07	12	
17	00 19	00 26	12 00	23 10	10 49	13	○
18	00 33	00 40	11 59	23 52	11 31	14	

- Determine the distance to the nearest standard meridian. In this case, the standard meridian is 60° W, and you are 4° 30' west (late) of the meridian.
- Convert the difference in arc to time. In this case, 4° 30' is 18 minutes.
- Apply the correction. In this case we are west (late) of the meridian, so the final time of meridian passage for the sun is 1159 + 18, or 1217.

Problem 2.3.7. Answer: 1217 local time.

Reference:

Bowditch, 2024 edition, Article 2013. https://msi.nga.mil/Publications/APN

Problem 2.3.8. What is the local time of meridian passage of the sun on 18 April at position 34° 13’N, 74° 30’W? You are observing ZD (+5).

Problem 2.3.9. Your DR position is 56° 12’ S and 18° 30’ E. You are observing ZD (-1). What is the local time of meridian passage of the sun on 18 April?

Problem 2.3.10. It is 24 July, and you desire the local time of meridian passage of the sun. Your position is 11° 13’ N and 118° 20’ E. You are observing ZD (-8).

Problem 2.3.8. Answer. 1157
Problem 2.3.9. Answer. 1145
Problem 2.3.10. Answer. 1213

Latitude by Meridian Passage of the Sun (Noon Sight)

The noon sight is a historic and important celestial navigation technique, and results in a line of latitude for the mariner. With daily noon latitudes from the sun and a good dead reckoning log, you can sail across an ocean!

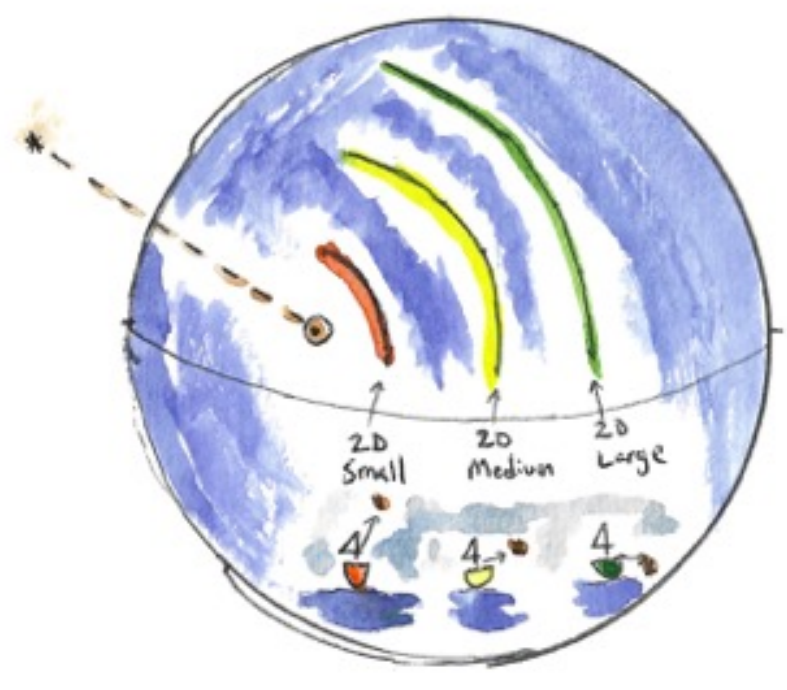
Given the proficiency you've obtained so far in this course, the noon sight is a summative experience that incorporates many of the skills described so far.

This observation is made when the sun reaches its highest point in the sky for you each day. This is local noon, when the sun passes your meridian due south or due north of you. At this moment, the spherical trigonometry we'll talk about in the next section of the course becomes simplified into a line.

The key requirements for this observation include:

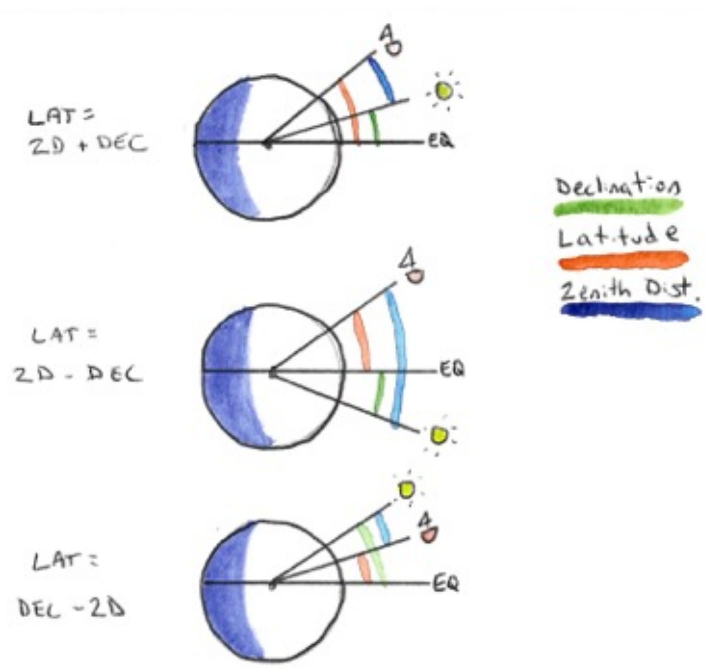
- Declination – determined via the Nautical Almanac for the time of sight.

- **Zenith Distance** – this term is equal to 90° minus your height observed (Ho). It corresponds to the degrees of arc you are away from the GP of the sun in any direction. It also represents the theoretical arc from the spot over your head (zenith) to the spot in the sky which is occupied by the sun.



There are a few situations you could find yourself in, and each has a different formula for determining latitude.

- Latitude = Zenith Distance + Declination. If you and the sun's GP are in the same hemisphere but you are further towards the pole than the sun.
- Latitude = Zenith Distance – Declination. If you and the sun's GP are in opposite hemispheres.
- Latitude = Declination – Zenith Distance. If you and the sun's GP are in the same hemisphere but you are closer to the equator than the sun.
- Latitude = Declination. If the sextant observation (Ho) is 90°, you are directly beneath the GP, and the math is as simple as it gets! This is rare though….
- Latitude = Zenith Distance. If the declination is zero, such as on the equinox, the math is also relatively easy! This is also rare though….



In the following example problems, the GMT and the declination are given to highlight the main learning objective of the noon sight for latitude. However, in subsequent problems you will need to use the skills learned in this part of the course to determine latitude.

The general steps to the problem:

- Determine the declination of the sun at the appropriate GMT.
- Determine the zenith distance (90° - Ho)
- Decide which latitude formula to use
- Calculate the latitude using declination and zenith distance

Example Problem 2.3.11. You observe the noon sight for latitude. The Height Observed (Ho) is 30° 30'. The declination of the sun at the time of sight is 15° 15' S. Your dead reckoning latitude is 44° N. What is the actual latitude?

- Determine the GMT of the sight. In this case it is not required.
- Determine the declination. It is given as 15° 15'S
- Determine the Zenith Distance.
  - 90° - 30° 30' = 59° 30'
- Decide which formula to use.
  - In this case, based on your DR latitude, you are in the opposite hemisphere as the sun's GP, so the formula is Latitude = ZD – Dec.
- Latitude = 59° 30' – 15° 15' = **44° 15' North.**

Problem 2.3.11. Answer. 44° 15' North

Example Problem 2.3.12. Your DR latitude is 37° N and you observe the noon sight for latitude with the sun. The Ho is 73° 15'. The declination of the sun at the time of sight is 20° 20'N. What is your exact latitude?

- GMT and declination are provided for this problem.
- ZD = 90° - 73° 15' = 16° 45'
- Since you and the sun are in the same hemisphere, but you are poleward, the correct formula is Latitude = ZD + Dec.
- Latitude = 16° 45' + 20° 20' = **37° 05' North**

Problem 2.3.12. Answer. 37° 05' North

Example Problem 2.3.13. You observe the sun for noon latitude with an Ho of 85° 12'. The declination at the time of sight is 15° 40'S, and your DR latitude is 11° S. What is the latitude?

- In this case, the ZD is 90° - 85° 12' = 4° 48'. The GMT and declination are given.
- Since you and the sun are in the same hemisphere, but you are closer to the equator, the formula is Latitude = Dec – ZD.
- Latitude = 15° 40' – 4° 48' = **10° 52' South**

Problem 2.3.13. Answer. 10° 52' South

Problem 2.3.14. On 22 February, you observe the sun at local apparent noon at 1447 GMT. The observed altitude (Ho) is 73° 33.3'. Your DR latitude is approximately 26° 30' S. What is the latitude?

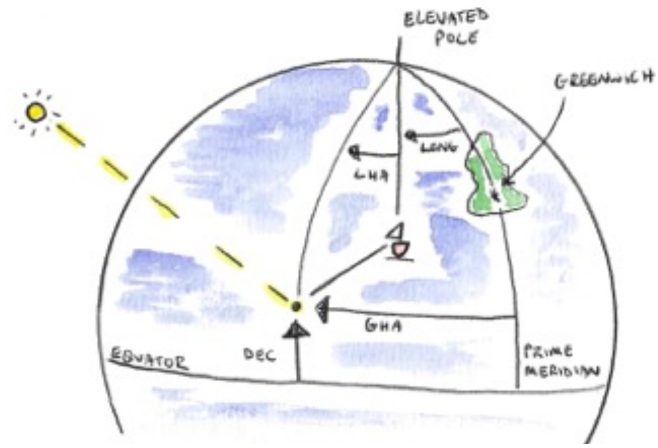
Problem 2.3.15. On 16 September your DR latitude is 29° N. You observe the sun at 1627 GMT for a noon sight of latitude. The Ho is 63° 25.3'. What is the latitude?

Problem 2.3.16. On 7 November you observe the sun at 1138 Local Time. The sun reaches its highest point, and you record Ho of 45° 35'. Your DR position is 27° 30'N and 163° W (ZD +11). What is the latitude by noon sight?

Problem 2.3.17. On 1 July at 1215 Zone Time (ZD +10), you observe the sun reach a peak of 42° 55.0'. Your DR latitude is 24° 30'S. What is the latitude?

Problem 2.3.14. Answer. 26° 31.4’ S
Problem 2.3.15. Answer. 29° 04.6’ N
Problem 2.3.16. Answer. 27° 57.2’ N
Problem 2.3.17. Answer. 24° 01.1’ S

Building a Celestial Triangle as a Proxy



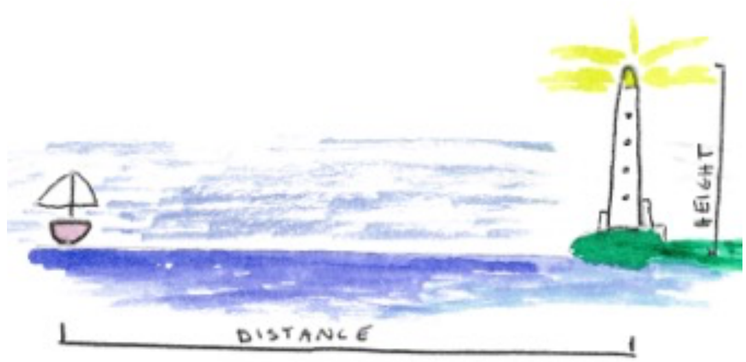
Remember that one problem with celestial navigation is the scales involved…. although we could measure our distance directly from the GP of any celestial object, we don't have charts or tools that would allow us to get a meaningful line of position for something thousands of miles away.

Instead, in this step of the 5-step celestial sight reduction process we will create a spherical triangle on the surface of the Earth using the GP, the elevated pole, and our assumed position.

Later, we'll solve the triangle and compare a calculated solution to an observed solution to determine the line of position in a way that will be helpful to our navigational plot.

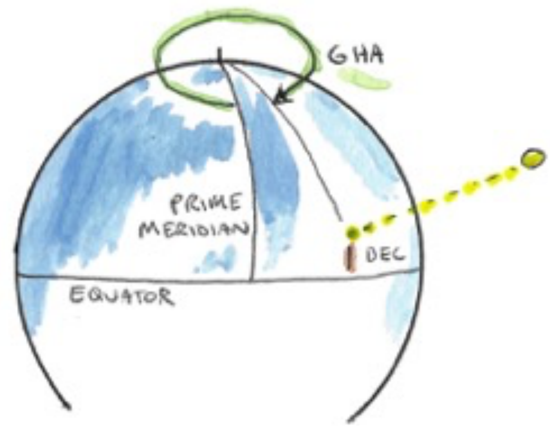
As a quick review of the power of triangles, take the example of a lighthouse. If you assumed that the lighthouse was straight (e.g. built 90° from the surface of Earth), and you knew the height of the lighthouse from the Light List or other publication, you could determine your distance from the light by measuring the angle from the surface of the sea to the top of the lighthouse with a sextant.

You would be creating a triangle and can use the rules of trigonometry to use known angles and sides to determine unknown sides (e.g. your distance off). The same general idea applies in spherical trigonometry.



Geographic Position Revisited – Triangle Point #1

In step 2 of our process, we learned how to calculate the geographic position (GP) of any celestial body at any time. This involved finding the **declination** and **Greenwich hour angle** of the body very precisely.



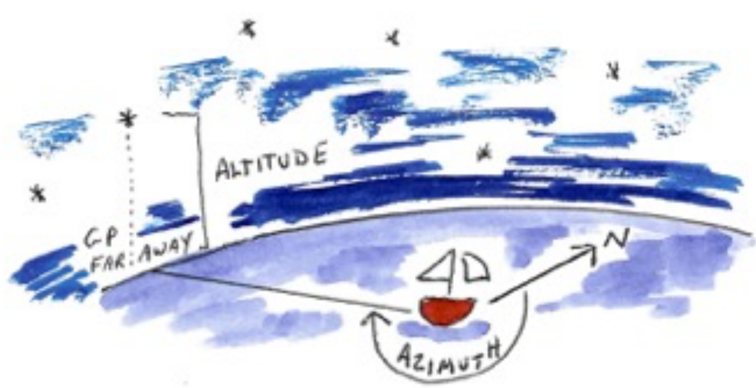
At this stage in the course, it's helpful to think of that GP as merely one point of a spherical triangle, and with some specific terminology:

- While the distance from the equator to the latitude of the GP is called declination, the distance from the GP to the pole is called **co-declination**. This represents one leg of a spherical triangle.

- The compass direction from your position to the GP is called **azimuth**. This is measured from 0° to 360° in a clockwise manner from due north.

- The distance from an observer to the GP is called **zenith distance**, or **co-altitude**. This was described earlier when discussing the noon sight for latitude – but in other cases its value represents a second leg of a spherical triangle.

Thinking of the GP in these theoretical terms is not critical for computing an individual sight but is helpful when working through celestial navigation problems.



Reference:

Bowditch, 2024 edition, Article 109. https://msi.nga.mil/Publications/APN

The Elevated Pole – Triangle Point #2

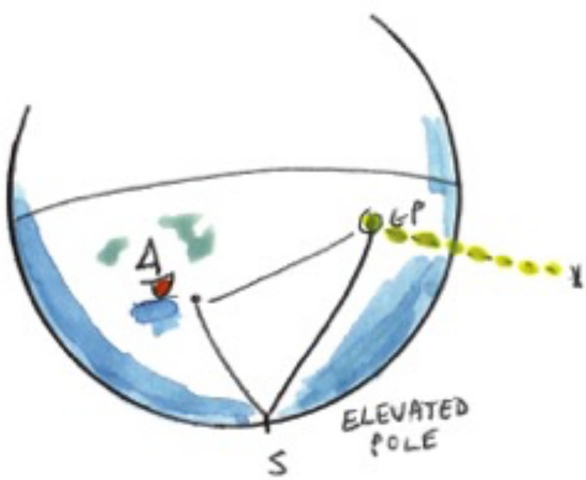
The second point in the navigational triangle is the **elevated pole**. This simply means the pole (north or south) which is in the same hemisphere as the observer.

Few people other than polar explorers, Santa Claus (north), and penguins (south) visit these poles, but they are important when constructing navigational triangles.

Although the sign of the declination of a body may vary, the key takeaway is that the elevated pole is always in the same hemisphere as the observer.

Reference:

Bowditch, 2024 edition, Article 1431. https://msi.nga.mil/Publications/APN



Dead Reckoning and the Assumed Position – Triangle Point #3

A **dead reckoning** position is a location that the mariner determines by observing past positions, wind, current, speed, and direction, and making a best guess at future positions. Any good navigator will always know roughly where they are. So, a reasonable DR is never a problem in celestial navigation.

Although it is possible to solve celestial triangles directly from a DR position using math formulae which we will discuss later, the common method of using tables means that we need to think critically about our DR and instead modify it slightly to become an **assumed position** which makes it easy to use tables.

This assumed position (AP) represents a position on Earth somewhat nearby our DR position, but with whole number values for latitude (e.g. 23° N or 45° S). This whole value of latitude allows us to easily use tabulated data without the tables being annoyingly large.

The latitude of the AP should be the closest whole latitude, within 30'.

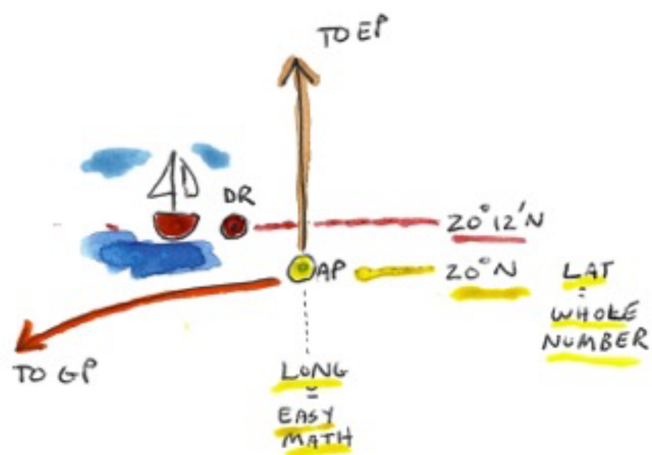
The longitude of the AP is also important, but in a different way.

Rather than compute the tables for every possible combination of latitude and longitude, we instead assume that a triangle is the same whether it is inside out, backwards, or upside down.

In this way, if we have the relative values for sides and angles, we can retrieve solutions from the table.

Because of that, we don't use longitude in the tables. Instead, we use a value called **local hour angle** or LHA, which we'll cover next.

The key takeaway is that we want to our assumed position to have a whole value of latitude, and a longitude that allows us to compute a whole value of LHA.



Reference:

Bowditch, 2024 edition, Article 1431 and 2002. https://msi.nga.mil/Publications/APN

Problem 3.1.1. Your DR Latitude is 23° 15’ N. What latitude would you use for the assumed position?

Problem 3.1.2. Your DR Latitude is 23° 45’ N. What latitude would you use for the assumed position?

Problem 3.1.3. Your DR Latitude is 36° 25’ S. What latitude would you use for the assumed position?

Problem 3.1.4. Your DR Latitude is 36° 55’ S. What latitude would you use for the assumed position?

Problem 3.1.1. Answer. 23° N
Problem 3.1.2. Answer. 24° N
Problem 3.1.3. Answer. 36° S
Problem 3.1.4. Answer. 37° S

Local Hour Angle

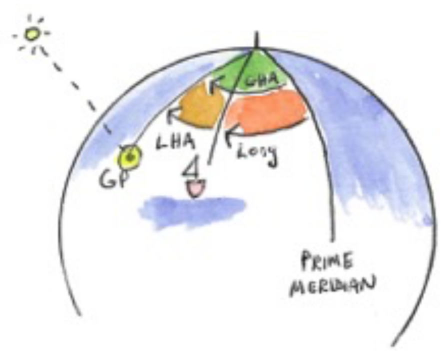
Local hour angle (LHA) is a relationship between the observer's longitude and the longitude of the GP (otherwise known as GHA),and somewhat standardizes the spherical triangles to keep our tables concise.

LHA, like latitude, is listed in whole units only (e.g. 23°, 45°, or 321°). LHA can be calculated as follows:

- In the western hemisphere, LHA = GHA – Longitude.
- In the eastern hemisphere, LHA = GHA + Longitude.
- If any value exceeds 360°, you can always subtract 360° (one circle) to make the numbers work in the tables.

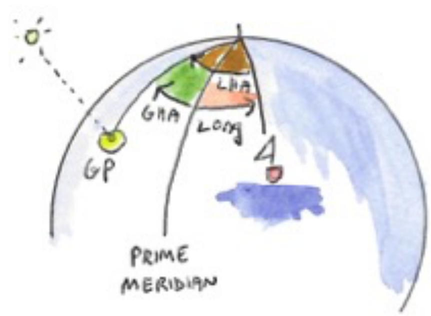
The key concept is to pick an assumed longitude which, when added to or subtracted from the GHA, results in a whole number of LHA. That assumed longitude should be within 30' of your DR longitude.

Example Problem 3.1.7. Your DR Longitude is 42° 15' W. The GHA of the sun is 95° 20'. What assumed longitude do you select and what is the resultant whole value of LHA?



- There are two rules for LHA:
  - LHA must be a whole value.
  - Our assumed longitude should be within 30' of our DR longitude.
- If we select 42° 20' W for our assumed longitude, both rules are met.
- GHA (95° 20') – W Longitude (42° 20') = 53° LHA

Problem 3.1.7. Answer. 42° 20'W assumed longitude, LHA =53°



Reference:

Bowditch, 2024 edition, Article 1425. https://msi.nga.mil/Publications/APN

Problem 3.1.5. Your DR Longitude is 45° 30' W. The GHA of the sun is 125° 30'. What is the LHA?

Problem 3.1.6. Your DR Longitude is 45° 30' E. The GHA of the sun is 125° 30'. What is the LHA?

<u>Western Hemisphere Practice:</u>

Problem 3.1.7. Your DR Longitude is 42° 15' W. The GHA of the sun is 95° 20'. What assumed longitude do you select and what is the resultant whole value of LHA?

Problem 3.1.8. Your DR Longitude is 42° 44.6' W. The GHA of the sun is 136° 02.1'. What assumed longitude do you select within 30' of your DR, and what is the resultant whole value of LHA?

Problem 3.1.9. Your DR Longitude is 125° 13.2 W. The GHA of the sun is 215° 22.6'. What assumed longitude do you select and what is the resultant LHA?

Problem 3.1.10. The GHA of Jupiter is 330° 26.9'. Your DR longitude is 36° 20'W. What is the assumed longitude and LHA?

Problem 3.1.11. The GHA of Acrux is 15° 16.3'. Your DR longitude is 136° 05'W. What is the assumed longitude and LHA?

<u>Eastern Hemisphere Practice:</u>

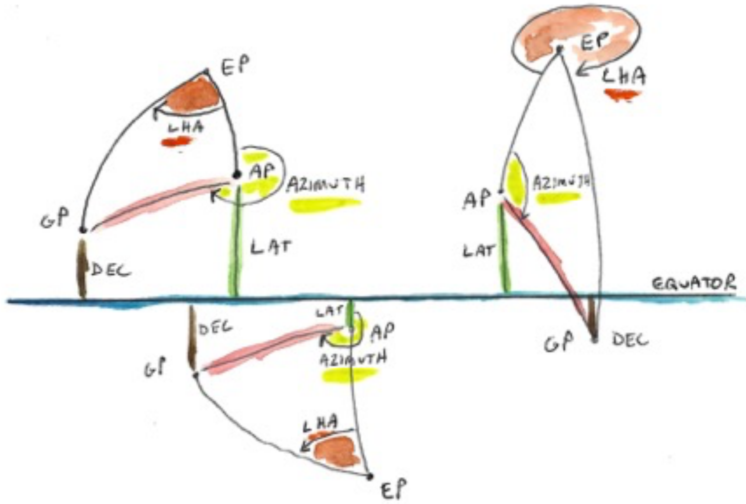
Problem 3.1.12. Your DR Longitude is 65° 20' E. The GHA of the sun is 125° 30'. What is the assumed longitude and LHA?

Problem 3.1.13. The GHA of Spica is 346° 12.2'. Your DR Longitude is 74° 40'E. What is the assumed longitude and LHA?

Problem 3.1.14. The GHA of Mars is 225° 15.3'. Your DR Longitude is 105° 02.1'E. What is the assumed longitude and LHA?

Problem 3.1.5. Answer. LHA = 80°
Problem 3.1.6. Answer. LHA = 171°
Problem 3.1.7. Answer. 42° 20'W assumed longitude, LHA =53°
Problem 3.1.8. Answer. Assumed Longitude = 43° 02.1'W, LHA = 93°
Problem 3.1.9. Answer. Assumed Longitude = 125° 22.6'W, LHA = 90°
Problem 3.1.10. Answer. Assumed Longitude = 36° 26.9'W, LHA = 294°
Problem 3.1.11. Answer. Assumed Longitude = 136° 16.3'W, LHA = 239° (add 360° to longitude)
Problem 3.1.12. Answer. Assumed Longitude = 65° 30'E, LHA = 191°
Problem 3.1.13. Answer. Assumed Longitude = 74° 47.8'E, LHA = 61° (subtract 360° from LHA)
Problem 3.1.14. Answer. Assumed Longitude = 104° 44.7'E, LHA = 330°

Entering Arguments for Solving Spherical Triangles



The three arguments needed for using tables to solve spherical triangles are:

- Whole hourly degrees of declination for the celestial body.
- Whole degrees of latitude for the assumed position (within 30' of DR).
- Whole degrees of LHA for the relationship between assumed position and celestial body.

It is also helpful to note whether the declination of the object and the observer are in the same or contrary (opposite) hemispheres.

With these values, you can use tables such as HO229 (sight reduction tables for marine navigation) or HO249 (air almanac) to solve any spherical triangle.

Example Problem 3.1.15. You have determined the GP of the sun as:

- Declination = 23° 12.5' N
- GHA = 144° 44.6'

Your DR position is 55° 15'N, 32° 30'W. What are the whole values of declination, GHA, and LHA selected for solving spherical triangles?

- The whole value of declination is 23°N.
- The whole value of latitude is 55° N (same hemisphere as declination)
- The whole value of LHA:
  - GHA: 144° 44.6'
  - Long: 32° 44.6' W (assumed longitude to ensure whole LHA)
  - LHA = GHA – West Longitude = 112°

Problem 3.1.15. Answer. Dec 23° N, Lat 55° N, LHA 112°

Problem 3.1.16. You have determined the GP of Venus to be declination 12° 55.9’S, GHA 49° 13.2’. Your DR position is 42° 58’N, 112° 55’W. What are the entering arguments selected?

Problem 3.1.17. Your DR position is 63° 20’S, 175° 10’E. You have determined the GP of the moon to be declination 12° 47’S, GHA 132° 14’. What are the entering arguments selected?

Problem 3.1.18. The GHA of the sun is 330° 12.7’. The declination is 12° 34.6’ S. Your DR position is 12° 02’ N, 33° 34’ E. What are the entering arguments selected?

Problem 3.1.16. Answer:

- Declination = 12° S (select the whole value and discard minutes)
- Latitude = 43° N/contrary(select the whole latitude within 30’ of DR)
- LHA = 296° (assumed longitude is 113° 13.2’W)

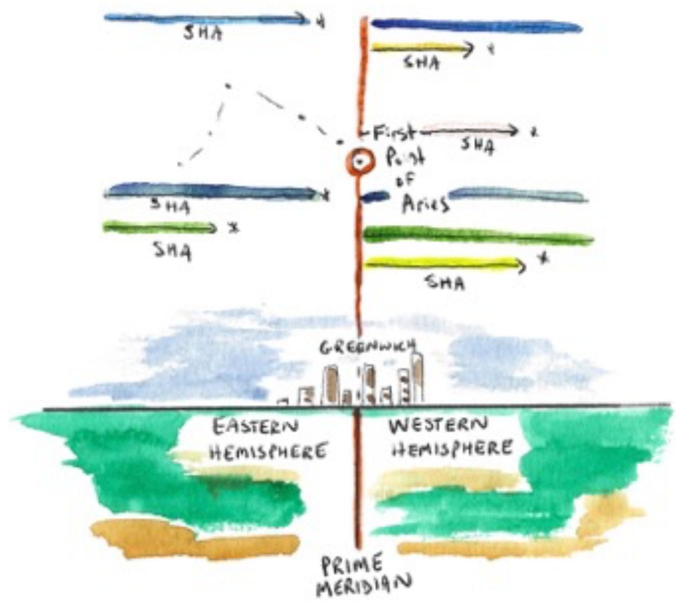
Problem 3.1.17. Answer:

- Declination = 12° S
- Latitude = 63° S/same
- LHA = 307° (assumed longitude is 174° 46.0’ E)

Problem 3.1.18. Answer:

- Declination = 12° S
- Latitude = 12° N/contrary
- LHA = 004° (assumed longitude is 33° 47.3’ E)

Astronomy for Mariners: Aries



As described briefly in part 2 of this course, the **first point of Aries** is the location that lies over Greenwich at the vernal equinox (at the time of its naming). You can consider Aries as the "Prime Meridian of the Sky" – the zero point for hour angles in the celestial sphere.

When determining the GP of stars, we earlier discussed that the GHA of a star is equal to the GHA of Aries plus a correcting SHA (sidereal hour angle).

At this stage in the course, it is helpful to introduce a few more concepts about hour angles in general:

- **Great circles** are planes that divide a sphere in two equal halves. In terms of navigation, great circles trace the shortest distance between any two points on the surface of the Earth.

- **Hour circles** are great circles on the celestial sphere that pass through the celestial poles and a celestial body. They are perpendicular to the celestial equator.

- **Hour angles** give an angle from a reference hour circle. There are three reference hour circles of note:

    - Aries – used for stars to determine SHA
    - The local meridian (DR longitude line) – used to determine LHA
    - Greenwich – used to determine GHA

This brings up the larger question of coordinate systems. When considering celestial navigation, it is helpful to remember that there are different ways to represent angles and lines on Earth.

For example, using the **horizon coordinate system**, in which the observer is the center of the universe, the term **zenith** represents the spot directly overhead. The term **nadir** represents the spot directly beneath.

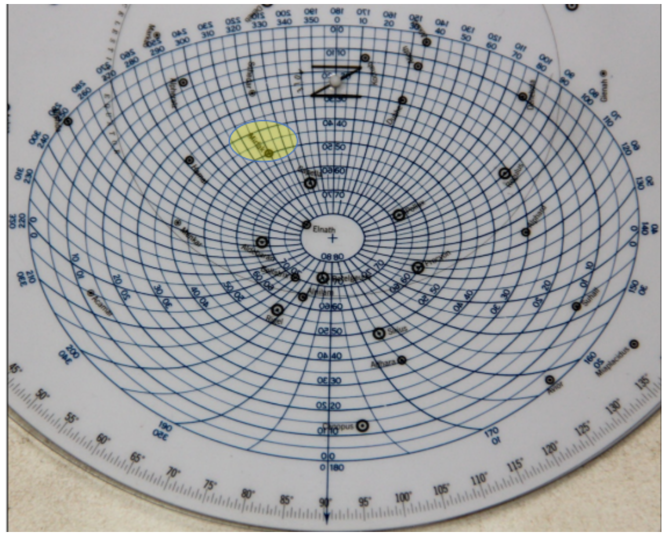
Likewise, the **Earth coordinate system** has key features such as **poles,** and **meridians** and **parallels**, the latter two used to describe longitude and latitude.

The **celestial coordinate system** is where we plot the hour circles, declination, and hour angles described earlier in the course.

Reference:

Bowditch, 2024 edition, Article 1425, 1429. https://msi.nga.mil/Publications/APN

Starfinder 2102-D



With a solid understanding of LHA and the concept of Aries, you can unlock the skill of using the 2102-D Starfinder.

This tool allows you to determine the approximate **altitude** (height above horizon) and **azimuth** (compass direction) towards many celestial bodies.

It allows you to identify a star or planet that you observed but were unable to identify by sight due to clouds or unfamiliarity. It also allows you to plan an observation session so that you can obtain a series of stars/planets spread out around the sky, enabling a pinpoint fix.

Here are some key tips when using the Starfinder:

- Use the correct hemisphere backing plate (north or south)
- Use the correct clear plate based on the DR latitude
- Find the LHA of Aries for the time of observation and spin the Starfinder to that setting
- Navigational stars are noted on the plates. When using the Starfinder for planets, they must be individually drawn in based on GHA/Dec.
- The best celestial objects are between 30° - 60° altitude and spread around in azimuth to allow a crossing fix.
- The level of precision is lower than usual – LHA to the nearest degree, and altitude/azimuth to the nearest 5 degrees is sufficient.

Example Problem 3.2.1. On 17 March, you observe an unknown star on an azimuth (bearing) of 320° T at an observed altitude (Ho) of approximately 50°. Your DR latitude is 25° N, and your DR longitude is 66° 48' W. The GHA of Aries is 156° 53.7'. What star did you observe?

- Set the Starfinder to the northern hemisphere and use the 25° N clear plate.
- Determine the LHA of Aries
  - GHA (Aries) = 156° 53.7'
  - DR Longitude = 66° 48' W
  - LHA (Aries) = 90° 05.7' (whole values are not necessary, but estimation is also ok).
- Search the Starfinder for altitude 50° and azimuth 320° T.
- Mirfak is the closest object (see image).

Problem 3.2.1. Answer. Mirfak

Reference:

Bowditch, 2024 edition, Article 1440-1441. https://msi.nga.mil/Publications/APN

Problem 3.2.2. The LHA of Aries is 185°. Your DR latitude is 25° N. What star is visible at azimuth 155° T at an altitude of 50°?

Problem 3.2.3. The LHA of Aries is 277°. Your DR latitude is 25° N. What star is visible at azimuth 210° T at an altitude of 30°?

Problem 3.2.4. The LHA of Aries is 80°. Your DR latitude is 25° S. What star is visible at azimuth 160° T at an altitude of 60°?

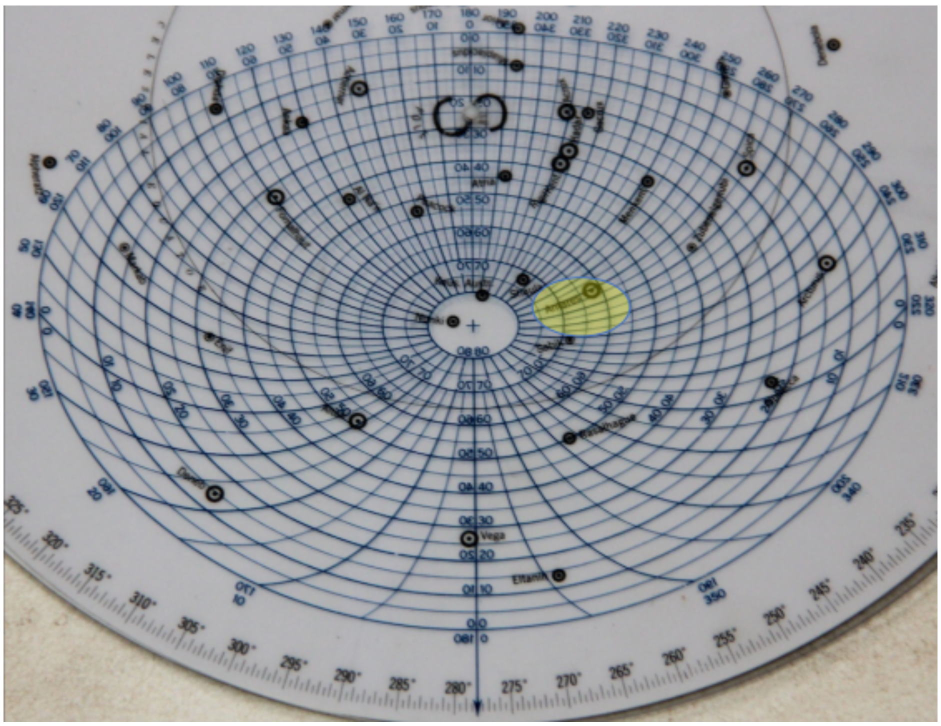
Problem 3.2.5. Which star is directly overhead if your DR latitude is 25° S and the LHA of Aries is 246° 30'?

Problem 3.2.6. Your DR latitude is 35° N. Your DR longitude is 45° 13' W. The GHA of Aries is 145° 13'. What is the altitude of Sirius?

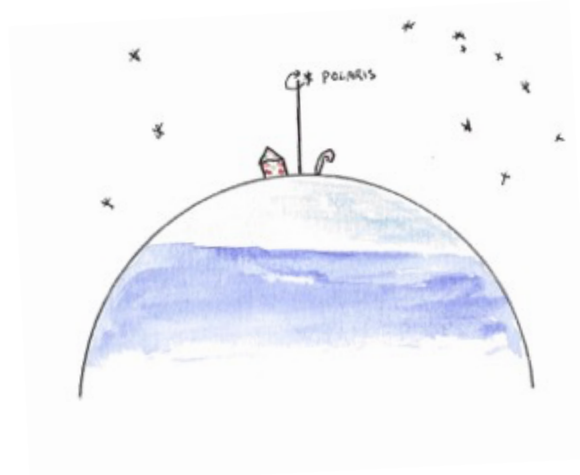
Problem 3.2.7. On 23 September at 07:36:06 GMT you observe an unknown star with an altitude of 261° T at Ho of 61° 35'. Your DR position is 25° 18'S, 162° 36'E. What star was observed?

Problem 3.2.8. It is 17 March at 05:20:30 local time when you observe an unknown star at an altitude of 45° and an azimuth of 050° T. Your DR position is 27° 23'N, 39° 42'W (ZD +3). What star was observed?

Problem 3.2.2. Answer. Spica
Problem 3.2.3. Answer. Antares
Problem 3.2.4. Answer. Canopus
Problem 3.2.5. Answer. Antares
Problem 3.2.6. Answer. Approximately 37-40°
Problem 3.2.7. Answer. Antares (see image)
Problem 3.2.8. Answer. Deneb



Latitude by Polaris



Because the Earth's orbit is not exactly circular and because it is inclined to the orbital plane of the Sun (e.g. we have seasons), the celestial north pole is subject to **precession** over the course of 28,500 years just like a spinning top that wiggles slightly on the surface of a table.

We live at a time which the star Polaris sits roughly above Earth's north pole, such that it would be directly overhead if you were standing at the pole.

If the star were exactly over the pole, a corrected sextant measurement (Ho) of the Polaris in the northern hemisphere would be exactly equal to the observer's latitude.

However, the star is not exactly over the pole, which means we need to make a couple of slight corrections to determine latitude. The Polaris tables in the Nautical Almanac make this process quite easy.

The required data for computing latitude by Polaris includes:

- LHA of Aries
- DR latitude to nearest 10°
- Month

The tables then compute the latitude using the formula:

Lat = Ho – 1° + LHA correction (Ao) + Latitude correction (A1) + Month correction (A2)

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POLARIS (POLE STAR) TABLES, 1981

FOR DETERMINING LATITUDE FROM SEXTANT ALTITUDE AND FOR AZIMUTH

L.H.A. ARIES	0°–9°	10°–19°	20°–29°	30°–39°	40°–49°	50°–59°	60°–69°	70°–79°	80°–89°	90°–99°	100°–109°	110°–119°
	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0	a_0
°	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′	° ′
0	0 17·8	0 13·7	0 10·9	0 09·7	0 09·9	0 11·7	0 14·9	0 19·5	0 25·3	0 32·1	0 39·7	0 47·9
1	17·3	13·3	10·7	09·6	10·1	12·0	15·3	20·0	25·9	32·8	40·5	48·7
2	16·9	13·0	10·6	09·6	10·2	12·2	15·7	20·6	26·6	33·5	41·3	49·6
3	16·4	12·7	10·4	09·6	10·3	12·5	16·2	21·1	27·2	34·3	42·1	50·4
4	16·0	12·4	10·3	09·6	10·5	12·8	16·6	21·7	27·9	35·0	42·9	51·3
5	0 15·6	0 12·1	0 10·1	0 09·6	0 10·6	0 13·1	0 17·1	0 22·3	0 28·6	0 35·8	0 43·7	0 52·1
6	15·2	11·9	10·0	09·7	10·8	13·5	17·5	22·8	29·3	36·6	44·6	53·0
7	14·8	11·6	09·9	09·7	11·0	13·8	18·0	23·4	29·9	37·3	45·4	53·8
8	14·4	11·4	09·8	09·8	11·2	14·2	18·5	24·0	30·6	38·1	46·2	54·7
9	14·0	11·2	09·7	09·8	11·5	14·5	19·0	24·7	31·4	38·9	47·0	55·5
10	0 13·7	0 10·9	0 09·7	0 09·9	0 11·7	0 14·9	0 19·5	0 25·3	0 32·1	0 39·7	0 47·9	0 56·4
Lat.	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1	a_1
°	′	′	′	′	′	′	′	′	′	′	′	′
0	0·5	0·6	0·6	0·6	0·6	0·5	0·5	0·4	0·3	0·3	0·2	0·2
10	·5	·6	·6	·6	·6	·6	·5	·4	·4	·3	·3	·2
20	·5	·6	·6	·6	·6	·6	·5	·5	·4	·4	·3	·3
30	·6	·6	·6	·6	·6	·6	·5	·5	·5	·4	·4	·4
40	0·6	0·6	0·6	0·6	0·6	0·6	0·6	0·5	0·5	0·5	0·5	0·5
45	·6	·6	·6	·6	·6	·6	·6	·6	·6	·5	·5	·5
50	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6	·6
55	·6	·6	·6	·6	·6	·6	·6	·6	·7	·7	·7	·7
60	·6	·6	·6	·6	·6	·6	·7	·7	·7	·7	·8	·8
62	0·7	0·6	0·6	0·6	0·6	0·6	0·7	0·7	0·7	0·8	0·8	0·8
64	·7	·6	·6	·6	·6	·6	·7	·7	·8	·8	·9	0·9
66	·7	·6	·6	·6	·6	·7	·7	·8	·8	0·9	0·9	1·0
68	0·7	0·6	0·6	0·6	0·6	0·7	0·7	0·8	0·9	1·0	1·0	1·0
Month	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2	a_2
	′	′	′	′	′	′	′	′	′	′	′	′
Jan.	0·7	0·7	0·7	0·7	0·7	0·7	0·7	0·7	0·7	0·6	0·6	0·6
Feb.	·6	·6	·7	·7	·7	·7	·8	·8	·8	·8	·8	·8
Mar.	·5	·5	·6	·6	·7	·7	·8	·8	·8	·9	·9	·9
Apr.	0·3	0·4	0·4	0·5	0·5	0·6	0·7	0·7	0·8	0·8	0·9	0·9
May	·2	·2	·3	·3	·4	·5	·5	·6	·7	·7	·8	·9
June	·2	·2	·2	·2	·3	·3	·4	·5	·5	·6	·7	·7
July	0·2	0·2	0·2	0·2	0·2	0·2	0·3	0·3	0·4	0·4	0·5	0·6
Aug.	·3	·3	·3	·2	·2	·2	·2	·3	·3	·3	·4	·4
Sept.	·5	·5	·4	·4	·3	·3	·3	·3	·3	·3	·3	·3
Oct.	0·7	0·6	0·6	0·5	0·5	0·4	0·4	0·3	0·3	0·3	0·3	0·3
Nov.	0·9	0·8	·8	·7	·6	·6	·5	·5	·4	·3	·3	·3
Dec.	1·0	1·0	0·9	0·9	0·8	0·8	0·7	0·6	0·6	0·5	0·4	0·4
Lat.	AZIMUTH											
°	°	°	°	°	°	°	°	°	°	°	°	°
0	0·4	0·3	0·1	0·0	359·8	359·7	359·6	359·5	359·4	359·3	359·2	359·2
20	0·4	0·3	0·1	0·0	359·8	359·7	359·5	359·4	359·3	359·2	359·2	359·1
40	0·5	0·3	0·2	0·0	359·8	359·6	359·4	359·3	359·2	359·1	359·0	358·9
50	0·6	0·4	0·2	0·0	359·7	359·5	359·3	359·1	359·0	358·9	358·8	358·7
55	0·7	0·5	0·2	0·0	359·7	359·5	359·2	359·0	358·9	358·7	358·6	358·6
60	0·8	0·5	0·2	0·0	359·7	359·4	359·1	358·9	358·7	358·5	358·4	358·4
65	0·9	0·6	0·3	359·9	359·6	359·3	359·0	358·7	358·4	358·3	358·1	358·1

Latitude = Apparent altitude (corrected for refraction) $- 1° + a_0 + a_1 + a_2$

The table is entered with L.H.A. Aries to determine the column to be used; each column refers to a range of 10°. a_0 is taken, with mental interpolation, from the upper table with the units of L.H.A. Aries in degrees as argument; a_1, a_2 are taken, without interpolation, from the second and third tables with arguments latitude and month respectively. a_0, a_1, a_2 are always positive. The final table gives the azimuth of *Polaris*.

Reference:

Bowditch, 2024 edition, Article 2015. https://msi.nga.mil/Publications/APN

Problem 3.2.9. The LHA of Aries is 110°. Using the Polaris tables, what is the A0 correction?

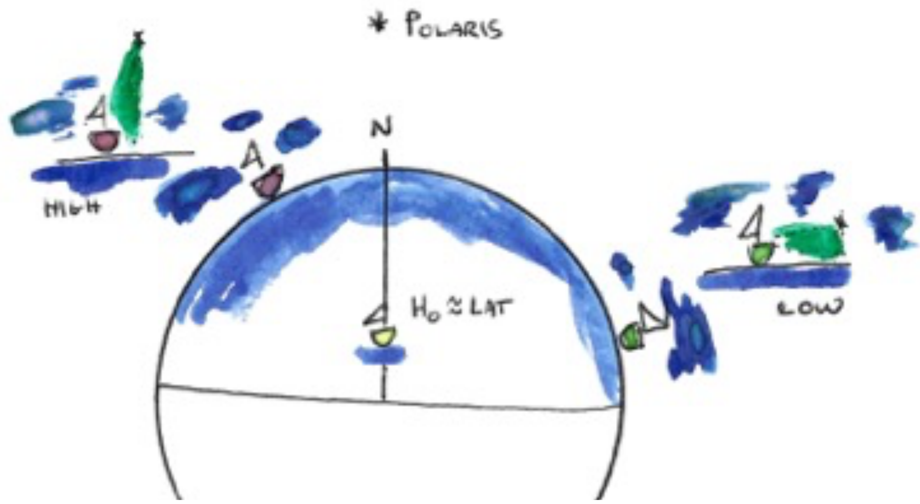
Problem 3.2.10. The LHA of Aries is 110°. Your DR latitude is 62°N and it is March. What are the A1 and A2 corrections from the Polaris tables?

Problem 3.2.11. The LHA of Aries is 115° 32'. Using the Polaris tables and mental interpolation (as described in Polaris table instructions) what is the A0 correction?

Problem 3.2.12. The LHA of Aries is 89° 27.2'. Your DR latitude is 29°N and the date is 15 March. You observe Polaris with Ho 29° 48.3'. What is your latitude?

Problem 3.2.13. The LHA of Aries is 003° 27.1'. Your DR latitude is 25°N and the date is 16 December. You observe Polaris with Ho 23° 50.1'. What is your latitude?

Problem 3.2.14. On 12 Feb at 01:32:40 GMT, your DR position is 25° 30' N and 110° 52.6' W (ZD +8).   You observe Polaris with Sextant Height (Hs) 26° 19.8'. The index error is 2.7' off the arc and the height of eye is 60.2 feet. What is your latitude?



Solutions within 10' are acceptable

Problem 3.2.9. Answer. 0° 47.9'
Problem 3.2.10. Answer. 0.8' for latitude, and 0.9' for month
Problem 3.2.11. Answer. 0° 52.6' (interpolated between value for 115° and 116°)
Problem 3.2.12. Answer. 29° 21.3'
Problem 3.2.13. Answer. 23° 07.8'
Problem 3.2.14. Answer

- Ho = 26° 13.0'
- LHA Aries = 54° 18.4'
- Latitude = 25° 27.2'

Solving the Celestial Triangle for a Computed Value

There are certain tools that can help us in our celestial navigation quest. One of them is HO229, the sight reduction tables for marine navigation.

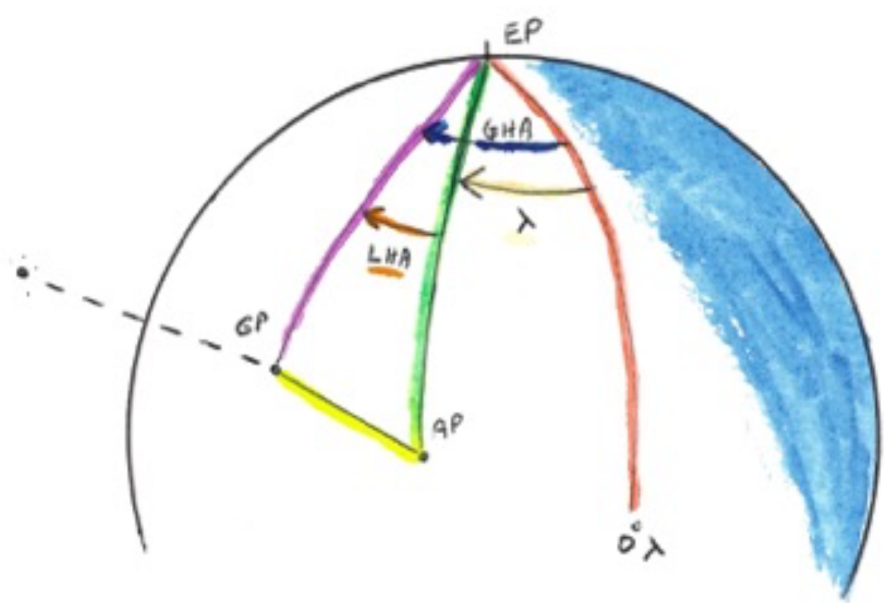
This book solves spherical triangles. If you feed it a whole value of declination of the GP, a whole value of latitude of the assumed position, and the LHA calculated in the previous step, HO229 (or a memorized formula, or the HO249 Air Almanac) will spit back two key pieces of information (after you make a couple of quick corrections):

- The **azimuth** (bearing) from the AP to the GP. This is measured on the 360° compass from the assumed position.

- The **computed height** of the celestial object if you were standing exactly at the AP.

Most often, you are not standing directly at the AP, and therefore in step 5 of the process, we will compare the computed height to the observed height to determine an **intercept** allowing us to plot the LOP on a human scale chart.

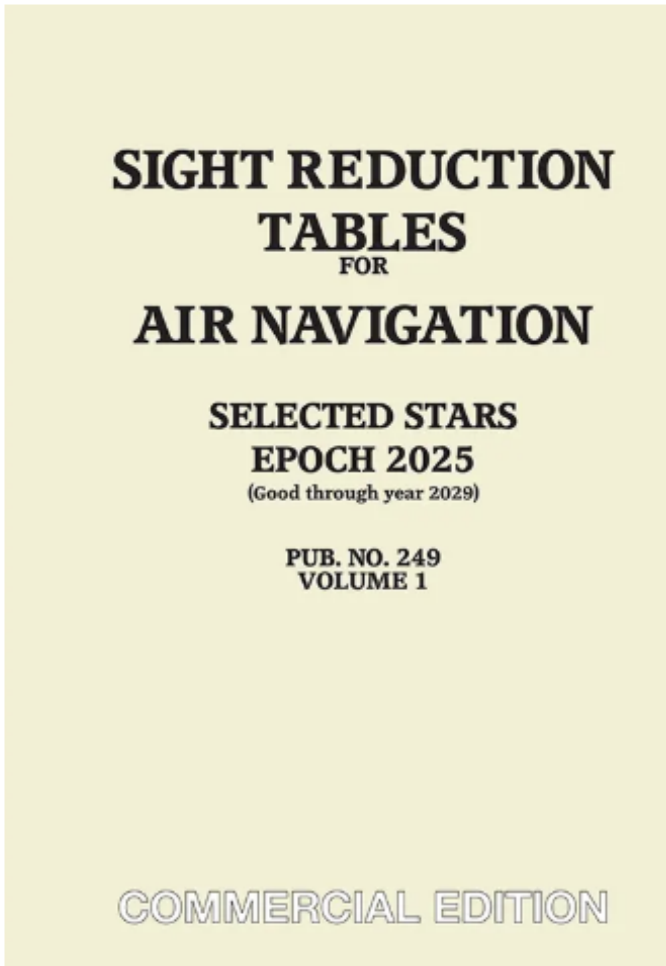
But for now, in step 4 of the process, we'll focus on using HO229 or direct calculation to obtain our azimuth (Z) and computed height (Hc) for a given spherical triangle.



HO249: Sight Reduction Tables for Air Navigation (Air Almanac)

This publication is designed for rapid reduction of astronomical sights as used on fast-moving aircraft and has a following among mariners as well as aviators.

Although this course relies primarily on HO229 for sight reduction – students are encouraged to review the Air Almanac and determine which works best for their own style.



Reference:

Bowditch, 2024 edition, Article 2005. https://msi.nga.mil/Publications/APN

Online edition of HO249. https://thenauticalalmanac.com/Pub_No_249_Epoch_2025.html

Online edition of HO249 (Government). https://aa.usno.navy.mil/publications/aira

HO229: Sight Reduction Tables for Marine Navigation

This series of publications is in six volumes, based on latitude; they provide solutions to navigational triangles where two legs and one included angle are known.

Although the tables can be used for other purposes (solving great circle sailing routes, etc.), they are primarily used to facilitate celestial navigation via the intercept method described in this course.

PUB. NO. 229
VOL. 2

SIGHT REDUCTION TABLES

FOR

MARINE NAVIGATION

LATITUDES 15°–30°, Inclusive

The front matter of the book is extremely helpful and provides background and formulae for navigation, and an excellent glossary. It also includes the direct-calculation formula described later in the course.

Most of the pages are dedicated to solutions for navigational triangles:

- The LHA is listed in the corners of each page.
- The latitude of the AP is listed along the top and bottom of the page
- The declination of the GP is listed along the left and right sides of the page
- The pages are also divided into sections for where the GP and AP are in the same or contrary hemispheres.
- There are **azimuth rules** located in the center margin at the top and bottom of each page.

By entering the tables with a whole value of declination (GP), a whole value of latitude (AP), the LHA, and the same/contrary argument, the user retrieves:

- Hc – the computed height of the object if you were standing at the AP.
- Z – the azimuth angle of the navigational triangle
- d – a correction factor (altitude difference) used to refine the solution of the triangle

The front and rear covers of the publication feature interpolation tables for refining the solution of the triangle if necessary.

Reference:

Bowditch, 2024 edition, Article 2005. https://msi.nga.mil/Publications/APN

Online edition of HO229. https://thenauticalalmanac.com/Pub_No_229.html

Online edition of HO229 (Government). https://msi.nga.mil/Publications/SRTMar

Retrieving Key Data from HO229

The process for using HO229 to solve celestial triangles is generally the same for any celestial body.

In step 3 of the process, we learned how to determine entering arguments of latitude (AP), declination (GP), and LHA. Another consideration is whether the latitude of the AP and the declination of the GP are in the same or contrary hemispheres.

Given this information, finding the correct page and value is relatively straightforward, but worth practice to determine the three key values:

- Hc – the computed height of the object if you were standing at the AP.
- Z – the azimuth angle of the navigational triangle
- d – a correction factor (altitude difference) used to refine the solution of the triangle
    - If the d value is italic and has a dot next to it (e.g. *47.7*●), this indicates an additional correction is required, which we will discuss later.

Example Problem 4.1.1. You have previously determined the following entering arguments for HO229:

LHA = 30°
Dec = 10° N
Lat = 15° N

What is the Hc, d, and Z values for these arguments?

Problem 4.1.1 answer.

Hc = 60° 18.8’
d = +12.4
Z = 96.1°

30°, 330° L.H.A. LATITUDE SAME NAME AS DECLINATION N. Lat. {L.H.A. greater than 180° .....Zn=Z; L.H.A. less than 180°............Zn=360°−Z}

Dec.	15°			16°			17°			18°		
°	Hc ° ′	d ′	Z °	Hc ° ′	d ′	Z °	Hc ° ′	d ′	Z °	Hc ° ′	d ′	Z °
0	56 46.4	+27.8	114.1	56 21.2	+29.3	115.5	55 54.8	+30.7	116.9	55 27.0	+32.2	118.2
1	57 14.2	+26.4	112.5	56 50.5	+28.0	113.9	56 25.5	+29.5	115.3	55 59.2	+31.0	116.7
2	57 40.6	+25.0	110.8	57 18.5	+26.7	112.3	56 55.0	+28.4	113.7	56 30.2	+29.9	115.1
3	58 05.6	+23.7	109.1	57 45.2	+25.4	110.6	57 23.4	+27.0	112.1	57 00.1	+28.6	113.5
4	58 29.3	+22.2	107.4	58 10.6	+24.0	108.9	57 50.4	+25.7	110.4	57 28.7	+27.3	111.9
5	58 51.5	+20.7	105.6	58 34.6	+22.5	107.2	58 16.1	+24.2	108.7	57 56.0	+26.0	110.2
6	59 12.2	+19.1	103.8	58 57.1	+21.0	105.4	58 40.3	+22.9	107.0	58 22.0	+24.6	108.5
7	59 31.3	+17.5	101.9	59 18.1	+19.4	103.6	59 03.2	+21.3	105.2	58 46.6	+23.2	106.8
8	59 48.8	+15.9	100.0	59 37.5	+17.8	101.7	59 24.5	+19.7	103.4	59 09.8	+21.6	105.0
9	60 04.7	+14.1	98.1	59 55.3	+16.2	99.8	59 44.2	+18.2	101.5	59 31.4	+20.1	103.2
10	60 18.8	+12.4	96.1	60 11.5	+14.5	97.9	60 02.4	+16.5	99.6	59 51.5	+18.5	101.3
11	60 31.2	+10.7	94.2	60 26.0	+12.7	95.9	60 18.9	+14.7	97.7	60 10.0	+16.7	99.4
12	60 41.9	+8.8	92.2	60 38.7	+10.9	93.9	60 33.6	+13.0	95.7	60 26.7	+15.1	97.5
13	60 50.7	+6.9	90.1	60 49.6	+9.1	91.9	60 46.6	+11.2	93.7	60 41.8	+13.3	95.5
14	60 57.6	+5.1	88.1	60 58.7	+7.2	89.9	60 57.8	+9.4	91.7	60 55.1	+11.5	93.5
15	61 02.7	+3.2	86.0	61 05.9	+5.4	87.8	61 07.2	+7.5	89.7	61 06.6	+9.7	91.5
16	61 05.9	+1.3	84.0	61 11.3	+3.4	85.8	61 14.7	+5.7	87.6	61 16.3	+7.8	89.4
17	61 07.2	−0.6	81.9	61 14.7	+1.6	83.7	61 20.4	+3.7	85.5	61 24.1	+5.9	87.3
18	61 06.6	−2.4	79.8	61 16.3	−0.3	81.6	61 24.1	+1.8	83.4	61 30.0	+4.0	85.3
19	61 04.2	−4.4	77.8	61 16.0	−2.3	79.5	61 25.9	−0.1	81.3	61 34.0	+2.1	83.2
20	60 59.8	[illegible]	75.7	61 13.7	[illegible]	77.5	61 25.8	[illegible]	79.3	61 36.1	[illegible]	81.1

19°			20°			21°			22°			Dec.
Hc ° ′	d ′	Z °	Hc ° ′	d ′	Z °	Hc ° ′	d ′	Z °	Hc ° ′	d ′	Z °	°
54 58.1	+33.6	119.4	54 28.1	+34.8	120.6	53 57.0	+36.1	121.8	53 24.8	+37.3	123.0	0
55 31.7	+32.4	118.0	55 02.9	+33.9	119.2	54 33.1	+35.1	120.5	54 02.1	+36.4	121.7	1
56 04.1	+31.4	116.5	55 36.8	+32.7	117.8	55 08.2	+34.1	119.1	54 38.5	+35.4	120.3	2
56 35.5	+30.1	114.9	56 09.5	+31.7	116.3	55 42.3	+33.1	117.6	55 13.9	+34.5	118.9	3
57 05.6	+29.0	113.3	56 41.2	+30.4	114.7	56 15.4	+32.0	116.1	55 48.4	+33.3	117.4	4
57 34.6	+27.6	111.7	57 11.6	+29.3	113.2	56 47.4	+30.8	114.6	56 21.7	+32.3	115.9	5
58 02.2	+26.3	110.1	57 40.9	+28.0	111.5	57 18.2	+29.5	113.0	56 54.0	+31.2	114.4	6
58 28.5	+25.0	108.4	58 08.9	+26.6	109.9	57 47.7	+28.3	111.4	57 25.2	+29.9	112.8	7
58 53.5	+23.4	106.6	58 35.5	+25.3	108.2	58 16.0	+27.0	109.7	57 55.1	+28.6	111.2	8
59 16.9	+22.0	104.8	59 00.8	+23.8	106.4	58 43.0	+25.6	108.0	58 23.7	+27.3	109.6	9
59 38.9	+20.4	103.0	59 24.6	+22.2	104.6	59 08.6	+24.1	106.3	58 51.0	+25.9	107.8	10
59 59.3	+18.8	101.1	59 46.8	+20.8	102.8	59 32.7	+22.6	104.5	59 16.9	+24.5	106.1	11
60 18.1	+17.1	99.2	60 07.6	+19.1	100.9	59 55.3	+21.1	102.6	59 41.4	+22.9	104.3	12
60 35.2	+15.3	97.3	60 26.7	+17.4	99.0	60 16.4	+19.4	100.7	60 04.3	+21.4	102.4	13
60 50.5	+13.7	95.3	60 44.1	+15.7	97.1	60 35.8	+17.7	98.8	60 25.7	+19.7	100.6	14
61 04.2	+11.8	93.3	60 59.8	+13.9	95.1	60 53.5	+16.1	96.9	60 45.4	+18.1	98.6	15
61 16.0	+9.9	91.2	61 13.7	+12.1	93.1	61 09.6	+14.2	94.9	61 03.5	+16.3	96.7	16
61 25.9	+8.1	89.2	61 25.8	+10.3	91.0	61 23.8	+12.4	92.9	61 19.8	+14.6	94.7	17
61 34.0	+6.2	87.1	61 36.1	+8.3	89.0	61 36.2	+10.5	90.8	61 34.4	+12.7	92.7	18
61 40.2	+4.2	85.0	61 44.4	+6.5	86.9	61 46.7	+8.7	88.7	61 47.1	+10.8	90.6	19
61 44.4	[illegible]	82.9	61 50.9	[illegible]	84.8	61 55.4	[illegible]	86.6	61 57.9	[illegible]	88.5	20

Problem 4.1.2. You have previously determined the following entering arguments for HO229. What are the Hc, d, and Z values for these arguments?
LHA = 30°
Dec = 20° N
Lat = 21° S (remember this means contrary pages)

Problem 4.1.3. You have previously determined the following entering arguments for HO229. What are the Hc, d, and Z values for these arguments?
LHA = 30°
Dec = 20° S
Lat = 25° N

Problem 4.1.4. You have previously determined the following entering arguments for HO229. What are the Hc, d, and Z values for these arguments?
LHA = 30°
Dec = 25° S
Lat = 23° S

Problem 4.1.5. You have previously determined the following entering arguments for HO229. What are the Hc, d, and Z values for these arguments?
LHA = 305°
Dec = 17° N
Lat = 22° N

Problem 4.1.6. You have previously determined the following entering arguments for HO229. What are the Hc, d, and Z values for these arguments?
LHA = 245°
Dec = 60° S
Lat = 25° S

Problem 4.1.7. You have previously determined the following entering arguments for HO229. What are the Hc, d, and Z values for these arguments?
LHA = 41°
Dec = 11° S
Lat = 26° N

Problem 4.1.8. You have previously determined the following entering arguments for HO229. What are the Hc, d, and Z values for these arguments?
LHA = 356°
Dec = 47° N
Lat = 24° N

Problem 4.1.2. Answer
Hc = 39° 34.9’
d = -47.9
Z = 142.4°

Problem 4.1.3. Answer
Hc = 36° 22.2’
d = -49.7
Z = 144.3°

Problem 4.1.4. Answer
Hc = 62° 34.5’
d = 1.3* (this dot indicates an additional correction that may be required)
Z = 79.7°

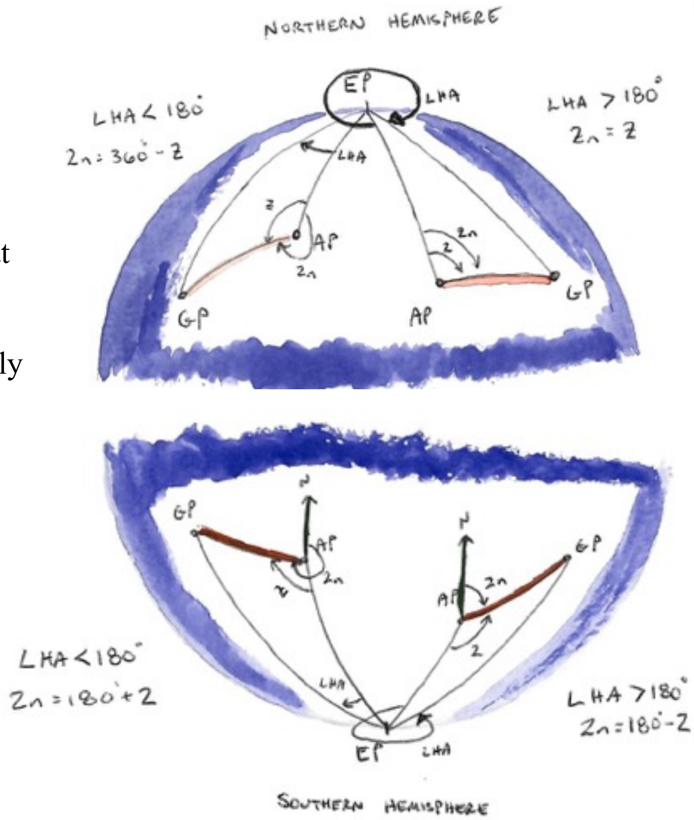
Problem 4.1.5. Answer
Hc = 38° 10.6’
d = 15.1
Z = 85.2°

Problem 4.1.6. Answer
Hc = 10° 02.9’
d = 33.1
Z = 27.4°

Problem 4.1.7. Answer
Hc = 35° 36.4
d = -41.5
Z = 127.6°

Problem 4.1.8. Answer
Hc = 66° 46.7’
d = -59.2
Z = 6.9°

Azimuth Rules



To save space in the publication, the authors of HO229 recognized that certain triangles are the same, whether viewed from the northern or southern hemisphere, or rotated in certain directions. After all, geometry says that angles are the same angles even when viewed upside down or inside out. Therefore, they rely on mariners to apply some corrections to standard triangles for our given situation/location.

The Z value given from HO229 is correctly defined as the **azimuth angle.** This is measured from 0° to 180° eastward or westward from the meridian. It is an intermediary value to help us get our actual azimuth (Zn).

It is important to note that the Z value (azimuth angle) from HO229 is not the final solution to the navigational triangle for a given celestial sight.

So instead – we take the azimuth angle (Z) and apply some rules to determine our specific azimuth (Zn). Zn is what we will plot to help obtain our LOP in step 5 and represents the true (xxx° T) compass bearing from the AP to the GP.

The azimuth rules are located near the center margin of each page in HO229 at the top and bottom, and are reproduced here:

For northern latitudes:

- if the LHA is greater than 180°, then Zn = Z
- if the LHA is less than 180°, then Zn = 360° - Z

For southern latitudes:

- if the LHA is greater than 180°, then Zn = 180° - Z
- if the LHA is less than 180°, then Zn = 180° + Z

**It is also possible to refine the azimuth value to the nearest tenth of a degree by interpolating the Z value as described in the front matter of HO229, but this is rarely important for realistic celestial navigation; to the nearest degree is sufficient. Refining azimuth angle values is discussed in Part 5 of the course.

Example problem 4.1.9. If the LHA is 92° and you are in the northern hemisphere, correct the Z (azimuth angle) value of 127° to a Zn (azimuth) value.

- The correct azimuth rule to apply (Northern hemisphere, LHA less than 180°) is
    - Zn = 360° - Z
    - Zn = 360° - 127°
    - Zn = 233° T

Problem 4.1.9 answer. 233° T

Reference:

Bowditch, 2024 edition, Article 1428-1430. https://msi.nga.mil/Publications/APN

Problem 4.1.10. If the LHA is 330° and you are in the northern hemisphere, correct a Z (azimuth angle) value of 46.3° to a Zn (azimuth) value.

Problem 4.1.11. If the LHA is 287° and you are in the southern hemisphere, correct a Z (azimuth angle) value of 52.8° to a Zn (azimuth) value.

Problem 4.1.12. You are in the northern hemisphere and the LHA is 74°. The Z value is 73.7°. What is the Zn?

Problem 4.1.13. You are in the southern hemisphere and the LHA is 26°. The Z value is 13.0°. What is the Zn?

Problem 4.1.14. You have obtained an azimuth angle from HO229 of 49.7°. The LHA is 341° and you are in the southern hemisphere. What is the azimuth of your sight (Zn)?

Problem 4.1.10 answer. 46.3° T

Problem 4.1.11 answer. 127.2° T

Problem 4.1.12 answer. 286.3° T

Problem 4.1.13 answer. 193° T

Problem 4.1.14 answer. 130.3° T

Altitude Difference (d) Issues

In step 3 of the process, we took the whole value of declination to obtain our entering argument for HO229. We ignored the declination increments and promised to deal with them later. The time is now.

Just like many processes in celestial navigation tables, the authors construct standard values and trust us to apply corrections for our specific case.

To account for the remainder of declination, HO229 provides a correction factor allowing us to fine-tune our results. This is called **altitude difference** and is noted as "d" in the tables. This difference is then added or subtracted (based on sign) to the computed height (Hc) obtained from HO229.

There are two ways to calculate the altitude difference. The first way is by using direct calculation, and the second way is to use the interpolation tables in the front and back of HO229.

The formula to account for any remainder of declination is:

- Altitude difference (d) x (Declination increment / 60')

The interpolation tables at the front and back of HO229 are constructed based on this formula. The table is set up with values of declination increment on the left, and values of tens and units for the altitude difference (d) along the top. The user can retrieve the correction in 2 steps.

Occasionally, for high elevation sights (over 60°), there may be an additional correction called **double second difference**. This is used to account for interpolations at high observation angle which are non-linear.

It is generally safe to ignore this correction, but for maximum accuracy, HO229 describes how to make the double second difference correction in the front of the publication. The altitude difference (d) value is noted in italics with a dot next to it in these cases and is always added. An example (Problem 4.1.21) is provided for completeness later.

Example problem 4.1.15. If the declination increment (remainder) from Step 3 of the process as 12.5', and the altitude difference (d) value from HO229 was +49.9, what is the correction you must apply to Hc?

- Correction = d x (dec inc / 60)
- Correction = +49.9 x (12.5'/60') = +10.4'
- 10.4' must be added to the Hc.

Problem 4.1.15. Answer. +10.4'

Reference:
Bowditch, 2024 edition, Article 2006-2007. https://msi.nga.mil/Publications/APN

Problem 4.1.16. If the declination increment (remainder) is 42.5' and the altitude difference (d) value from HO229 is 22.2, what is the correction you must apply to Hc?

Problem 4.1.17. If the declination increment (remainder) is 26.6' and the altitude difference (d) value from HO229 is -30.3, what is the correction you must apply to Hc?

Problem 4.1.18. The declination of the GP is 13° 34.9'. You used the whole value of declination (13°) to obtain results from HO229 which included Hc = 22° 13.2', d = 44.2, and Z = 129°. What is the correction to Hc?

Problem 4.1.19. Using the interpolation tables in HO229, if the declination increment is 24.0', and the altitude difference (d) is 20.5, what is the correction in tens and units (and total correction)?

Problem 4.1.20. The altitude difference (d) from HO229 is +13.6*. What does this mean?

Double Second Difference Example

**Note double second differences rarely make a significant difference in celestial sight reductions, but an example is included here for completeness of the course.

Example Problem 4.1.21. The declination of the GP is 13° 24.2'. Using the table on the next page, determine the linear (normal) correction for altitude difference (d) and the also the double second difference (nonlinear) correction. The latitude is 23° N and the LHA is 21°.

- Tabular d = +28.7*
- Linear (normal) correction = d x (dec inc/60') = 28.7 x (24.2/60) = +11.6'
- Double second difference:
  - Subtract the d value immediately prior to the desired d value from the d value immediately following the desired d value, for the same latitude/LHA.
    - (+26.6) (immediately following desired value) - (+30.6) (immediately prior to desired value) = -4.0
  - Enter the compartment in the interpolation table furthest right which corresponds to the declination increment.
    - Use the left part of the table to bracket the result you found in the last step (e.g. -4.0 is bracketed by 2.5 and 4.1. Pull a result of 0.2'.
  - Add this correction to the normal (linear) corrected value and then apply it to Hc.
- Linear correction (normal) = +11.6'
- Nonlinear correction (double second difference) = +0.2'

Example Problem 4.1.21. Answer. Total +11.8'
For more about double second differences, refer to the front matter of HO229.

21°, 339° L.H.A. **LATITUDE SAME NAME AS DECLINATION** N. Lat. {L.H.A. greater than 180° .....Zn=Z; L.H.A. less than 180°............Zn=360°–Z}

Dec.	23° Hc	23° d	23° Z	24° Hc	24° d	24° Z	25° Hc	25° d	25° Z	26° Hc	26° d	26° Z
°	° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°
0	59 14.7	+45.5	135.5	58 31.5	+46.4	136.7	57 47.5	+47.2	137.8	57 02.7	+48.1	138.8
1	60 00.2	+44.7	134.2	59 17.9	+45.7	135.4	58 34.7	+46.6	136.6	57 50.8	+47.4	137.7
2	60 44.9	+43.8	132.9	60 03.6	+44.9	134.1	59 21.3	+45.9	135.4	58 38.2	+46.8	136.5
3	61 28.7	+42.9	131.5	60 48.5	+44.0	132.8	60 07.2	+45.1	134.1	59 25.0	+46.1	135.3
4	62 11.6	+41.9	130.0	61 32.5	+43.1	131.4	60 52.3	+44.3	132.7	60 11.1	+45.3	134.0
5	62 53.5	+40.9	128.4	62 15.6	+42.2	129.9	61 36.6	+43.3	131.3	60 56.4	+44.5	132.7
6	63 34.4	+39.6	126.8	62 57.8	+41.0	128.4	62 19.9	+42.4	129.9	61 40.9	+43.6	131.3
7	64 14.0	+38.5	125.1	63 38.8	+40.0	126.7	63 02.3	+41.3	128.3	62 24.5	+42.6	129.8
8	64 52.5	+37.1	123.3	64 18.8	+38.7	125.0	63 43.6	+40.2	126.7	63 07.1	+41.6	128.3
9	65 29.6	+35.6	121.4	64 57.5	+37.3	123.3	64 23.8	+38.9	125.0	63 48.7	+40.4	126.7
10	66 05.2	+34.1	119.5	65 34.8	+35.9	121.4	65 02.7	+37.6	123.2	64 29.1	+39.2	125.0
11	66 39.3	+32.3	117.4	66 10.7	+34.3	119.4	65 40.3	+36.2	121.4	65 08.3	+37.9	123.2
12	67 11.6	+30.6	115.3	66 45.0	+32.7	117.4	66 16.5	+34.6	119.4	65 46.2	+36.4	121.3
13	67 42.2	+28.7•	113.0	67 17.7	+30.9	115.2	66 51.1	+33.0	117.3	66 22.6	+34.9	119.4
14	68 10.9	+26.6•	110.7	67 48.6	+28.9•	113.0	67 24.1	+31.1	115.2	66 57.5	+33.3	117.3
15	68 37.5	+24.4•	108.2	68 17.5	+26.9•	110.6	67 55.2	+29.2	112.9	67 30.8	+31.4	115.2
16	69 01.9	+22.0•	105.7	68 44.4	+24.6•	108.2	68 24.4	+27.2•	110.6	68 02.2	+29.5	112.9
17	69 23.9	+19.7•	103.1	69 09.0	+22.4•	105.7	68 51.6	+25.0•	108.1	68 31.7	+27.5•	110.6
18	69 43.6	+17.0•	100.4	69 31.4	+19.9•	103.0	69 16.6	+22.6•	105.6	68 59.2	+25.2•	108.1
19	70 00.6	+14.4•	97.6	69 51.3	+17.3•	100.3	69 39.2	+20.2•	103.0	69 24.4	+22.9•	105.6
20	70 15.0	+11.6•	94.7	70 08.6	+14.6•	97.5	69 59.4	+17.5•	100.2	69 47.3	+20.5•	102.9
21	70 26.6	+8.8•	91.8	70 23.2	+11.9•	94.6	70 16.9	+14.9•	97.4	70 07.8	+17.8•	100.2
22	70 35.4	+5.8•	88.9	70 35.1	+8.9•	91.7	70 31.8	+12.1•	94.5	70 25.6	+15.1•	97.3
23	70 41.2	+2.8•	85.9	70 44.0	+6.0•	88.7	70 43.9	+9.1•	91.6	70 40.7	+12.3•	94.4
24	70 44.0	−0.1•	82.8	70 50.0	+3.0•	85.7	70 53.0	+6.2•	88.6	70 53.0	+9.4•	91.5

27° Hc	27° d	27° Z	28° Hc	28° d	28° Z	29° Hc	29° d	29° Z	30° Hc	30° d	30° Z	Dec.
° ′	′	°	° ′	′	°	° ′	′	°	° ′	′	°	°
56 17.2	+48.8	139.8	55 31.1	+49.5	140.7	54 44.3	+50.2	141.6	53 57.0	+50.8	142.5	0
57 06.0	+48.3	138.7	56 20.6	+49.0	139.7	55 34.5	+49.7	140.7	54 47.8	+50.3	141.6	1
57 54.3	+47.6	137.6	57 09.6	+48.4	138.7	56 24.2	+49.2	139.7	55 38.1	+49.9	140.6	2
58 41.9	+47.0	136.5	57 58.0	+47.9	137.6	57 13.4	+48.6	138.6	56 28.0	+49.4	139.6	3
59 28.9	+46.3	135.3	58 45.9	+47.2	136.4	58 02.0	+48.1	137.5	57 17.4	+48.8	138.6	4
60 15.2	+45.5	134.0	59 33.1	+46.5	135.2	58 50.1	+47.4	136.4	58 06.2	+48.3	137.5	5
61 00.7	+44.8	132.7	60 19.6	+45.7	134.0	59 37.5	+46.7	135.2	58 54.5	+47.6	136.4	6
61 45.5	+43.8	131.3	61 05.3	+45.0	132.6	60 24.2	+46.0	133.9	59 42.1	+47.0	135.2	7
62 29.3	+42.8	129.8	61 50.3	+44.1	131.2	61 10.2	+45.2	132.6	60 29.1	+46.2	133.9	8
63 12.1	+41.9	128.3	62 34.4	+43.1	129.8	61 55.4	+44.3	131.2	61 15.3	+45.4	132.6	9
63 54.0	+40.7	126.7	63 17.5	+42.1	128.3	62 39.7	+43.4	129.8	62 00.7	+44.6	131.2	10
64 34.7	+39.4	125.0	63 59.6	+40.9	126.7	63 23.1	+42.3	128.3	62 45.3	+43.6	129.8	11
65 14.1	+38.2	123.2	64 40.5	+39.8	125.0	64 05.4	+41.2	126.7	63 28.9	+42.6	128.3	12
65 52.3	+36.7	121.3	65 20.3	+38.4	123.2	64 46.6	+40.1	125.0	64 11.5	+41.5	126.7	13
66 29.0	+35.2	119.4	65 58.7	+37.0	121.3	65 26.7	+38.7	123.2	64 53.0	+40.3	125.0	14
67 04.2	+33.5	117.3	66 35.7	+35.5	119.4	66 05.4	+37.3	121.3	65 33.3	+39.1	123.2	15
67 37.7	+31.8	115.2	67 11.2	+33.8	117.3	66 42.7	+35.8	119.4	66 12.4	+37.6	121.4	16
68 09.5	+29.8•	112.9	67 45.0	+32.1	115.2	67 18.5	+34.1	117.3	66 50.0	+36.1	119.4	17
68 39.3	+27.7•	110.5	68 17.1	+30.1•	112.9	67 52.6	+32.4	115.2	67 26.1	+34.4	117.4	18
69 07.0	+25.6•	108.1	68 47.2	+28.0•	110.5	68 25.0	+30.4•	112.9	68 00.5	+32.7	115.2	19
69 32.6	+23.2•	105.5	69 15.2	+25.9•	108.1	68 55.4	+28.4•	110.5	68 33.2	+30.7•	112.9	20
69 55.8	+20.7•	102.9	69 41.1	+23.5•	105.5	69 23.8	+26.1•	108.1	69 03.9	+28.7•	110.6	21
70 16.5	+18.1•	100.1	70 04.6	+21.0•	102.8	69 49.9	+23.8•	105.5	69 32.6	+26.4•	108.1	22
70 34.6	+15.4•	97.3	70 25.6	+18.4•	100.0	70 13.7	+21.3•	102.8	69 59.0	+24.1•	105.5	23
70 50.0	+12.5•	94.3	70 44.0	+15.6•	97.2	70 35.0	+18.6•	100.0	70 23.1	+21.6•	102.8	24

INTERPOLATION TABLE

8′	9′	Dec. Inc.	Tens 10′	20′	30′	40′	50′	Decimals ⇓	Units 0′	1′	2′	3′	4′	5′	6′	7′	8′	9′
′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′	′
2.2	2.5	**24.0**	4.0	8.0	12.0	16.0	20.0	.0	0.0	0.4	0.8	1.2	1.6	2.0	2.4	2.9	3.3	3.7
2.2	2.5	**24.1**	4.0	8.0	12.0	16.0	20.1	.1	0.0	0.4	0.9	1.3	1.7	2.1	2.5	2.9	3.3	3.7
2.3	2.5	**24.2**	4.0	8.0	12.1	16.1	20.1	.2	0.1	0.5	0.9	1.3	1.7	2.1	2.5	2.9	3.3	3.8
2.3	2.6	**24.3**	4.0	8.1	12.1	16.2	20.2	.3	0.1	0.5	0.9	1.3	1.8	2.2	2.6	3.0	3.4	3.8
2.3	2.6	**24.4**	4.1	8.1	12.2	16.3	20.3	.4	0.2	0.6	1.0	1.4	1.8	2.2	2.6	3.0	3.4	3.8
2.3	2.6	**24.5**	4.1	8.2	12.3	16.3	20.4	.5	0.2	0.6	1.0	1.4	1.8	2.2	2.7	3.1	3.5	3.9
2.4	2.6	**24.6**	4.1	8.2	12.3	16.4	20.5	.6	0.2	0.7	1.1	1.5	1.9	2.3	2.7	3.1	3.5	3.9
2.4	2.7	**24.7**	4.1	8.3	12.4	16.5	20.6	.7	0.3	0.7	1.1	1.5	1.9	2.3	2.7	3.1	3.6	4.0
2.4	2.7	**24.8**	4.2	8.3	12.4	16.6	20.7	.8	0.3	0.7	1.1	1.6	2.0	2.4	2.8	3.2	3.6	4.0
2.4	2.7	**24.9**	4.2	8.3	12.5	16.6	20.8	.9	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6	4.0
2.3	2.6	**25.0**	4.1	8.3	12.5	16.6	20.8	.0	0.0	0.4	0.8	1.3	1.7	2.1	2.5	3.0	3.4	3.8
2.4	2.7	**25.1**	4.2	8.3	12.5	16.7	20.9	.1	0.0	0.5	0.9	1.3	1.7	2.2	2.6	3.0	3.4	3.9
2.4	2.7	**25.2**	4.2	8.4	12.6	16.8	21.0	.2	0.1	0.5	0.9	1.4	1.8	2.2	2.6	3.1	3.5	3.9
2.4	2.7	**25.3**	4.2	8.4	12.6	16.9	21.1	.3	0.1	0.6	1.0	1.4	1.8	2.3	2.7	3.1	3.5	4.0
2.4	2.7	**25.4**	4.2	8.5	12.7	16.9	21.2	.4	0.2	0.6	1.0	1.4	1.9	2.3	2.7	3.1	3.6	4.0
2.5	2.8	**25.5**	4.3	8.5	12.8	17.0	21.3	.5	0.2	0.6	1.1	1.5	1.9	2.3	2.8	3.2	3.6	4.0
2.5	2.8	**25.6**	4.3	8.5	12.8	17.1	21.3	.6	0.3	0.7	1.1	1.5	2.0	2.4	2.8	3.2	3.7	4.1
2.5	2.8	**25.7**	4.3	8.6	12.9	17.2	21.4	.7	0.3	0.7	1.1	1.6	2.0	2.4	2.8	3.3	3.7	4.1
2.6	2.9	**25.8**	4.3	8.6	12.9	17.2	21.5	.8	0.3	0.8	1.2	1.6	2.0	2.5	2.9	3.3	3.7	4.2
2.6	2.9	**25.9**	4.4	8.7	13.0	17.3	21.6	.9	0.4	0.8	1.2	1.7	2.1	2.5	2.9	3.4	3.8	4.2

Double Second Diff. and Corr. (left)

′	′
1.0	
	0.1
3.0	
	0.2
4.9	
	0.3
6.9	
	0.4
8.9	
	0.5
10.8	
	0.6
12.8	
	0.7
14.8	
	0.8
16.7	
	0.9
18.7	
	1.0
20.7	
	1.1
22.7	
	1.2
24.6	
	1.3
26.6	
	1.4
28.6	
	1.5
30.5	
	1.6
32.5	
	1.7
34.5	

Double Second Diff. and Corr. (right)

′	′
0.8	
	0.1
2.5	
	0.2
4.1	
	0.3
5.8	
	0.4
7.4	
	0.5
9.1	
	0.6
10.7	
	0.7
12.3	
	0.8
14.0	
	0.9
15.6	
	1.0
17.3	
	1.1
18.9	
	1.2
20.6	
	1.3
22.2	
	1.4
23.9	
	1.5
25.5	
	1.6
27.2	
	1.7
28.8	
	1.8
30.4	
	1.9
32.1	
	2.0
33.7	
	2.1
35.4	

Problem 4.1.16. Answer. +15.7’

Problem 4.1.17. Answer. -13.4’

Problem 4.1.18. Answer. +25.7’

Problem 4.1.19. Tens (8.0’), units (0.2’), total 8.2’

Problem 4.1.20. +13.6 is the altitude difference, and the dot/star indicates a double second difference must be applied.

Solving Spherical Triangles

In this part of the course, we've learned how to retrieve values of Hc, d, and Z from HO229 given whole values of latitude, declination, and LHA.

Then, we learned how to tweak the Z value (azimuth angle) to arrive at a useable Zn (azimuth), and how to apply the d value (altitude difference) to remaining declination increments to eventually modify our Hc (computed height).

In this section, we'll put it all together and unlock the key skill of step 4: solving spherical triangles.

Example Problem 4.1.22. Given the following data from previous steps in the celestial navigation process, retrieve and correct values of Hc, d, and Z from HO229. Then, correct these values to obtain a final computed height (Hc) and azimuth (Zn).

LHA = 30°
Lat = 15° N
Dec =10° 12.7'N

- Retrieve values for Hc, d, and Z from HO229 based on whole values of LHA, Latitude, and Declination
  - Hc = 60° 18.8'
  - d = +12.4
  - Z = 96.1°

- Correct azimuth angle (Z) to azimuth (Zn)
  - N lat, LHA less than 180° → Zn = 360° - Z
  - Zn = 360° - 96.1° = <u>263.9° T</u>

- Apply altitude difference (d) to declination increment
  - Correction = d x (dec inc)/60'
  - Correction = +12.4 x (12.7'/60') = +2.6'

- Correct computed height (Hc) for altitude difference
  - Hc = 60° 18.8'
  - Correction = +2.6'
  - Hc = 60° 18.8' + 2.6' = <u>60° 21.4'</u>

Example Problem 4.1.22. Answer. Hc = 60° 21.4', Zn = 263.9° T

Problem 4.1.23. Given the following data from previous steps in the celestial navigation process, retrieve and correct values of Hc, d, and Z from HO229. Then, correct these values to obtain a final computed height (Hc) and azimuth (Zn).

LHA = 35°
Lat = 16° N
Dec =15° 44.2’N

Problem 4.1.24. Given the following data from previous steps in the celestial navigation process, retrieve and correct values of Hc, d, and Z from HO229. Then, correct these values to obtain a final computed height (Hc) and azimuth (Zn).

LHA = 293°
Lat = 15° S
Dec =25° 30.9’S

Problem 4.1.25. Given the following data from previous steps in the celestial navigation process, retrieve and correct values of Hc, d, and Z from HO229. Then, correct these values to obtain a final computed height (Hc) and azimuth (Zn).

LHA = 336°
Lat = 25° N
Dec =12° 13.2’S

Problem 4.1.26. Given the following data, what is the computed height (Hc) and azimuth (Zn) for this sight?

LHA = 318°
Lat = 17° S
Dec =50° 58.8’S

Problem 4.1.27. Given the following data, what is the computed height (Hc) and azimuth (Zn) for this sight?

LHA = 11°
Lat = 22° S
Dec =38° 39.1’ N

Answers to the nearest 0.5' for Hc and 1.0° for Zn are acceptable.

Problem 4.1.23. Answer.
(intermediate solution provided to check your work)

Hc = 56° 17.8'
d = +6.0
Z = 86.8°
--
Zn = 360° - Z = 273.2° T
Hc = 56° 17.8' + 4.4' = 56° 22.2'

Problem 4.1.24. Answer.
(intermediate solution provided to check your work)

Hc = 26° 50.2'
d = +4.7
Z = 69.2°
--
Zn = 180 – Z = 110.8° T
Hc = 26° 50.2' + 2.4' = 26° 52.6'

Problem 4.1.25. Answer.
(intermediate solution provided to check your work – remember to use contrary page)

Hc = 46° 13.2'
d = -51.0
Z = 144.9°
--
Zn = Z = 144.9° T
Hc = 46° 13.2' – 11.1' = 46° 02.1'

Problem 4.1.26. Answer.

Zn = 180° - Z = 144° T
Hc = 42° 54.3' – 29.0' = 42° 25.3'

Problem 4.1.27. Answer.

Zn = 350.1° T
Hc = 28° 28.7'

Astronomy: Stars

The background of stars could be considered an unchanging sphere that circles the Earth – and for the most part, the night sky is steady – a "celestial sphere."

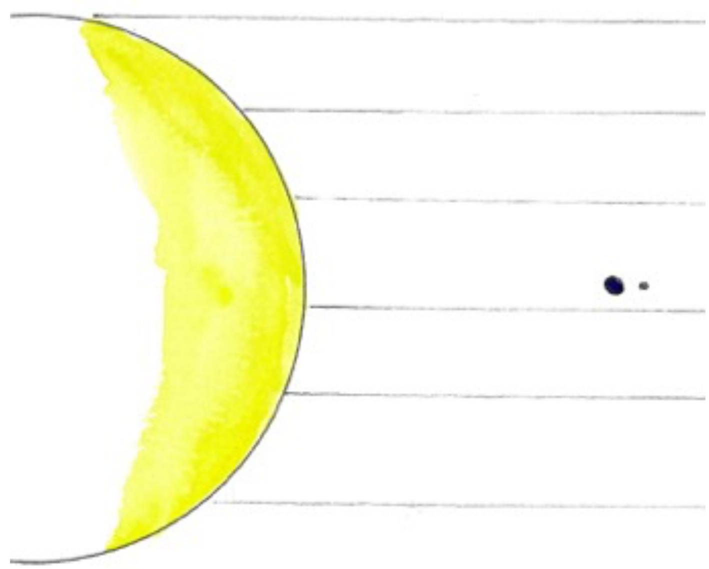
The Earth's year is 365 days and since Earth is a sphere (360°) the night sky will tend to shift forever westward by about 1° per day… if you locate a specific star in a certain point tonight, it will be about 1° west of that tomorrow.

The stars are extremely far away, so you can consider their light rays as parallel when they reach the Earth.

The brightness of the stars (magnitude) varies, but the 57 navigational stars listed in the Nautical Almanac are selected for being generally bright enough to use at twilight and distributed throughout the night sky such that several are always available for navigation, anywhere on Earth.

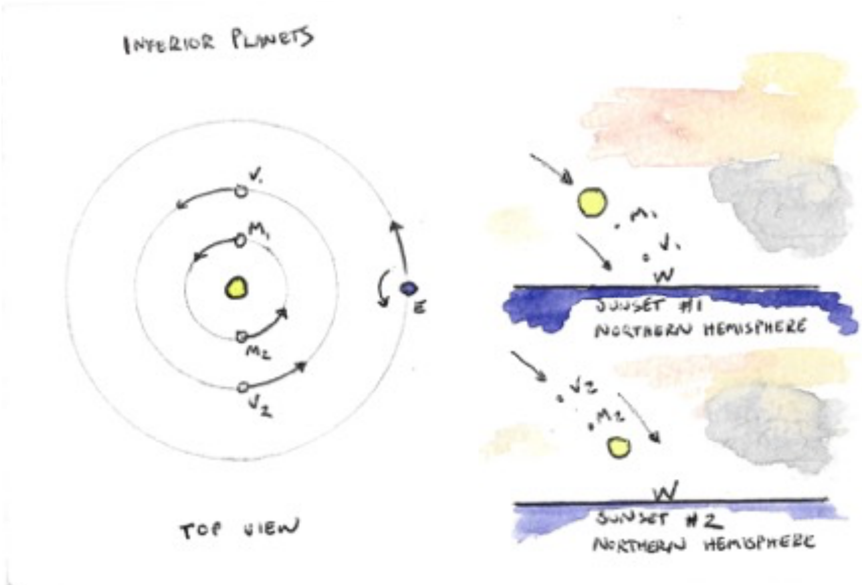
Reference:

Bowditch, 2024 edition, Article 1401-1402, 1404, 1433-1438.
https://msi.nga.mil/Publications/APN



Astronomy: Solar System

There are four navigational planets – Venus, Mars, Jupiter and Saturn. Although other planets orbit the sun, these four are frequently visible and bright enough for routine celestial navigation work.

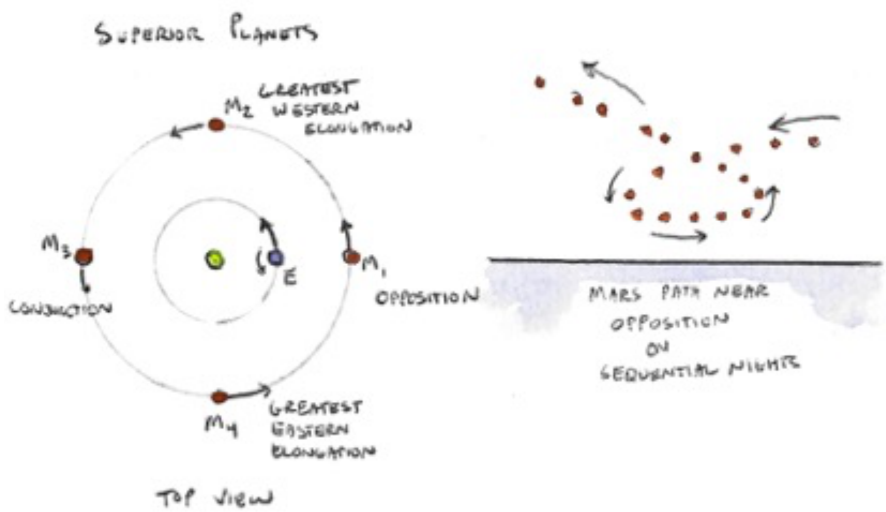


Venus is an inferior planet, meaning their orbit lies inside the Earth's, and these planets' year is shorter than Earths.

To celestial navigators on Earth, Venus tends to always be near the sun, either rising shortly before the Sun, or setting shortly after it.

Mars, Jupiter, and Saturn are superior planets, meaning they orbit the sun slower than the Earth. Consequently, they can be found anywhere along the ecliptic (roughly the path the sun takes through the sky) during the day or night.

These four navigational planets are generally very bright, hard to mistake for a star, and often visible at times of twilight – making them exceptional targets for celestial navigation.



Reference:

Bowditch, 2024 edition, Article 1408, 1410-1411.

https://msi.nga.mil/Publications/APN

Astronomy: Moon

The moon orbits the sun every 27.3 days, resulting in the phases that so beautifully grace the night (and day) sky.

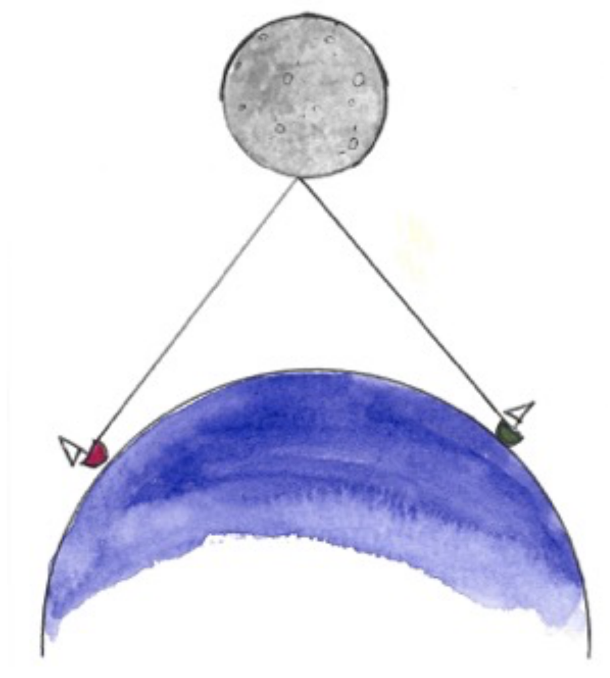
Because it is so close to the Earth, the rays of reflected light from the moon cannot be considered parallel – and therefore induce some parallax to viewers. This is generally accounted for in step 2 of the celestial navigation process when applying Horizontal Parallax to the GP of the moon.

Another consideration when using the moon for celestial navigation is the tilt, phases, and illumination of the moon; sometimes it is only possible to observe the upper limb of the moon.

Being an unmistakable object, the moon is also useable for celestial navigation during the day approximately half the month, resulting in excellent sun/moon fixes.

Reference:

Bowditch, 2024 edition, Article 1412.
https://msi.nga.mil/Publications/APN



Direct Calculations of HP and Z Using Formulae

So far in the course, we have used a step-by-step process to determine the GP of a celestial object, build a triangle using an assumed position (AP), and then solved that triangle using HO229.

Navigators can also use direct calculation to achieve these tasks – taking a shortcut through the tables.

Importantly, when using this process, no AP is required….you may calculate the computed height of an object directly from your DR.

The formulae needed for this process are given in the front matter of HO229, in Bowditch, or could be memorized:

sin Hc = (sin L x sin d) + (cos L x cos d x cos LHA)

tan Z = (cos d x sin LHA) / ((cos L x sin d) – (sin L x cos d x cos LHA))

where:
L is latitude
d is declination

- For calculations of Hc: when declination and latitude are contrary, treat declination as a negative number.

- For calculations of Z: when declination and latitude are contrary, treat declination as a negative number. When LHA is greater than 180°, treat it as a negative number, but add 180° if the result is negative.

Using a scientific calculator, and setting all units to degrees, allows for relatively straightforward calculation.

Example Problem 4.1.28. Given the following data from a celestial sight, determine the computed height (Hc) and Azimuth (Zn) of the sight using direct calculation methods.

DR Latitude 21° 05' N
DR Longitude 143° 27' E
GHA 301° 52.3
Declination 23° 13.2' N

- Determine the LHA: 301° 52.3' + 143° 27' E = 85° 19.3'

- Convert all values to decimal degree notation (divide minutes by 60):
  - Latitude = 21.08° N
  - Declination = 23.22° N
  - LHA = 85.32°

- Calculate Hc
  - sin Hc = (sin L x sin d) + (cos L x cos d x cos LHA)
  - sin Hc = (0.3597 x 0.3943) + (0.9331 x 0.9190 x 0.0816)
  - sin Hc = 0.1418 + 0.0700 = 0.2118
  - Hc = sin-1 (0.2118) = 12.226° = 12° 13.6'

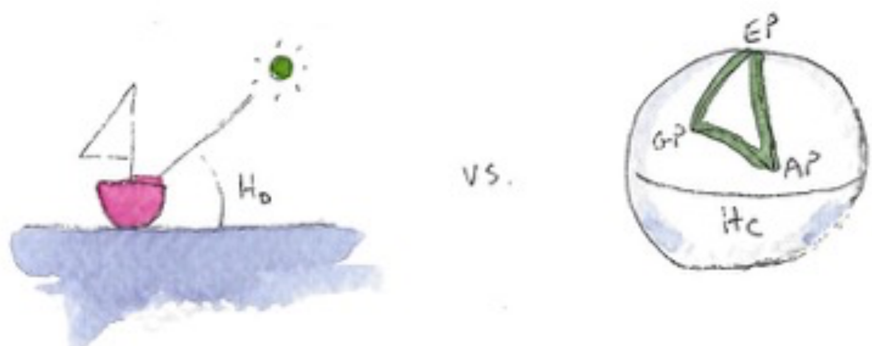
- Calculate Zn
  - tan Z = (cos d x sin LHA) / ((cos L x sin d) – (sin L x cos d x cos LHA))
  - tan Z = (0.9190 x 0.9967) x ((0.9331 x 0.3943) – (0.3597 x 0.9190 x 0.0816))
  - tan Z = (0.9160) / (0.3679 – 0.0270)
  - tan Z = 0.9160/0.3409 = 2.6870
  - Z = tan-1 (2.6870) = 69.6°
  - Zn = 360° - Z = 290.4°

Section Introduction

There are five steps to the basic celestial sight reduction process. In step 1, we observed and corrected the sighting. In step 2, we found the geographic position of the observed body. In step 3, we chose an assumed position to yield an effective navigational triangle.

In step 4, we solved that navigational triangle to achieve a height computed (Hc) and an azimuth (Zn) for the assumed position.

In this part of the course, part 5, we'll compare the computed solution to the observed solution and plot the difference on a chart or universal plotting sheet to determine our celestial line of position. We'll also learn about running fixes and three-body fixes to round out our understanding.



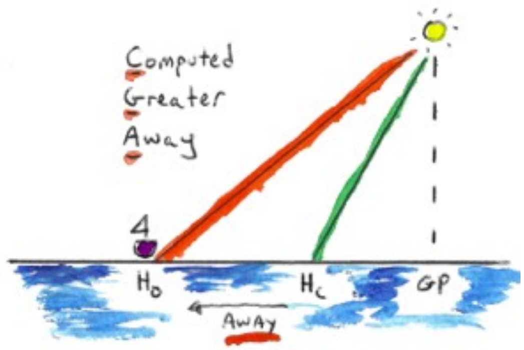
Comparing Hc (Height Computed) to Ho (Height Observed)

In step 4 of our process, we determined Hc, or height computed. This represented the height the celestial object would appear if we were standing directly at the assumed position chosen in step 3.

In step 1 of our process, we measured the height of the celestial object from our actual position and corrected it for index error, height of eye (dip), and apparent altitude (e.g. refraction, semidiameter, parallax, etc.).

We are now at the point where we can compare the two values and determine a difference by subtraction.

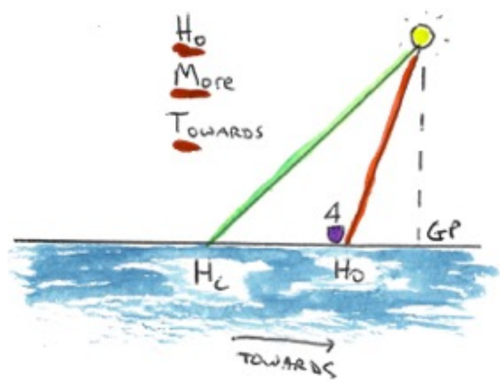
It is easy to make math errors and ruin an otherwise good sight here, so pay close attention – a good benchmark is that the Hc and Ho should almost always be less than 30 minutes of arc apart. If they are greater than 30'you may have made a math error somewhere in the process or made a bad sight.



The magnitude of this difference is called **intercept**. Once the intercept is noted you must also determine which value is greater (Ho or Hc).

- Case 1: If the computed value is greater, you are further away from the celestial object's GP than the assumed position. Remember "computed, greater, away."

- Case 2: If the observed value is greater, you are closer to the celestial object's GP than the assumed position. Remember "Ho, more, towards."



The final action is to plot the AP on a chart, plot the azimuth direction, and then plot the intercept either away from the GP (case 1), or towards the GP (case 2).

Once the intercept is plotted, a perpendicular line to the azimuth represents the celestial line of position.

Example Problem 5.1.1. You previously sighted the Sun at a height observed (Ho) of 49° 12.7'. The computed height (Hc) is 49° 04.9'. What is the intercept and direction to plot?

- Ho = 49° 12.7'
- Hc = 49° 04.9'
- Difference = 7.8'
    - Ho is "more", so we plot "towards" the GP.

Example Problem 5.1.1. Answer. 7.8' Towards

Reference:

Bowditch, 2024 edition, Article 2001-2002. https://msi.nga.mil/Publications/APN

Problem 5.1.2. You previously sighted the Sun at a height observed (Ho) of 36° 55.8'. The computed height (Hc) is 36° 41.2'. What is the intercept and direction to plot?

Problem 5.1.3. You previously sighted the Sun at a height observed (Ho) of 15° 02.6'. The computed height (Hc) is 15° 29.6'. What is the intercept and direction to plot?

Problem 5.1.4. The Hc from a sighting of Venus is 22° 25.6'. The observed height was 22° 20.9'. What is the intercept and direction to plot?

Problem 5.1.5. You observed Acrux at a height of 63° 42.9', and the computed height from HO229 was 63° 29.4'. What is the intercept and direction to plot?

Problem 5.1.6. The Hc for Antares is 14° 02.5'. The Ho is 13° 55.6'. What is the intercept and direction to plot?

Problem 5.1.7. Earlier, you observed Jupiter at a height of 15° 13.6'. You used HO229 to determine the height computed of Jupiter as 14° 49.1'. What is the intercept and direction to plot?

Problem 5.1.2. Answer. 14.6’ Towards
Problem 5.1.3. Answer. 27.0’ Away
Problem 5.1.4. Answer. 4.7’ Away
Problem 5.1.5. Answer. 13.5’ Towards
Problem 5.1.6. Answer. 6.9’ Away
Problem 5.1.7. Answer. 24.5’ Towards

Azimuth, Revisited

In step 4, we mentioned that an azimuth value to the nearest degree is more than sufficient for real celestial navigation practice at sea.

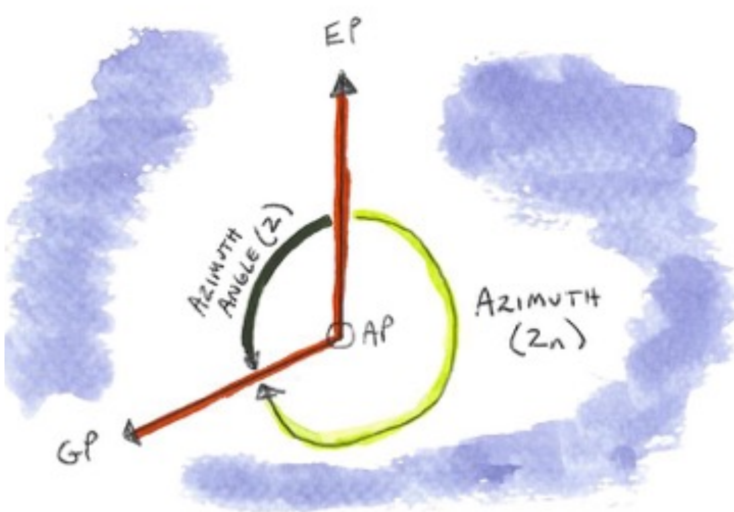
However, there are academic or operational cases in which an extremely accurate azimuth is desired.

For example, an **azimuth circle** is a device that can sit on top of marine gyroscopic compasses enabling extremely accurate measurement of the bearing to a celestial body. By comparing this measurement to a calculated value, we can determine the gyro compass error. This type of problem is outside the scope of this course but is available in Coast Guard license preparation courses.

Direct calculation of the azimuth is possible (reference the formula in step 4 of this course), but a tabular interpolation is another technique for more accurate azimuth measurements.

For maximum accuracy, in traditional celestial navigation, this interpolation also serves to refine the HO229 azimuth value for leftover declination values. This is like how we adjusted Hc using the declination increment and altitude difference (d value) in step 4.

The tabular technique involves creating a table using the "base" azimuth figure, and then inspecting the tables for increases in value of declination for the next highest whole value. By applying a quick interpolation for the declination value, you can "tweak" the azimuth value slightly for maximum accuracy.



Example Problem 5.1.8. Given the following base values, use an interpolation to refine the azimuth value to account for declination increment for maximum accuracy.

- Latitude 25° N
- Declination N 23° 23.1'
- LHA 300°
- Base Z value: 78.7°

- The first step is to create a table to account for each base value, as well as 1 increment greater for each. Fill in known data.

Argument	Base Value	Z value	Next incremental Z	Difference in Z	Increment	Correction (Zdiff x Inc) / 60
Latitude	25°	78.7°				
Declination	**23°**	**78.7°**			**23.1'**	
LHA	300°	78.7°				
Total						

- Next, look up the Z value for each sequential increment of declination. In this case, the Z value for a declination of 24°, with an accompanying latitude of 25° and LHA of 300° is 77.5°. Calculate the difference between this value and the base value and then update the table as appropriate.

Argument	Base Value	Z value	Next incremental Z	Difference in Z	Increment	Correction (Zdiff x Inc) / 60
Latitude	25°	78.7°				
Declination	23°	78.7°	**77.5°**	**-1.2°**	23.1'	
LHA	300°	78.7°				
Total						

- Finally, multiply the declination increment (23.1') by the difference in Z (-1.2° because the values are decreasing), and divide by 60 to obtain a correction. Update the table as appropriate.

Argument	Base Value	Z value	Next incremental Z	Difference in Z	Increment	Correction (Zdiff x Inc) / 60
Latitude	25°	78.7°				
Declination	23°	78.7°	77.5°	-1.2°	23.1'	**-0.46°**
LHA	300°	78.7°				
Total						**-0.46°**

- In this case, the base Z value of 78.7° needs to be corrected by -0.46° for maximum accuracy – resulting in a new Z value of 78.2°.

Example Problem 5.1.8. Answer. **Z = 78.2°**

Of note, this tedious process can be memorized and completed relatively quickly without a table. However, the tabular method is helpful when completing future academic exercises or compass error problems. In these cases, you must often interpolate for each of latitude, declination, and LHA, rather than just declination as in this specific case.

A final reminder – this process resulted in a correction of less than 0.5°, which is nearly impossible to differentiate on a chart when plotting the final celestial solution. However, the technique is available.

Reference:

Bowditch, 2024 edition, Article 2016. https://msi.nga.mil/Publications/APN

Problem 5.1.9. Given the following base values, use an interpolation to refine the azimuth angle (Z) value to account for declination increment for maximum accuracy. As an option, follow up your work with a Zn calculation.

- Latitude 28° S
- Declination N 4° 22.0’
- LHA 289°
- Base Z value 102.8°

Problem 5.1.10. Given the following base values, use an interpolation to refine the azimuth angle (Z) value to account for declination increment for maximum accuracy. As an option, follow up your work with a Zn calculation.

- Latitude 28° N
- Declination N 0° 37.7’
- LHA 80°
- Base Z value 94.7°

Problem 5.1.9. Answer. Next incremental Z is 103.7. Correction to base Z is 0.33°. Final Z is 103.1°.
If calculated, Zn = 76.9° T

Problem 5.1.10. Answer. Next incremental Z is 93.8°. Correction to base Z is -0.57°. Final Z is 94.1°
If calculated, Zn = 265.9° T

Universal Plotting Sheets

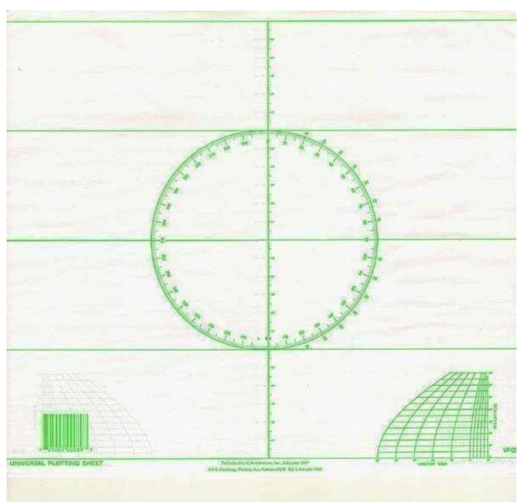
Part of the challenge with celestial navigation is that the distances from the GP to the AP are so vast, that it is difficult to plot on an appropriate scale chart.

The altitude-intercept method that we are learning in this course allows us to plot on a useful scale chart. In some cases, if we don't have a good chart for a region of the ocean, we can make our own using a universal plotting sheet (UPS).

Universal plotting sheets allow us to define a center latitude, plot the appropriate longitude values, and set the scale to one that is useful for our navigation goals.

Here are the steps to setting up a plotting sheet:

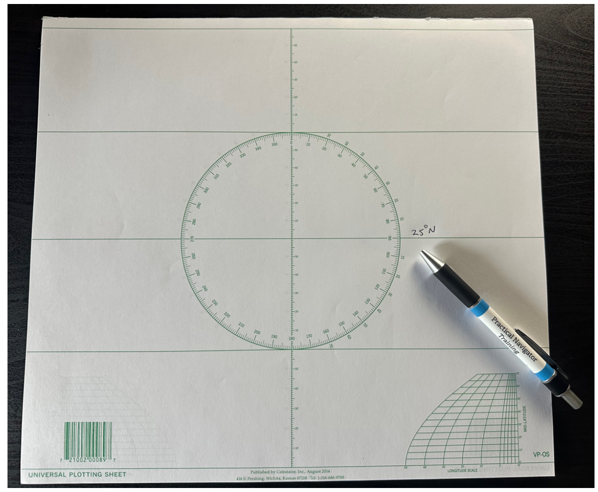
- Determine and label the latitude values.
- Determine the center longitude value.
- Measure and plot the nearby longitude values
  - You can use the value on center circle to reference a given longitude mark
  - You can also use the interpolation tool at the bottom right of the sheet
- Modify the sheet if necessary – for example you can zoom in or out on the sheet by setting values to 0.5x or 2x manually.



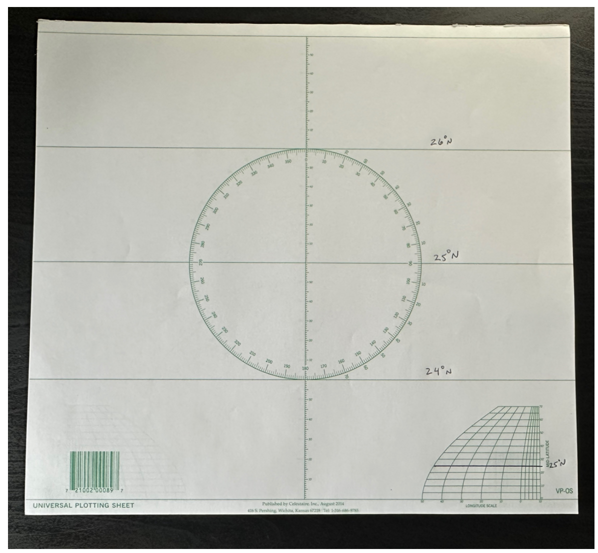
Reference:

Bowditch, 2024 edition, Article 522, 2903. https://msi.nga.mil/Publications/APN

Step 1: Plot the center latitude based on your desired scale for the UPS. Consider the direction of travel to allow yourself maximum "runway" if you are underway. In this case we chose 25° N.

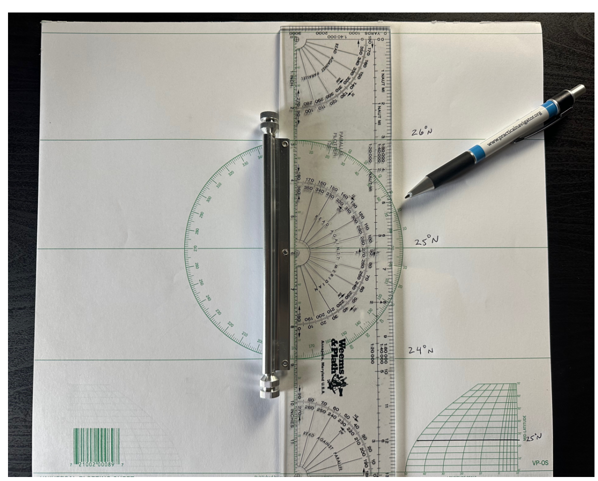


Step 2: Label the interpolation diagram in the bottom right. Choose the correct middle latitude and draw a line. This is where you will measure minutes of longitude when plotting. Label the remaining latitude lines above and below your middle latitude. For southern hemisphere plots, you may also cross out and re-label the minutes of latitude, so they represent southerly latitudes.

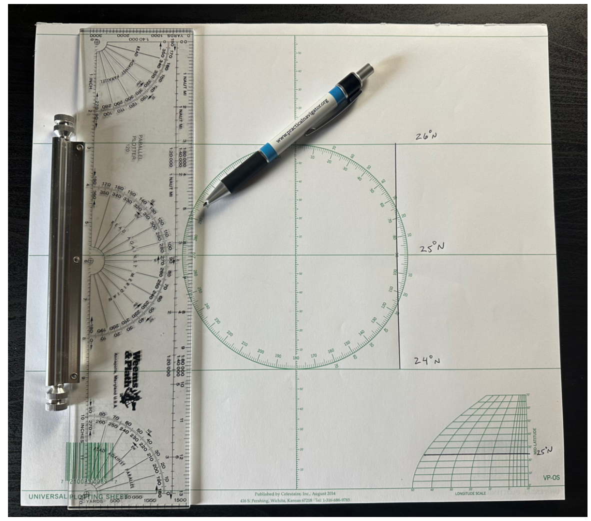


Step 3: Note the middle latitude and choose the same value on the outer ring on the right side of the plotting sheet. In this case, 25°N is the middle latitude, so we make marks at 25°. Then, use a parallel rule or triangle to connect the two marks to represent the longitude.

Alternatively, you can use the interpolation diagram at the bottom right, measure 60' along the 25° line, and use that to represent the longitude interval.

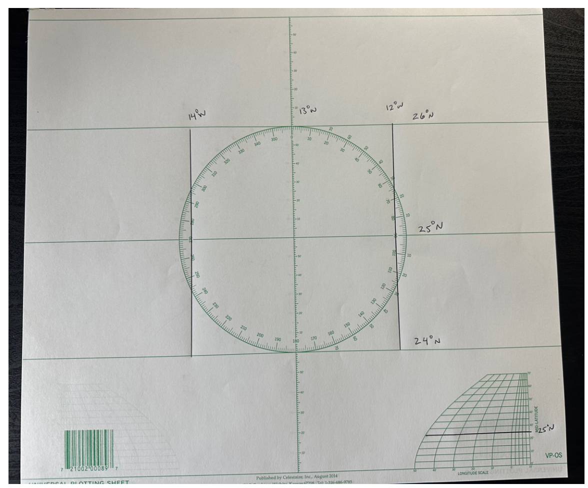


Step 4: Repeat this process on the left side of the plotting sheet. The numbers represented on the outer ring will be different, but the interval should be the same (e.g. 25 units above and below the middle latitude. Draw the longitude line.



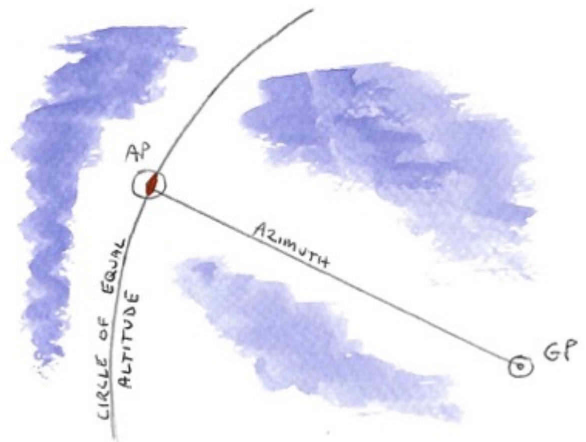
Step 5: Finish labelling the plotting sheet with longitude labels. In the eastern hemisphere, remember that longitude increases to the right. In the southern hemisphere, remember that latitude increases towards the bottom.

Alternatively, you can set the scale of the plotting sheet to a different unit. For instance, in this example, we set each latitude mark to equal one degree. You could choose to represent 5 degrees per mark, or 0.5 degrees per mark. If doing so, you must also change the longitude minutes on the interpolation diagram, and the latitude minutes along the top to bottom center line.



With practice, the universal plotting sheet is a great aid to plotting celestial lines of position, ocean tracks, and running fixes.

Plotting the Celestial Line of Position



Plotting the line of position is the last task in our 5-step sight reduction process.

To practice this skill, you need a height observed (Ho) from step 1, an assumed position (AP) from step 3, and a computed height (Hc) and azimuth (Zn) from step 4.

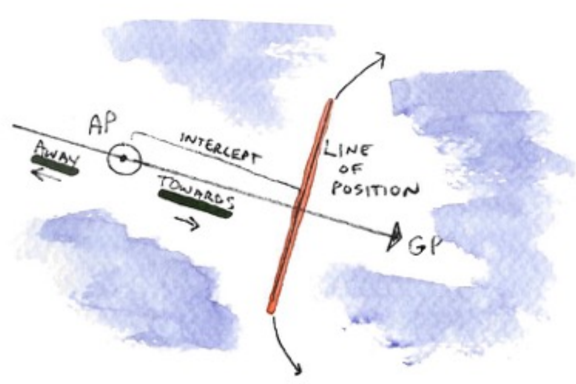
- Plot the assumed position (AP). It will generally be on a whole value of latitude, and a longitude nearby to your actual DR. This was deduced in step 3 of the sight reduction process.

- Compare Ho to Hc and determine the intercept and the direction, as discussed earlier in step 5 of the sight reduction process. The direction will either be towards the GP, or away from the GP.

- Plot the azimuth line from the AP either towards the GP (e.g. if the azimuth is 140° T, plot a line from the AP towards 140° T), or away (reciprocal) from the GP (e.g. if the azimuth line is 140° T, plot a line towards the reciprocal 320° T).

- From the AP, measure the distance in nautical miles equal to the intercept (the difference between Hc and Ho). Make a mark on the azimuth/reciprocal line.

- Draw a perpendicular line to the azimuth line.

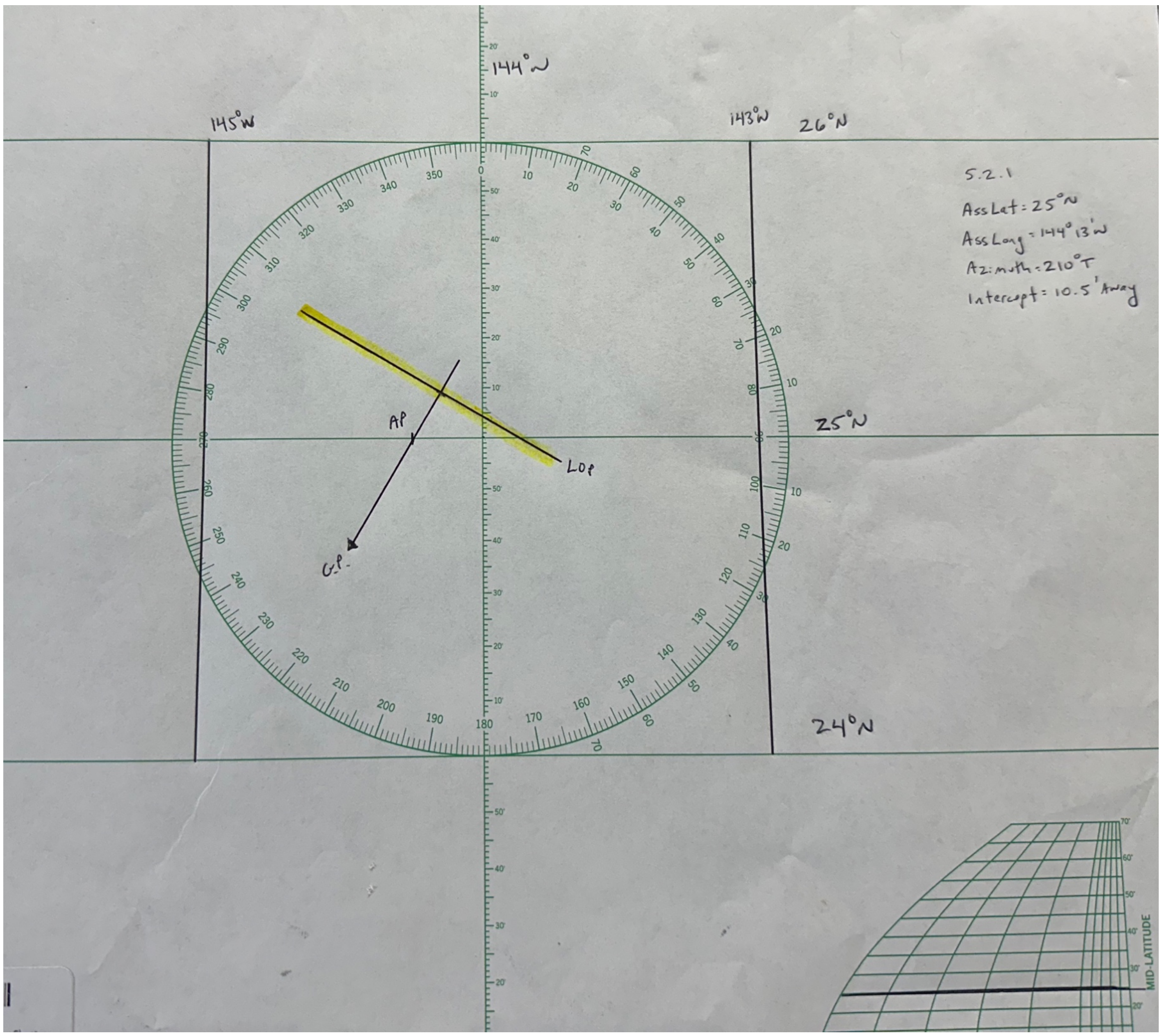


This line of position is a tangent to the large circle of equal altitude surrounding the GP of the celestial body. That circle likely extends for thousands of miles – by plotting this tangent to the circle of equal altitude on a useful scale chart, you've completed the altitude/intercept method of determining a line of position from a celestial object.

It is merely one piece of information used in navigation – later we will learn to plot multiple lines of position or advance this line of position to generate a running fix. But your first line of position based on a celestial observation is a significant achievement!

Example Problem 5.2.1. You have previously determined the following values for a celestial sight. Set up a Universal Plotting Sheet and plot the LOP.

- Assumed Latitude 25° N
- Assumed Longitude 144° 13’W
- Azimuth 210° T
- Intercept 10.5’ Away



Reference:

Bowditch, 2024 edition, Article 2003. https://msi.nga.mil/Publications/APN

Problem 5.2.2. Problem 5.2.2. You have previously determined the following values for a celestial sight. Set up a Universal Plotting Sheet and plot the LOP.

- Assumed Latitude 17° N
- Assumed Longitude 26° 27’E
- Azimuth 143° T
- Intercept 6.8’ Towards

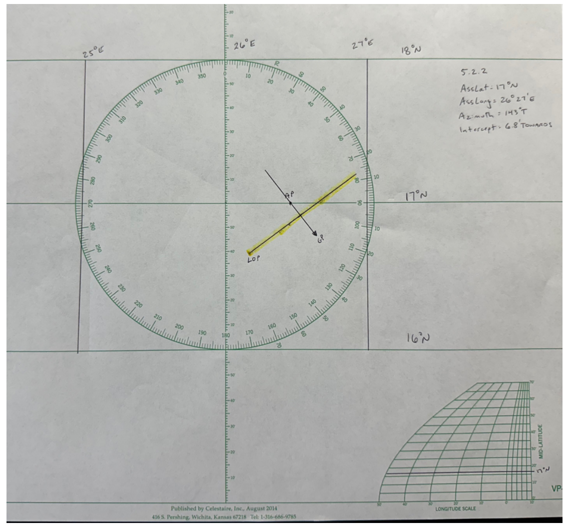
Problem 5.2.3. You have previously determined the following values for a celestial sight. Set up a Universal Plotting Sheet and plot the LOP.

- Assumed Latitude 22° N
- Assumed Longitude 47° 05.6’W
- Azimuth 085° T
- Intercept 21.4’ Towards

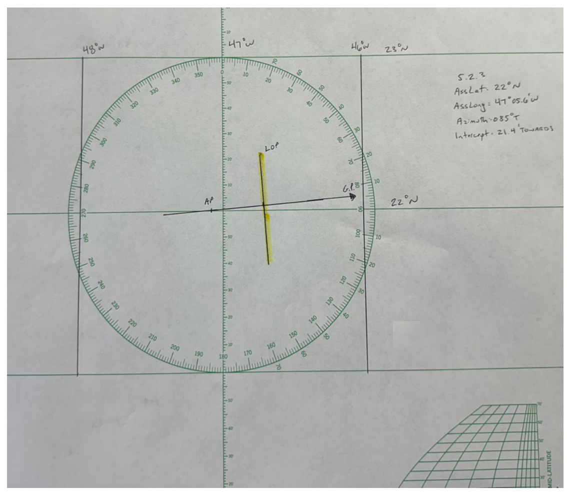
Problem 5.2.4. You have previously determined the following values for a celestial sight. Set up a Universal Plotting Sheet and plot the LOP.

- Assumed Latitude 15° S
- Assumed Longitude 126° 13’ W
- Azimuth 261° T
- Intercept 8.4’ Away

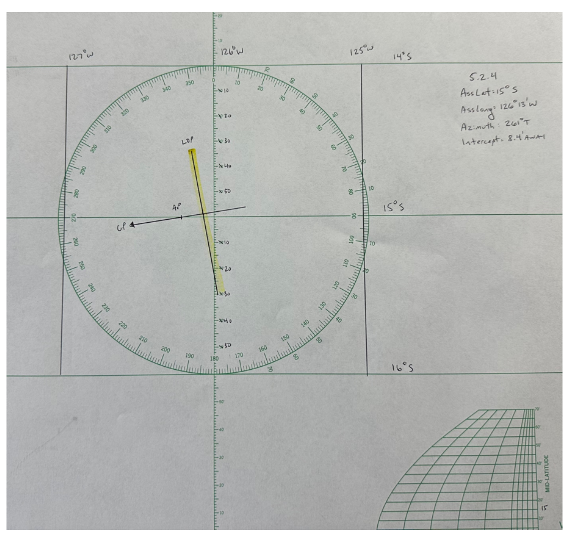
Problem 5.2.2. Answer.



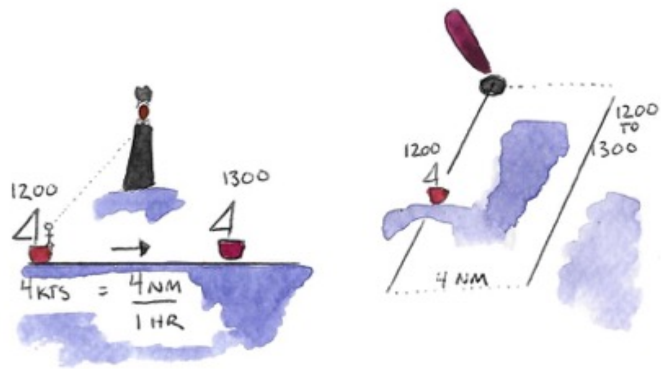
Problem 5.2.3. Answer.



Problem 5.2.4. Answer.



Running Fixes



In marine navigation, a line of position represents a line drawn on a chart, typically originating from some known object, which represents all possible locations for your ship.

In other words, you have a known bearing to an object and therefore must be somewhere along that line of position.

If you had two or more lines of position, you could identify exactly where you are, by crossing the bearings to obtain a **fix**.

One way to determine a useful position without multiple crossed bearings, is to use the same object at different times – and then advance or retard one of the lines of position to generate a **running fix.**

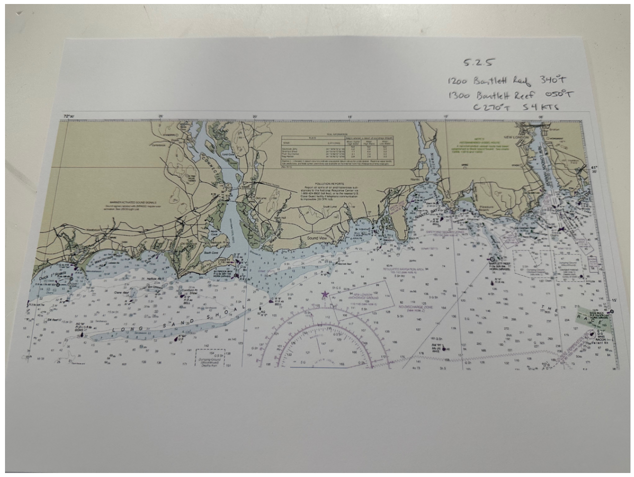
The running fix is not as valuable as a traditional crossed bearings fix but is still better than an estimated position or dead reckoning plot.

To generate a running fix:

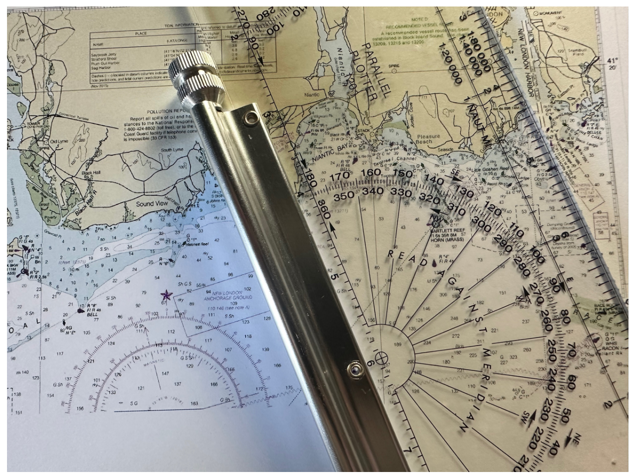
- Draw a line of position (LOP #1) from an object. Label it with the time observed.
- Draw a second line of position (LOP #2) from an object later. Label it with the time observed.
- Pick a point on the original line of position (LOP #1).
- Advance that point in the direction of travel, for a distance equal to the speed of advance divided by the time elapsed between lines of position. Mark a new point.
  - For example, if the first LOP was taken at 1200, and the second at 1230, and the speed of advance was 10 knots on a course of 120° T, advance the first LOP 5 miles in a direction of 120° T.
- Parallel the first LOP to the new point you just drew and draw a parallel line to the first LOP. This represents the advanced original LOP.
- Label the location where the two LOPs cross as a "RFIX" with the appropriate time.

This general process is helpful for terrestrial objects but is equally useful for celestial running fixes.

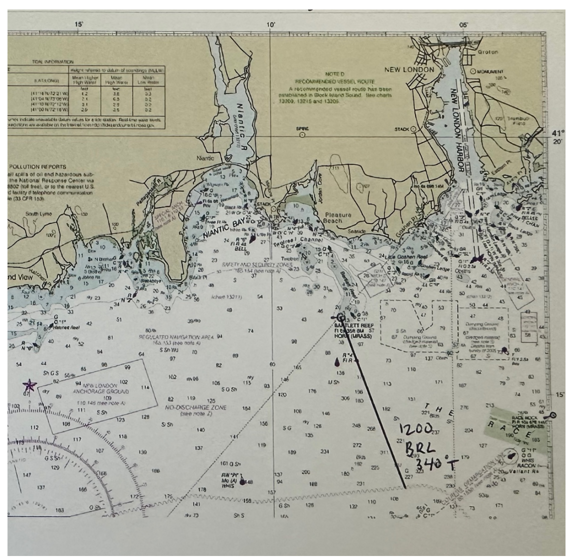
Example Problem 5.2.5. You are underway in Long Island Sound. Reference the chartlet included in the lesson. Your course is 270° T and your speed is 4 knots. At 1200 you sight Bartlett Reef Light at 340° T. At 1300 you sight Bartlett Reef Light at 050° T. What is your running fix position?



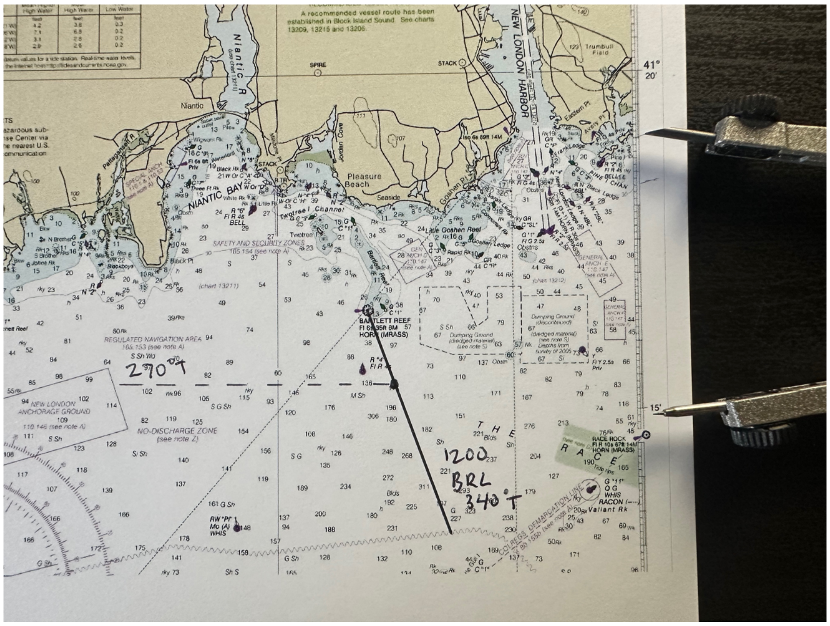
Step 1: Use plotting tools to measure a bearing of 340° T. Triangles or rolling rulers using the meridian tool is a great way to do this, but instruction is outside the scope of this course.

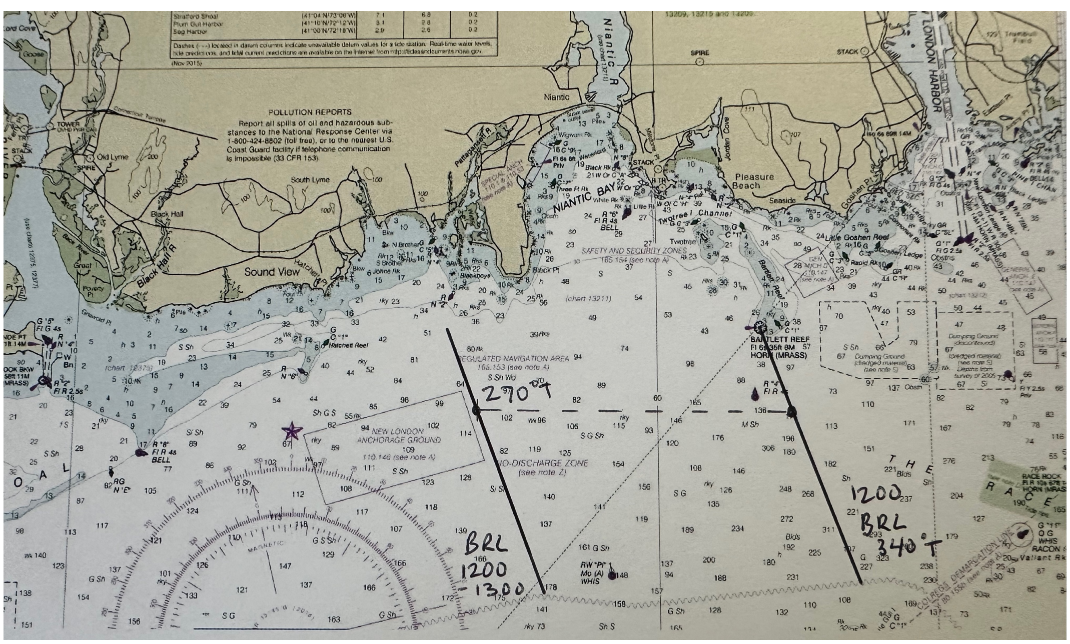


Step 2: Plot a bearing of 340° T from Bartlett Reef Light.

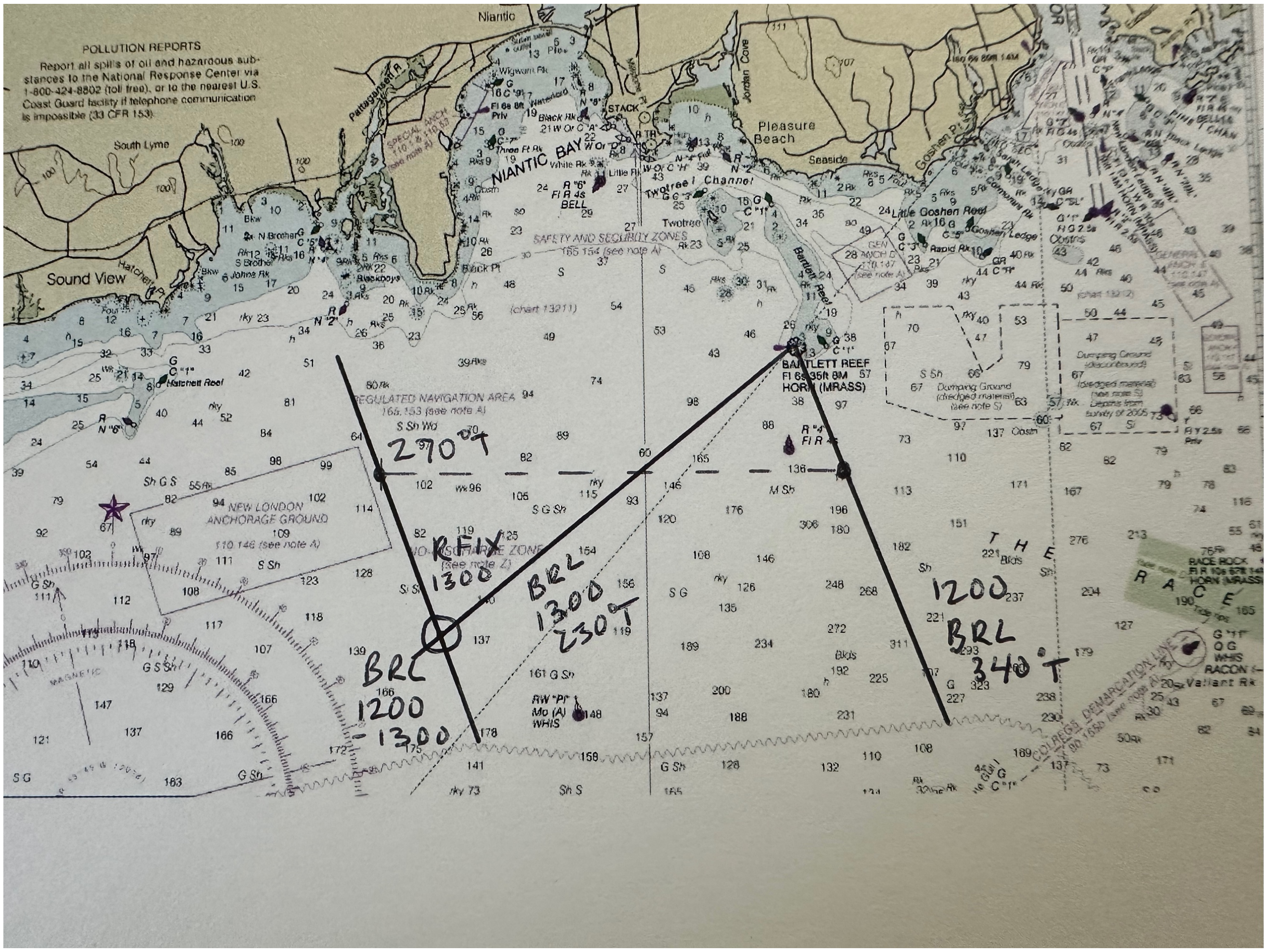


Step 3: Advance the 1200 LOP to 1300. Note that you can alternatively plot the 1300 LOP first, but for clarity of instruction we are advancing the 1200 LOP first. Pick a point on the 1200 LOP. Plot 270° T for 4 nautical miles from that position. Dashed lines are drawn for clarity. Parallel the 1200 LOP to the new position and label it.





Step 4: Plot the 1300 LOP of 050° T from Bartlett Reef Light. The location where the 1300 LOP and the advanced 1200-1300 LOP cross is the running fix position for 1300.



Example Problem 5.2.5. Answer. 41° 14.0'N, 72° 12.4'W. Answers within 0.3nm are acceptable.

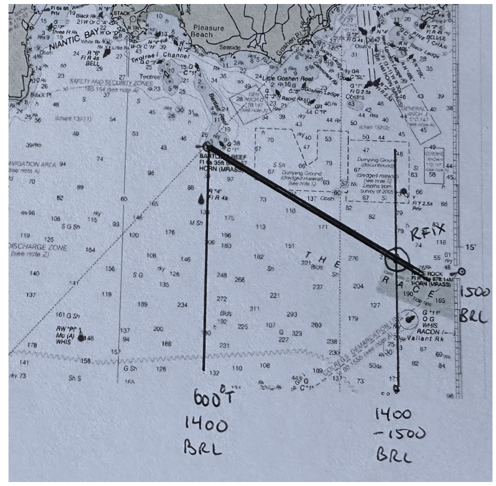
Reference:

Bowditch, 2024 edition, Article 1112, 2003. https://msi.nga.mil/Publications/APN

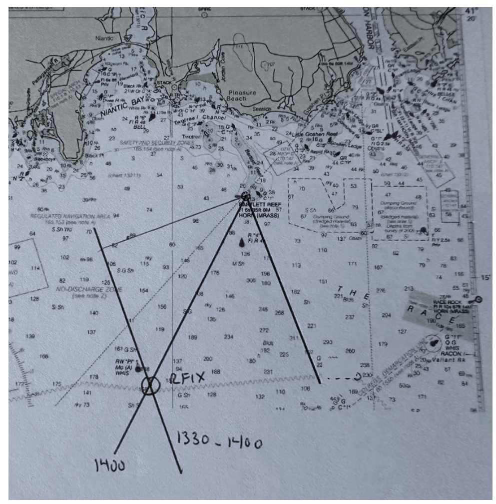
Problem 5.2.6. You are underway in Long Island Sound on course 090° T at 3 knots. You sight Bartlett Reef Light at 1400 at 000° T. At 1500, Bartlett Reef Light bears 300° T. What is your 1500 Running Fix Location?

Problem 5.2.7. While sailing in Long Island Sound, you sight Bartlett Reef Light at 1330 bearing 340° T. 30 minutes later, at 1400, you sight Barlett Reef Light at 033° T. Your course and speed during the 30-minute interval were 250° T at 5.5 knots.

Problem 5.2.6. Answer. 41° 14.8’N, 072° 04.2’W. Answers within 0.3nm are acceptable.



Problem 5.2.7. Answer. 41° 13.0’N, 072° 10.5’W. Answers within 0.3nm are acceptable. You should have advanced the 1330 LOP 2.75nm in the direction of 250° T to account for the 30-minute time interval.



Plotting the Celestial Running Fix

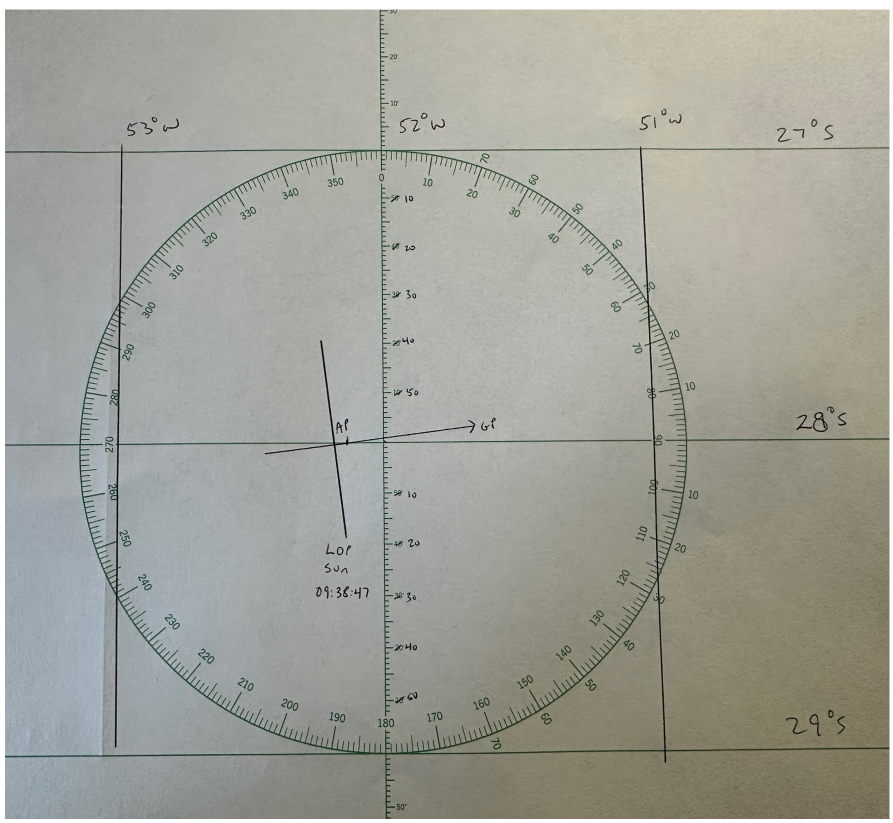
When plotting a celestial running fix, the same principles apply as when navigating terrestrially (so it’s a good idea to get the fundamentals down first).

However, with celestial lines of position, there tend to be several extra lines that can clutter up the plotting sheet, so labelling it is extremely important.

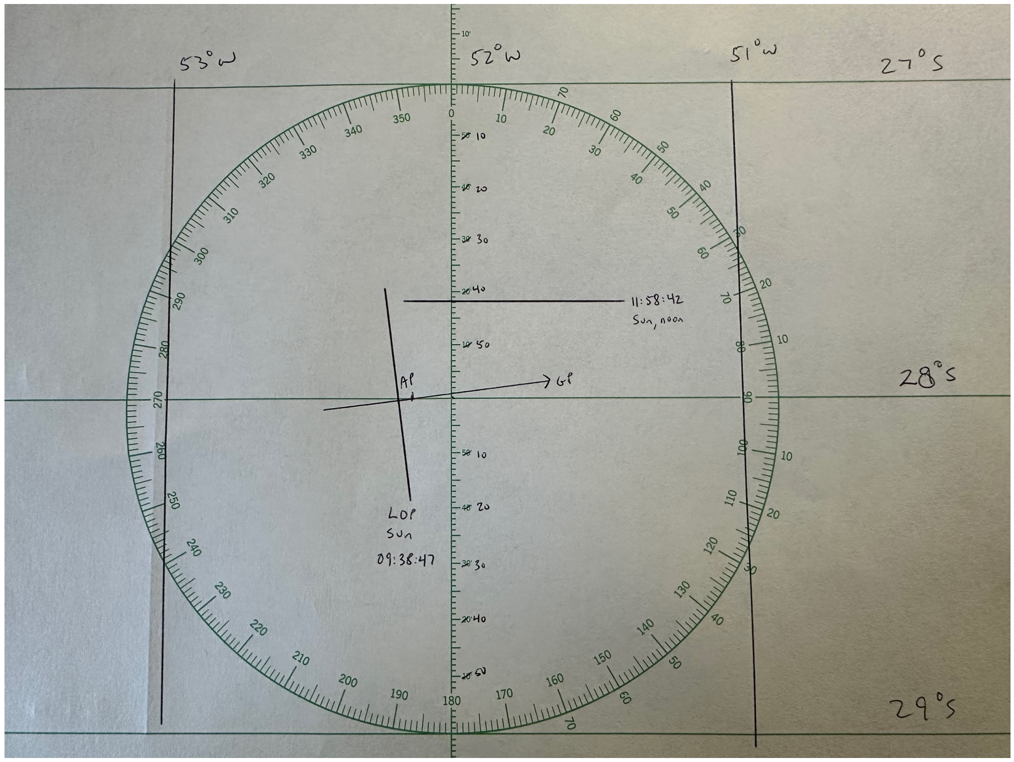
Traditionally, a morning sunline is advanced to a noon latitude, and then a noon latitude is advanced to an afternoon sunline. The reason is that as the sun progresses across the sky during the day, the celestial line of position will change its relative angle – so it is important to let several hours pass between sights used for running fixes of the same celestial body.

Example Problem 5.2.8. At 09:38:47, you observe the sun. The AP for the sight is 28° S, 52° 08.2’W. The azimuth is 082.8’ and the intercept is 2.1’away. Later, at 11:58:42, you observe the noon sight for latitude and determine your latitude to be 27° 42.1’S. During the interval between sights, you were travelling on course 036° T at 19 knots. What is the running fix position at 11:58:42?

Step 1: Set up the UPS, plot the AP, the azimuth, the intercept, and the LOP for the morning sunline at 09:38:47.

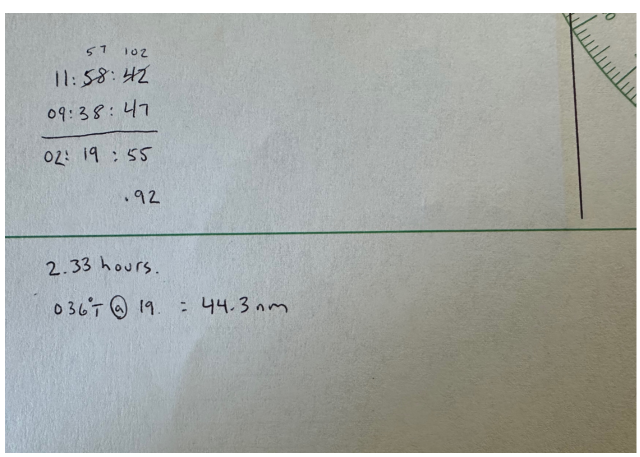


Step 2: Plot the noon sight for latitude at 11:58:42.



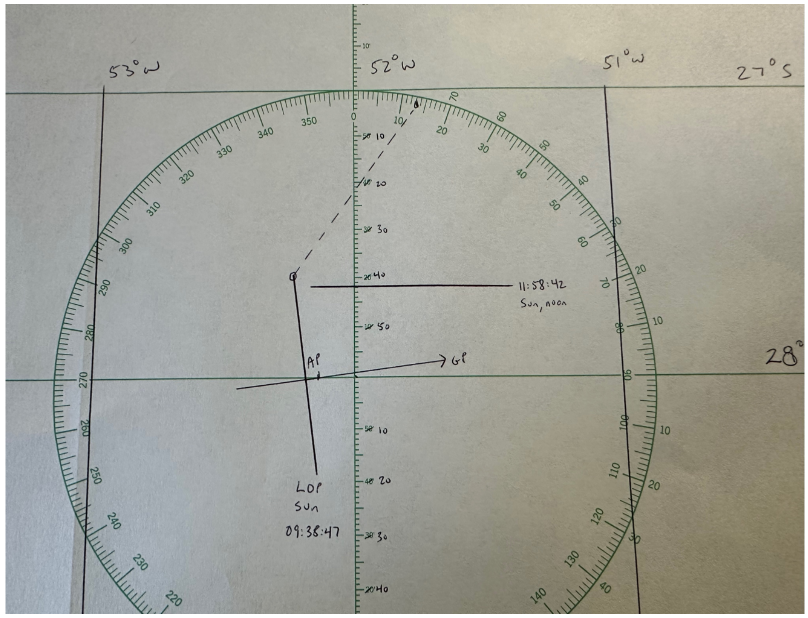
Step 3: Complete the running fix speed/distance/time calculation.

- The time from morning sunline to noon sight is 02:19:55.
- This is equal to 2.33 hours
    - 55/60 seconds is 0.92 minute, and 19.92/60 minutes is 2.33 hours.
- 2.33 hours at 19 knots is 44.3 nautical miles to advance the LOP.

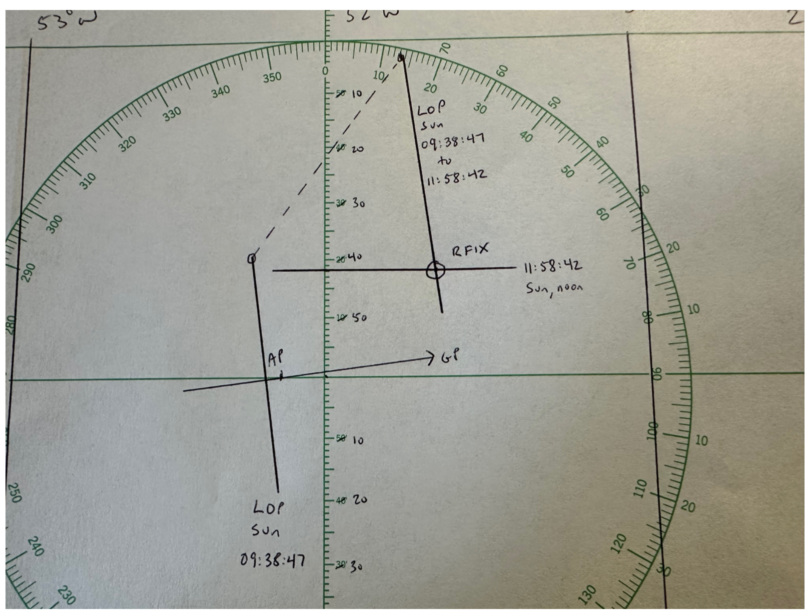


Step 4: Advance the morning sunline.

- Pick a point on the original LOP (we chose the top in this case).
- Draw a dashed line in the direction of 036° T for 44.3nm. You can omit the dashed line if you wish, it is presented here for illustration purposes.
- Parallel the original morning sunline to the new location and label it as having been advanced.



Example Problem 5.2.8. Answer. 27° 42.1’S, 51° 38.2’W. Longitude answers within 5nm are acceptable.



Reference:

Bowditch, 2024 edition, Article 2003. https://msi.nga.mil/Publications/APN

Problem 5.2.9. You are underway on course 280° T at 12 knots. What is your 1200 running fix position given the following celestial sights?

- At 0900 you observe the sun:
  - AP 12° N, 125° 10.0' W
  - Zn = 120° T
  - Int = 6.5' Towards

- At 1200 you observe the noon site for latitude with a result of 12° 10.0'N.

Problem 5.2.10. While underway in the North Pacific at 0930 you sight the Moon with the below celestial data. Later, at 1230, you observe the sun for a noon latitude with a result of 56° 25.0' N. What is your 1230 running fix position if you are travelling 295° T at 8 knots?

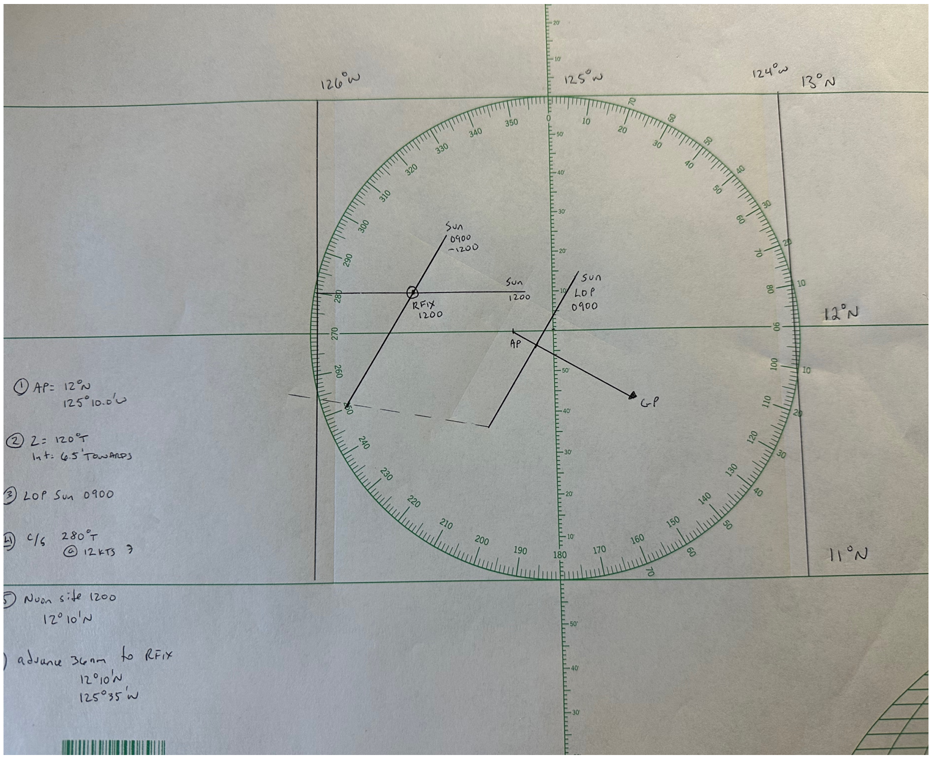
- Moon, 0930
  - AP = 56° N, 166° 12.5' E
  - Zn = 221.5° T
  - Int = 23.6' Away

Problem 5.2.11. You are en route St. Helena Island in the South Atlantic on course 315° T at 6 knots. It is partly cloudy and conditions for celestial navigation are marginal. You manage to get a sight of a star at 0435 and reduce it as below. At 0917, you can get a sight of the moon and reduce it as below. What is your 0917 running fix position?

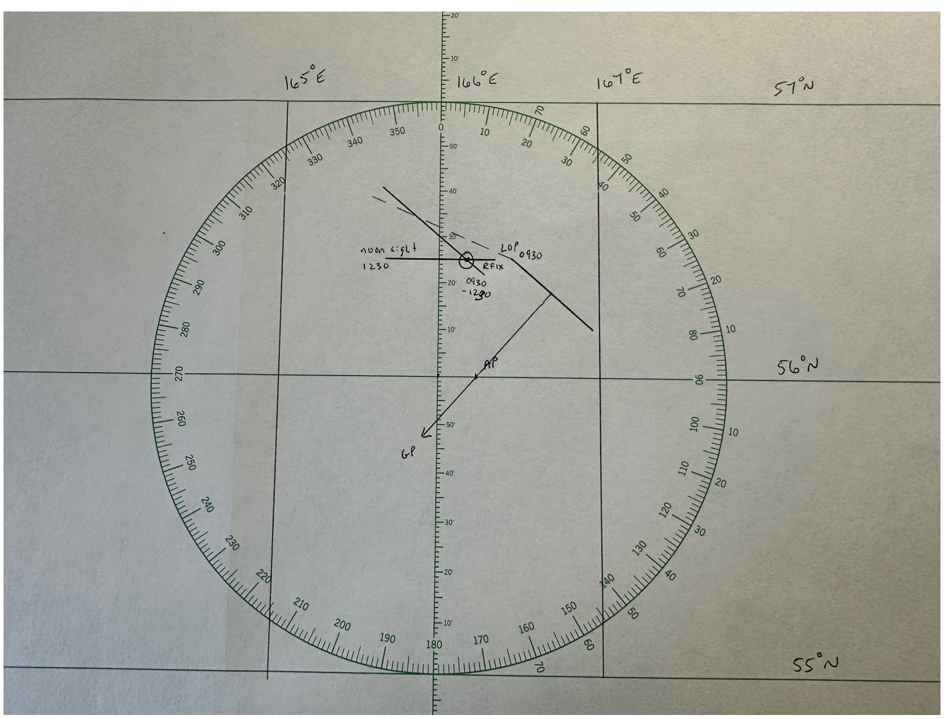
- Star, 0435
  - AP = 16° S, 3° 12.0'E
  - Zn = 121° T
  - Int = 16.5' Towards

- Moon, 0917
  - AP = 15° S, 3° 17.0'E (Note the difference in latitude from AP1)
  - Zn = 220° T
  - Int = 26.0' Towards

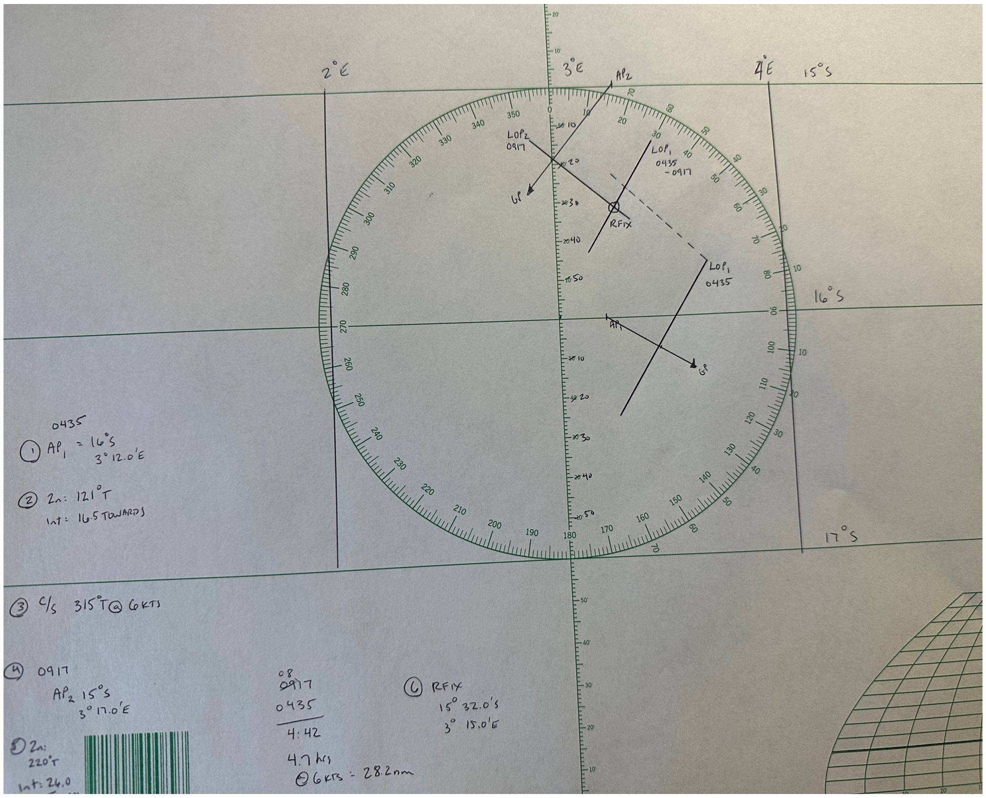
Problem 5.2.9. Answer. 12° 10.0’N, 125° 35.0’W. Answers within 5 nm of longitude are acceptable.



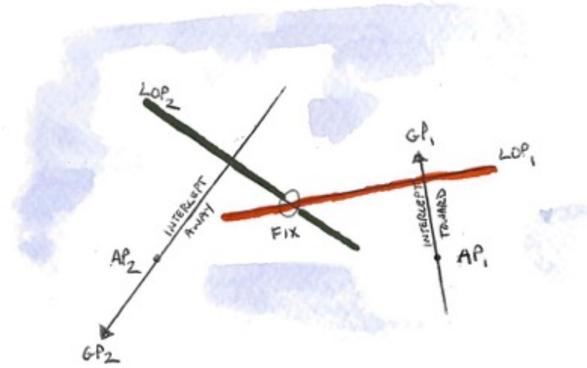
Problem 5.2.10. Answer. 56° 25.0’N, 166° 11.0’E. Answers within 5 nm of longitude are acceptable.



Problem 5.2.11. Answer. 15° 32’S, 3° 15.0’E. Answers within 5 nm of longitude are acceptable.



Plotting the Three Body Fix



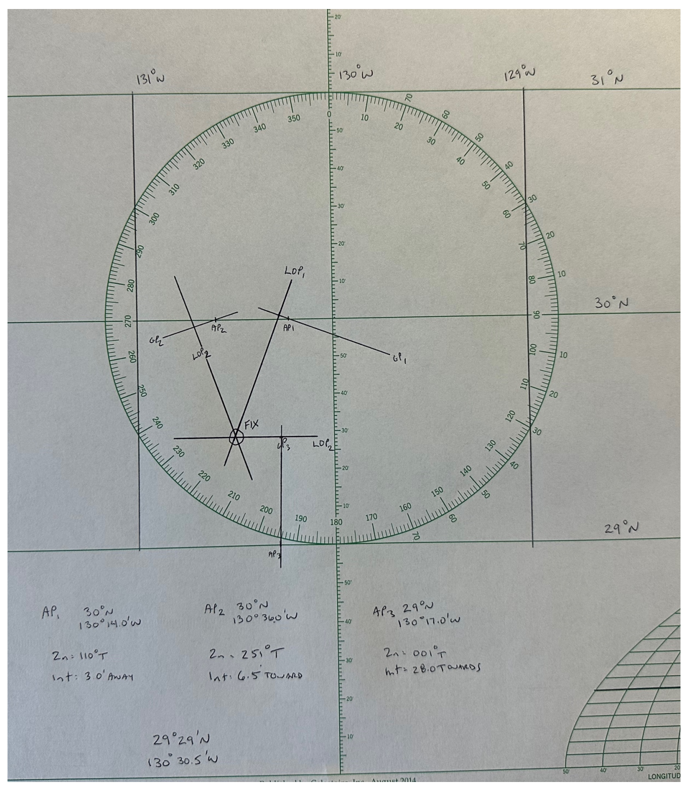
One of the best feelings in celestial navigation is observing three celestial bodies in rapid succession, reducing their sights through the 5-step process, and plotting a pinpoint fix on a plotting sheet – proudly exclaiming to the world: "here I am!"

This achievement marks the culmination of the course, and proves that you can reduce multiple objects successfully, and plot them on a cluttered plotting sheet to determine your exact position.

A word about accuracy – as you develop your skills, plotting objects that cross within 3 nm of your actual position is considered perfectly acceptable for ocean navigation, and 0.1 nm would be the gold standard. Strive for these marks as you grow in celestial navigation.

The steps to plotting a three-body fix are the same as plotting individual bodies – but the key is to keep organized so the plotting sheet doesn't get out of control.

Example Problem 5.2.12. You are underway in the North Pacific and take a round of sights as described below. What is your fix position?



- Star 1
  - AP = 30° N, 130° 14' W
  - Zn = 110° T
  - Int = 3.0' Away
- Star 2
  - AP = 30° N, 130° 36'W
  - Zn = 251° T
  - Int = 6.5' Towards
- Star 3
  - AP = 29° N, 130° 17'W
  - Zn = 001° T
  - Int = 28.0 Towards

Plot each sight in turn and be careful to avoid common errors such as incorrect AP latitudes, incorrect towards/away plots, and incorrect measurements. Another best practice is to keep lines relatively short and only extend them when it makes sense.

Example Problem 5.2.12. Answer. 29° 29' N, 130° 30.5' W. Answers within 5 nm are acceptable.

Reference:

Bowditch, 2024 edition, Article 2003, 2021. https://msi.nga.mil/Publications/APN

Problem 5.2.13. You shoot a round of three celestial sights with the information listed below. What is your fix position?

- Star 1
  - AP = 23° S, 111° 19.8'W
  - Zn = 045° T
  - Int = 5.1' Towards

- Star 2
  - AP = 23° S, 110° 58.0'W
  - Zn = 096° T
  - Int = 7.9' Away

- Star 3
  - AP = 23° S, 111° 27.8' W
  - Zn = 180° T
  - Int = 5.1' Toward

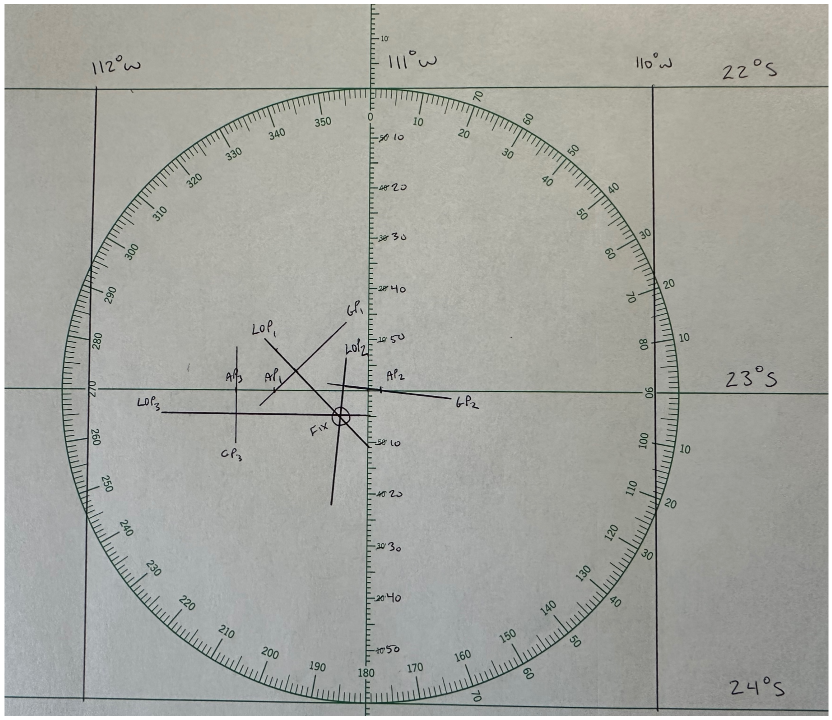
Problem 5.2.14. After shooting three celestial bodies and reducing them, what is your celestial fix?

- Star 1
  - AP = 26° N, 134° 26.2' E
  - Zn = 071.5° T
  - Int = 19.0' Towards

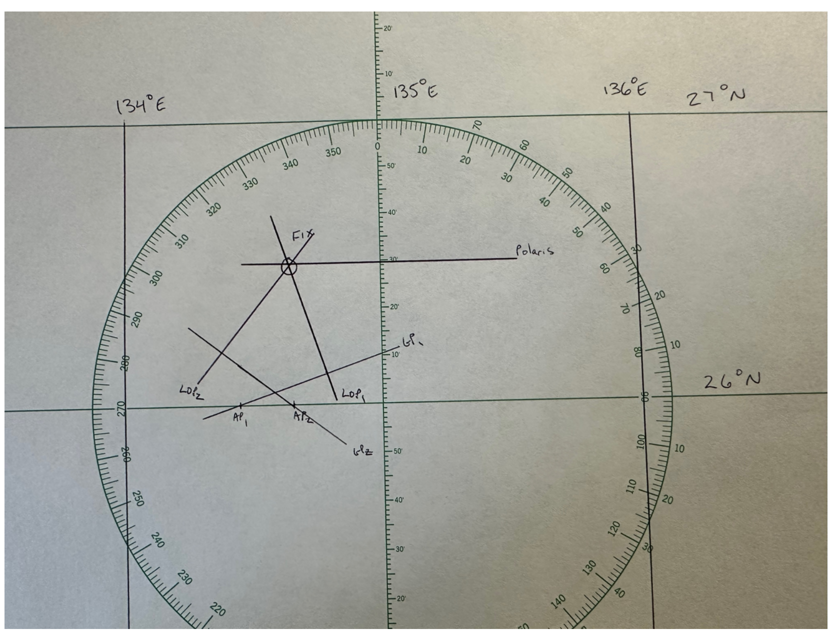
- Star 2
  - AP = 26° N, 134° 38.3' E
  - Zn = 127° T
  - Int = 18.5' Away

- Star 3 (Polaris)
  - Latitude = 26° 29.3'N

Problem 5.2.13. Answer. 23° 05'S, 111° 06'W. Answers within 5 nm are acceptable.



Problem 5.2.14. Answer. 26° 29.3' N, 134° 37.1' E. Answers within 5 nm are acceptable.



Section Introduction

In this section, we'll present example full sight reductions and extra practice problems. If you get stuck on a particular area, be sure to go back and review the lessons for that step in the process.

Sight Reduction of the Sun Example

Problem 6.1.1. At 09:15:36 local zone time on 10 January, you observe the sun's lower limb. The sextant height is 23° 00.7'. The index error is 2.6' off the arc. The height of eye is 55 feet. Your DR position at the time of sight is 25° 47.8'N, 124° 33.6'W. Reduce this sight and plot the resultant LOP on a Universal Plotting Sheet.

- Step 1: Observe and correct Hs to Ho.

  - Height of Sextant (Hs) = 23° 00.7'
    - IE = +2.6'
    - Dip = -7.2' (55 ft HOE)
    - Height Apparent (Ha) = 22° 56.1'
    - Apparent Altitude Correction = +14.0 (normal conditions)

  - **Height Observed (Ho) = 23° 10.1'**

- Step 2: Determine the GP of the Sun at the time of sight.

  - Correct zone time to GMT.
    - 09:15:36 zone time.
    - DR longitude is 124° 33.6', which correlates to (ZD +8)
    - 09:15:36 + 8 = **17:15:36, 10 January**

  - Determine declination of the sun for the time of sight.
    - Declination (hourly) = S 21° 53.6' (d = 0.4)
    - Declination (increments) = -0.1'
    - **Declination (total) = S 21° 53.5'**

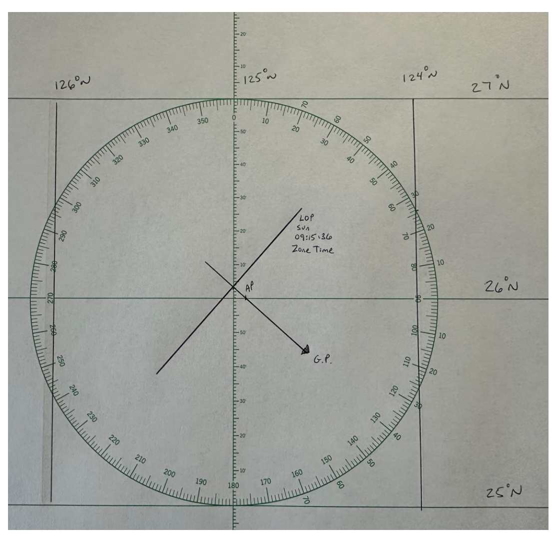
  - Determine the GHA of the sun for the time of sight.
    - GHA (hourly) = 73° 04.5'
    - GHA (increment) = 3° 54.0'
    - **GHA (total) = 76° 58.5'**

- Step 3: Determine the entering arguments for HO229.

  - Determine the AP and LHA.
    - **AP Latitude = 26° N.**
    - LHA = GHA – W Longitude
    - LHA = 76° 58.5' (+360) = 436° 58.5' – 124° 58.5' = 312°
    - **AP Longitude = 124° 58.5'** (chosen to make LHA whole number)

  - Entering arguments for HO229:
    - LHA = 312° (Contrary Pages)
    - Declination = S 21° (increment = 53.5')
    - Latitude = 26° N

- Step 4: Solve the spherical triangle using HO229.
    - Retrieve base values from HO229.
        - Computed height (Hc) = 23° 51.1’
        - Altitude difference (d) = -41.1
        - Azimuth Angle (Z) = 130.7’
    - Correct Hc for declination increment using altitude difference (d).
        - Altitude difference (d) = -41.1
        - Declination increment = 53.5’
        - Correction = -41.1 x (53.5/60) = -36.6’
        - Hc = 23° 51.1’
        - Correction = -36.6’
        - **Hc = 23° 14.5’**
    - Correct azimuth angle for declination increment (if desired) and rules.
        - Z = 130.7°
        - Declination increment = 53.5’
        - Next tabular Z = 131.5°
        - Z difference = + 0.8°
        - Correction = 0.8 x (53.5/60) = + 0.7°
        - Corrected azimuth angle = 131.4° (this correction can usually be omitted)
        - Azimuth rules: Lat - N, if LHA is greater than 180°, Zn = Z.
        - **Azimuth = 131.4° T**
- Step 5: Compare calculations and observations to plot the LOP.
    - Determine the Intercept.
        - Hc = 23° 14.5’
        - Ho = 23° 10.1’
        - **Intercept = 4.4’ Away**
    - Determine the plotting data.
        - AP = 26° N, 124° 58.5’ W
        - Azimuth = 131.4° T
        - Intercept = 4.4’ Away
    - Plot the LOP.



Sight Reduction of the Moon Example

Problem 6.1.2. On 25 February at 06:22:06 zone time, you observe the moon's upper limb with a sextant height of 59° 58.6'. The height of eye is 59 feet, and the index error is 2.5' on the arc. Your DR position is 30° 28.3' S, 102° 39.3' E. Reduce this sight and plot the resultant LOP on a Universal Plotting Sheet.

- Step 1: Observe and correct Hs to Ho.
  - Height of Sextant (Hs) = 59° 58.6
    - IE = -2.5'
    - Dip = -7.5' (59 ft HOE)
    - Height Apparent (Ha) = 59° 48.6'
  - Determine the horizontal parallax.
    - HP = 54.2'
  - Determine the altitude correction.
    - Apparent altitude correction #1 = +39.1'
    - Upper limb correction = -30.0'
    - Apparent altitude correction #2 = +2.7'
    - Total correction = +11.8'
  - **Height Observed (Ho) = 60° 00.4'**
- Step 2: Determine the GP of the moon at the time of sight.
  - Correct zone time to GMT.
    - 06:22:06 zone time.
    - DR longitude is 102° 39.3' E, which correlates to (ZD -7)
    - 06:22:06 - 7 = **23:22:06 GMT, 24 February**
  - Determine declination of the moon for the time of sight.
    - Declination (hourly) = S 10° 52.8' (d = 8.8)
    - Declination (increments) = + 3.3'
    - **Declination (total) = S 10° 56.1'**
  - Determine the GHA of the moon for the time of sight.
    - GHA (hourly) = 277° 10.3' (v = 14.8)
    - GHA (increment) = 5° 16.4'
    - GHA (v correction) = 5.6'
    - **GHA (total) = 282° 32.3'**

- Step 3: Determine the entering arguments for HO229.

    - Determine the AP and LHA.
        - **AP Latitude = 30° S**
        - LHA = GHA + E Longitude
        - **LHA =** 282° 32.3 + 102° 27.7' E - (360°) = **25°**
        - **AP Longitude = 102° 27.7'** (chosen to make LHA whole number)

    - Entering arguments for HO229:
        - LHA = 25° (Same pages)
        - Declination = S 10° (increment = 56.1')
        - Latitude = 30° S

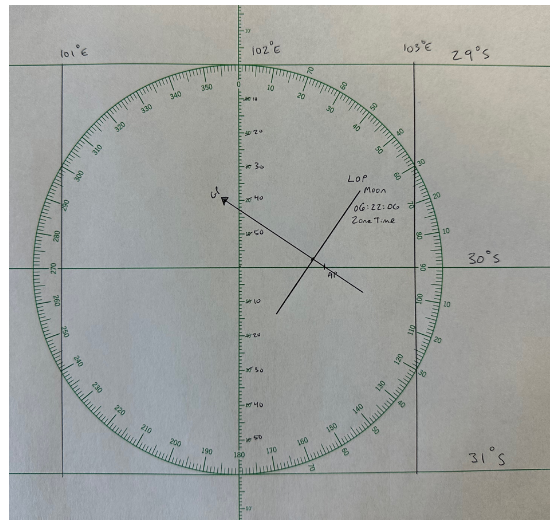
- Step 4: Solve the spherical triangle using HO229.

    - Retrieve base values from HO229.
        - Computed height (Hc) = 59° 17.6'
        - Altitude difference (d) = +41.3
        - Azimuth Angle (Z) = 125.4°

    - Correct Hc for declination increment using altitude difference (d).
        - Altitude difference (d) = +41.3
        - Declination increment = 56.1'
        - Correction = +38.6' (d x inc/60)
        - Hc = 59° 17.6'
        - Correction = +38.6'
        - **Hc = 59° 56.2'**

    - Correct azimuth angle for declination increment (if desired) and rules.
        - Z = 125.4°
        - Declination increment = 56.1'
        - Next tabular Z = 124.0°
        - Z difference = -1.4°
        - Correction = -1.3° (diff x inc/60)
        - Corrected azimuth angle = 124.1° (this correction can usually be omitted)
        - Azimuth rules: Lat S, - if LHA less than 180°, Zn = 180° + Z
        - **Azimuth = 304.1° T**

- Step 5: Compare calculations and observations to plot the LOP.
    - Determine the Intercept.
        - Hc = 59° 56.2’
        - Ho = 60° 00.4’
        - **Intercept = 4.2’ Towards**
    - Determine the plotting data.
        - AP = 30° S, 102° 27.7’ E
        - Azimuth = 304.1° T
        - Intercept = 4.2’ Towards
    - Plot the LOP.

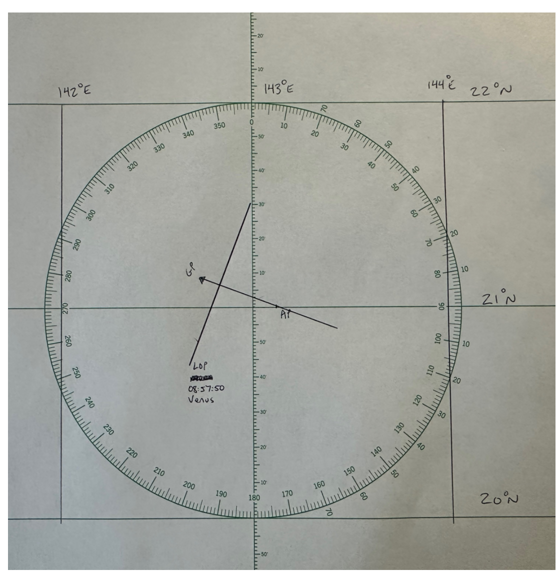


Sight Reduction of a Planet Example

Problem 6.1.3. On 25 May, at 08:57:50 GMT, you observe Venus at a height of 12° 53.4'. The index error is 4.5' off the arc and the height of eye is 55 feet. Your DR position is 21° 05' N, 143° 27' E. Reduce this sight and plot the resultant LOP on a Universal Plotting Sheet.

- Step 1: Observe and correct Hs to Ho.
    - Height of Sextant (Hs) = 12° 53.4'
        - IE = +4.5'
        - Dip = -7.2' (55 ft HOE)
        - Height Apparent (Ha) = 12° 50.7'
        - Apparent Altitude Correction = -4.2'
        - Additional Venus Correction = +0.1'
    - **Height Observed (Ho) = 12° 46.6'**
- Step 2: Determine the GP of Venus at the time of sight.
    - Correct zone time to GMT.
        - 08:57:50 is given as GMT.
    - Determine declination of the Venus for the time of sight.
        - Declination (hourly) = N 23° 12.8' (d = +0.4)
        - Declination (increments) = +0.4'
        - **Declination (total) = N 23° 13.2'**
    - Determine the GHA of the Venus for the time of sight.
        - GHA (hourly) = 287° 25.7' (v = -0.9)
        - GHA (increment) = 14° 27.5'
        - GHA (v correction) = -0.9'
        - **GHA (total) = 301° 52.3'**
- Step 3: Determine the entering arguments for HO229.
    - Determine the AP and LHA.
        - **AP Latitude = 21° N**
        - LHA = GHA + E Longitude
        - **LHA =** 301° 52.3' + 143° 07.7' - (360°) = **85°**
        - **AP Longitude = 143° 07.7'** (chosen to make LHA whole number)
    - Entering arguments for HO229:
        - LHA = 85° (Same pages)
        - Declination = N 23° (increment = 13.2')
        - Latitude = 21° N

- Step 4: Solve the spherical triangle using HO229.
    - Retrieve base values from HO229.
        - Computed height (Hc) = 12° 24.7’
        - Altitude difference (d) = +18.2
        - Azimuth Angle (Z) = 69.9°
    - Correct Hc for declination increment using altitude difference (d).
        - Altitude difference (d) = +18.2
        - Declination increment = 13.2’
        - Correction = +4.0’ (d x inc/60)
        - Hc = 12° 24.7’
        - Correction = +4.0’
        - **Hc = 12° 28.7’**
    - Correct azimuth angle for declination increment (if desired) and rules.
        - Z = 69.9°
        - Declination increment = 13.2’
        - Next tabular Z = 68.9°
        - Z difference = -1.0°
        - Correction = -0.2° (diff x inc/60)
        - Corrected azimuth angle = 69.7° (this correction can usually be omitted)
        - Azimuth rules: Lat - N, if LHA less than 180°, Zn = 360° - Z
        - **Azimuth = 290.3°**
- Step 5: Compare calculations and observations to plot the LOP.
    - Determine the Intercept.
        - Hc = 12° 28.7’
        - Ho = 12° 46.6’
        - **Intercept = 17.9’ Towards**
    - Determine the plotting data.
        - AP = 21° N, 143° 07.7’ E
        - Azimuth = 290.3° T
        - Intercept = 17.9’ Towards
    - Plot the LOP.



Sight Reduction of a Star Example

Problem 6.1.4. You are underway in the North Atlantic and take a sight of Antares on 4 September. Your DR position at the time of sight is 17° 12.1'N, 46° 30.5'W. The time of sight is 20:15:04 zone time and the sextant height is 32° 22.5'. Index error is 2.0' off the arc and height of eye is 10 feet. Reduce this sight and plot the resultant LOP.

- Step 1: Observe and correct Hs to Ho.
  - Height of Sextant (Hs) = 32° 22.5'
    - IE = +2.0'
    - Dip = -3.1' (10 ft HOE)
    - Height Apparent (Ha) = 32° 21.4'
    - Apparent Altitude Correction = -1.5' (normal conditions)
  - **Height Observed (Ho) = 32° 19.9'**
- Step 2: Determine the GP of the Antares at the time of sight.
  - Correct zone time to GMT.
    - 20:15:04 zone time.
    - DR longitude is 46° 30.5' W, which correlates to (ZD +3)
    - 20:15:04 + 3 = **23:15:04, 4 September**
  - Determine declination of the Antares for the time of sight.
    - **Declination (total) = S 26° 23.5'**
  - Determine the GHA of the sun for the time of sight.
    - GHA (Aries, hourly) = 328° 58.2'
    - GHA (increment) = 3° 46.6'
    - GHA (Aries, total) = 332° 44.8'
    - SHA (Antares) = 112° 56.1'
    - GHA (Antares) = 85° 40.9' (-360°)
- Step 3: Determine the entering arguments for HO229.
  - Determine the AP and LHA.
    - **AP Latitude = 17° N**
    - LHA = GHA – W Longitude
    - LHA = 85° 40.9' – 46° 30.5' = 39°
    - **AP Longitude = 46° 40.9'** (chosen to make LHA whole number)
  - Entering arguments for HO229:
    - LHA = 39° (Contrary Pages)
    - Declination = S 26° (increment = 23.5')
    - Latitude = 17° N

- Step 4: Solve the spherical triangle using HO229.

    - Retrieve base values from HO229.
        - Computed height (Hc) = 32° 40.2’
        - Altitude difference (d) = -42.1
        - Azimuth Angle (Z) = 137.8°

    - Correct Hc for declination increment using altitude difference (d).
        - Altitude difference (d) = -42.1
        - Declination increment = 23.5’
        - Correction = -16.5’ (d x inc/60)
        - Hc = 32° 40.2’
        - Correction = -16.5’
        - **Hc = 32° 23.7’**

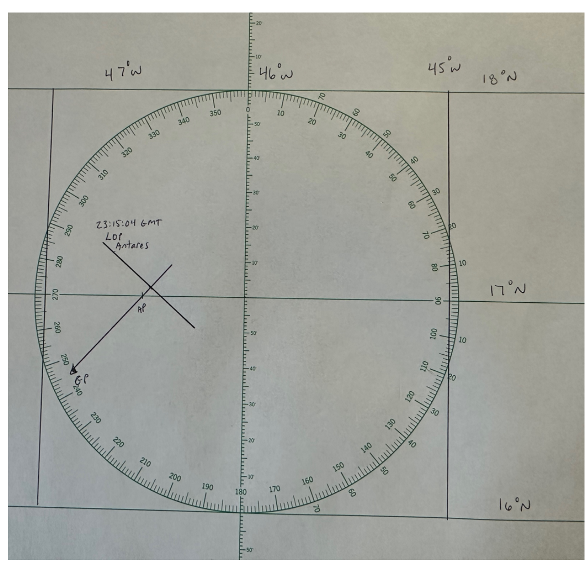
    - Correct azimuth angle for declination increment (if desired) and rules.
        - Z = 137.8°
        - Declination increment = 23.5’
        - Next tabular Z = 138.6°
        - Z difference = +0.8°
        - Correction = +0.3° (diff x inc/60)
        - Corrected azimuth angle = 138.1° (this correction can usually be omitted)
        - Azimuth rules: Lat – N, if LHA less than 180°, Zn = 360° - Z
        - **Azimuth = 221.9° T**

- Step 5: Compare calculations and observations to plot the LOP.

    - Determine the Intercept.
        - Hc = 32° 23.7’
        - Ho = 32° 19.9’
        - **Intercept = 3.8’ Away**

    - Determine the plotting data.
        - AP = 17° N, 46° 40.9’ W
        - Azimuth = 221.9° T
        - Intercept = 3.8’ Away

    - Plot the LOP.



Practice Problems

The solutions to these practice problems provide key data to check your understanding. Refer to the appropriate section of the course if you are getting stuck.

6.2.1. On 30 November at 13:58:12 zone time you sight the sun at 60° 46.7'. There is no index error, and the height of eye is 15 feet. Your DR position is 29° 12' S and 133° 30' E and you are observing (ZD -9). Reduce this sight.

6.2.1. Answer.

- Step 1:
  - Hs = 60° 46.7'
  - IC = 0.0
  - Dip = -3.8'
  - App Alt = +15.7'
  - Ho = 60° 58.6'

- Step 2:
  - 13:58:12 zone time is 04:58:12 GMT
  - GHA = 257° 24.2'
  - Dec = S 21° 37.7'

- Step 3:
  - Assumed Lat = 29° S
  - Declination = S 21° (same)
  - LHA = GHA + E. Long = 31°
  - Assumed Long = 133° 35.8' E

- Step 4:
  - Hc = 60° 53.1'
  - d = +21.8
  - Z = 98.8°
  - Hc (corrected) = 61° 06.8'
  - Z (corrected) = 97.6°

- Step 5:
  - Azimuth = 277.6° T
  - Intercept = 8.2' Away

6.2.2. You observe the sun at an altitude of 21° 58.6' above the horizon on 2 July at 11:06:14 GMT. Your DR position at the time of sight was 16° 12.5' S, 42° 15.0' W. The height of eye was 10 feet and the index error was 3.1 off the arc. Reduce this sight.

6.2.2. Answer.

- Step 1:
  - Hs = 21° 58.6'
  - IC = +3.1'
  - Dip = -3.1'
  - App Alt = +13.6'
  - Ho = 22° 12.2'

- Step 2:
  - 11:06:14 GMT
  - GHA = 345° 34.5'
  - Dec = N 23° 01.7'

- Step 3:
  - Assumed Lat = 16° S
  - Declination = N 23° (contrary)
  - LHA = GHA – W Long = 303°
  - Assumed Long = 42° 34.5' W

- Step 4:
  - Hc = 21° 58.6'
  - d = -29.8
  - Z = 123.6°
  - Hc (corrected) = 21° 57.8'
  - Z (corrected) = 123.6°

- Step 5:
  - Azimuth = 56.4° T
  - Intercept = 14.4' Towards

6.2.3. You take a sighting of the moon's lower limb at 08:21:21 GMT on 24 February. The moon's altitude is 30° 20.0', the height of eye is 16 feet, and the index error is 1.0 on the arc. Your DR position is 26° 02.5' N, 13° 45.0'W. Reduce this sight. 6.3.1. Answer.

- Step 1:
  - Hs = 29° 20.0'
  - IC = -1.0'
  - Dip = -3.9'
  - Ha = 29° 15.1', Horizontal Parallax = 54.2
  - App Alt #1 = +58.8'
  - App Alt #2 = +1.1'
  - Ho = 30° 15.0'

- Step 2:
  - 08:21:21 GMT
  - GHA = 63° 52.3' (remember v correction)
  - Dec = S 8° 39.2'

- Step 3:
  - Assumed Lat = 26° N
  - Declination = S 8° (contrary)
  - LHA = GHA – Long = 50°
  - Assumed Long = 13° 52.3' W

- Step 4:
  - Hc = 30° 44.2'
  - d = -36.1
  - Z = 118.0°
  - Hc (corrected) = 30° 20.9'
  - Z (corrected) = 118.7°

- Step 5:
  - Azimuth = 241.3° T
  - Intercept = 5.9' Away

Appendix 1

1.1

Step	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		40	05.0’	Observed by sextant
Index Correction	-		02.0’	On the arc needs to be subtracted
Height Apparent (Ha)		40	03.0’	Final answer

1.2	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		40	05.0	Observed by sextant
Index Correction	+		02.0	Off the arc needs to be added
Height Apparent (Ha)		40	07.0	Final answer

1.3	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		44	13.5	Observed by sextant
Index Correction	+		02.5	Off the arc needs to be added
Height Apparent (Ha)		44	16.0	Final answer

1.4	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		40	59.5	Observed by sextant
Index Correction	+		02.0	Off the arc needs to be added
Height Apparent (Ha)		41	01.5	Final answer

1.5	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		66	01.0	Observed by sextant
Index Correction	-		01.5	On the arc needs to be subtracted
Height Apparent (Ha)		65	59.5	Final answer

1.6	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		40	00.5	Observed by sextant
Index Correction	-		01.0	On the arc needs to be subtracted
Height Apparent (Ha)		39	59.5	Final answer

1.7	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		40	05.0	
Index Correction				
Dip/Height of Eye Correction	-		03.1	Height of Eye is 10 feet
Height Apparent (Ha)		40	01.9	Final Answer

1.8	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		55	30.0	
Index Correction				
Dip/Height of Eye Correction	-		04.3	Height of Eye is 20 feet
Height Apparent (Ha)		55	25.7	Final Answer

1.9	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		25	45.0	
Index Correction				
Dip/Height of Eye Correction	-		02.2	Height of Eye is 1.5 meters
Height Apparent (Ha)		25	42.8	Final Answer

1.10	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		33	25.0	
Index Correction				
Dip/Height of Eye Correction	-		03.4	Height of Eye is 12 feet
Height Apparent (Ha)		33	21.6	Final Answer

1.11	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		46	01.5	
Index Correction				
Dip/Height of Eye Correction	-		02.8	Height of Eye is 2.5 meters
Height Apparent (Ha)		45	58.7	Final Answer

1.12	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		35	13.2	Observed by Sextant
Index Correction	-		02.0	On the arc needs to be subtracted
Dip/Height of Eye Correction	-		02.9	Height of Eye is 9 feet
Height Apparent (Ha)		35	08.3	Final Answer

1.13	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		66	00.5	Observed by Sextant
Index Correction	+		02.2	Off the arc needs to be added
Dip/Height of Eye Correction	-		03.4	Height if Eye is 3.7 meters
Height Apparent (Ha)		65	59.3	Final Answer

1.14	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		43	13.0	Starting with Ha in this problem
Apparent Altitude Correction	+		15.2	Lower Limb, October 11th.
Additional Correction				
Height Observed (Ho)		43	28.2	Final Answer

1.15	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		62	45.0	Starting with Ha in this problem
Apparent Altitude Correction	+		15.5	Lower Limb, June 27th
Additional Correction				
Height Observed (Ho)		63	00.5	Final Answer

1.16	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		10	12.0	Starting with Ha in this problem
Apparent Altitude Correction	+		11.1	Lower Limb, October 31st
Additional Correction				
Height Observed (Ho)		10	23.1	Final Answer

1.17	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		13	43.7	Starting with Ha in this problem
Apparent Altitude Correction	+		12.4	Lower Limb, November 2nd
Additional Correction				
Height Observed (Ho)		13	56.1	Final Answer

1.18	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		27	12.5	Starting with Ha in this problem
Apparent Altitude Correction	+		14.1	Lower Limb, April 13th
Additional Correction				
Height Observed (Ho)		27	26.6	Final Answer

1.19	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		65	30.0	Starting with Ha in this problem
Apparent Altitude Correction	-		16.5	UPPER LIMB, October 2nd
Additional Correction				
Height Observed (Ho)		65	13.5	Final Answer

1.20	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		45	30.0	
Index Correction -			02.0	On the arc is subtracted
Dip/Height of Eye Correction	-		03.1	Height of eye is 10 feet
Height Apparent (Ha)		45	24.9	
Apparent Altitude Correction	+		15.3	Lower limb, December 12th
Additional Correction				
Height Observed (Ho)		45	40.2	Final Answer

1.21	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		53	59.5	
Index Correction	+		03.5	Off the arc is added
Dip/Height of Eye Correction	-		02.8	Height of eye is 2.5 meters
Height Apparent (Ha)		54	00.2	
Apparent Altitude Correction	+		15.3	Lower Limb, April 15th
Additional Correction				
Height Observed (Ho)		54	15.5	Final Answer

1.22	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		54	28.2	
Index Correction	+		01.5	Off the arc is added
Dip/Height of Eye Correction	-		07.3	Height of eye is 56 feet
Height Apparent (Ha)		54	22.4	
Apparent Altitude Correction	+		15.3	Lower Limb, 22 July
Additional Correction				
Height Observed (Ho)		54	37.7	Final Answer

1.23	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		30	15.0	Apparent altitude of Sirius
Apparent Altitude Correction	-		01.7	
Additional Correction				
Height Observed (Ho)		30	13.3	Final answer

1.24	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		55	12.9	Apparent altitude of Mirfak
Apparent Altitude Correction	-		00.7	
Additional Correction				
Height Observed (Ho)		55	12.2	Final answer

1.25	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		18	33.6	Apparent Altitude of Venus, December 15th
Apparent Altitude Correction	-		02.9	
Additional Correction	+		0.5	
Height Observed (Ho)		18	31.2	Final answer

1.26	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		12	15.9	Apparent Altitude of Venus, January 5
Apparent Altitude Correction	-		04.4	
Additional Correction	+		00.1	
Height Observed (Ho)		12	11.6	Final answer

1.27	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		15	30.0	Apparent Altitude, Sun Lower Limb, 2 October
Apparent Altitude Correction	+		12.8	
Intermediate Step		15	42.8	
Additional Correction	-		00.4	Pressure is 1020 and Temperature is +10F (Zone B)
Height Observed (Ho)		15	42.4	Final answer

1.28	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		22	13.9	Apparent Altitude, Sun Lower Limb, 5 April
Apparent Altitude Correction	+		13.7	
Intermediate Step		22	27.6	
Additional Correction	+		00.2	Pressure is 972 and Temperature is 70F (Zone K)
Height Observed (Ho)		22	27.8	Final answer

1.29	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		06	14.5	Sun, Lower Limb, October 20th
Apparent Altitude Correction	+		08.1	Special table for low altitude sights
Additional Correction				
Height Observed (Ho)		06	22.6	Final answer

1.30	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		08	11.0	Jupiter, September 5th
Apparent Altitude Correction	-		06.4	Special table for low altitude sights
Additional Correction				
Height Observed (Ho)		08	04.6	Final answer

1.31	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		42	12.6	
Index Correction -			01.0	On the arc is subtracted
Dip/Height of Eye Correction	-		02.5	Height of eye is 2 meters
Height Apparent (Ha)		42	09.1	
Apparent Altitude Correction	-		01.1	Venus, 3 January
Additional Correction	+		00.1	
Intermediate Step		42	08.1	
Additional Correction	-		00.1	Pressure is 1030 and Temperature is 20F (Zone C)
Height Observed (Ho)		42	08.0	Final answer

1.32	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		06	13.0	
Index Correction	+		01.1	Off the arc is added
Dip/Height of Eye Correction	-		06.2	Height of eye is 12.5 meters
Height Apparent (Ha)		06	07.9	Sun, Lower Limb, March 5th
Apparent Altitude Correction	+		08.0	Special Table for low altitude sights
Intermediate Step		06	15.9	
Additional Correction	-		00.9	Pressure 1010, Temperature 15F (Zone C)
Height Observed (Ho)		06	15.0	Final answer

1.33	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		29	10.0	
Apparent Altitude Correction	+		59.3	Given
Additional Correction (HP)	+		00.7	Horizonal Parallax given as 54.0 LL
Height Observed (Ho)		30	10.0	Final Answer

1.34	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		37	41.0	Given
Apparent Altitude Correction	+		55.0	
Additional Correction (HP)	+		05.9	Horizonal Parallax given as 58.5 LL
Height Observed (Ho)		38	41.9	Final Answer

1.35	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)				
Index Correction				
Dip/Height of Eye Correction				
Height Apparent (Ha)		61	29.0	Given
Apparent Altitude Correction	+		37.7	
Additional Correction (HP)	+		03.2	Horizontal Parallax is 55.5 Upper Limb
Additional Correction (Moon)	-		30.0	Due to Upper Limb
Height Observed (Ho)		61	39.9	Final Answer

1.36	Sign	Degrees	Minutes	Notes
Height of Sextant (Hs)		18	04.6	
Index Correction	+		03.2	Off the arc is added
Dip/Height of Eye Correction	-		05.5	32 feet height of eye
Height Apparent (Ha)		18	02.3	Calculated
Apparent Altitude Correction	+		62.5	
Additional Correction (HP)	+		07.5	HP is 59.7
Height Observed (Ho)		19	12.3	Final Answer

Appendix 2

2.0.1.
S 23° 01.3’ – directly from nautical almanac

2.0.2.
S 23° 01.1’– directly from nautical almanac

2.0.3.
75° 10.8’ – directly from nautical almanac

2.1.1.
28 per table

2.1.2.
29 per table

2.1.3.
29.5 by interpolation

2.1.4.
30 per table

2.1.5.
82 by interpolation

2.1.6.
82.5 by interpolation

2.1.7.
1200, 1 hour west of Greenwich Time

2.1.8.
1030; 4 hours west of Greenwich Time

2.1.9.
1330; 2 hours east of Greenwich Time

2.1.10.
0200; 3 hours east of Greenwich Time

2.1.11.
1500; 1 hour west of Greenwich Time

2.1.12.
1330; 2 hours east of Greenwich Time

2.1.13.
16 minutes per table

2.1.14
5 hours, 48 minutes per table

2.1.15.
12 seconds per table

2.1.16.
1 minute per table

2.1.17.
34 seconds per table interpolation or use the right side for seconds of arc (0.5 minutes arc = 2 minutes time)

2.1.18.
2 hours 20 minutes for 35 degrees of arc (whole value)
2 minutes 2 seconds for 30.5 minutes of arc (increments)
2h 22m 2s total

2.1.19.
2h 40m for whole value (40°)
3m 05s for increments (46.25')
2h 43m 05s total

2.1.20.
1' per table

2.1.21.
6.3' per table

2.1.22.
9.5' per table (Moon)

2.1.23.
22.6' per table (Aries)

2.1.24.
1° 57.3' per table (Planets)

2.1.25.
1° 59.8' per table (Sun)

2.1.26.

		Hours	Minutes	
		12	51	
	+	1	12	
Subtotal		13	63	
Notes	change 60 minutes into 1 extra hour			
Total		14	03	

2.1.27.

		Hours	Minutes	
		13	06	
	+	4	55	
Subtotal		17	61	
Notes	change 60 minutes into 1 extra hour			
Total		18	01	

2.1.28.

		Hours	Minutes	
		23	50	
	+	1	12	
Subtotal		24	62	
Notes	Change 60 minutes into 1 extra hour and change 24 hours into 1 day			
Total		1	02	the following day

2.1.29.

		Hours	Minutes	
		10	44	
	-	1	03	
Subtotal		09	41	
Total		1	02	

2.1.30.

		Hours	Minutes	
		22	12	
Notes	Since minutes are small, convert 1 hour into 60 minutes			
		21	72	
	-	2	47	
Subtotal		19	25	
Total		19	25	

2.1.31.

		Hours	Minutes	
		2	14	
Notes	Since minutes are small, convert 1 hour into 60 minutes and 24 hours into 1 day			
		25	74	
	-	11	51	
Subtotal		14	23	
Total		14	23	the previous day

2.1.32.

		Degrees	Minutes
		14	55.2
		2	06.9
Subtotal		16	62.1
Notes	60 minutes into 1 degree		
Total		17	02.1

2.1.33.

		Degrees	Minutes
		47	12.2
Notes	Since minutes are small, convert 1 hour into 60 minutes and 24 hours into 1 day		
		46	72.2
		22	56.8
Subtotal		24	15.4
Total		24	15.4

2.1.34.
0.1’ per table

2.1.35.
0.2’ per table

2.1.36.
0.5’ per table

2.1.37.
2.0’ per table

2.1.38.
2.1’ per table

2.1.39.
10.9’ per table

2.2.1.
S 23° 01.3' per table

2.2.2.
S 23° 01.1' per table

2.2.3.
S 23° 01.2' by mental interpolation (half the interval)

2.2.4.
S 23° 01.1' by metal interpolation (nearly 0200)

2.2.5.
Towards the equator, decreasing in value towards zero (equator)

2.2.6.
20 March – the spring equinox

2.2.7.
21 June – the summer solstice in the northern hemisphere

2.2.8.
224° 08.0' per table

2.2.9.
239° 07.7' per table

2.2.10.
15 degrees per hour, which is normal for the sun

2.2.11.
S16° 05.2' per table

2.2.12.
0.7 per table (look at the bottom of the declination column).

2.2.13.
0.1 using the increments and corrections table for 11 minutes

2.2.14.
Decreasing, the declination gets smaller from hour to hour

2.2.15.

Declination	1800	S16° 05.2'
D value/Corr	0.7/0.1'	
Trend	Decreasing	
Final Answer	S16° 05.1'	

2.2.16.

Declination	1800	S16° 05.2'
D value/Corr	0.7/0.4'	
Trend	Decreasing	
Final Answer	S16° 04.8'	

2.2.17.

Declination	1800	S16° 05.2’
D value/Corr	0.7/0.5’	
Trend	Decreasing	
Final Answer	S16° 04.7’	

2.2.18.
N 19° 46.1’

2.2.19.

Declination	1900	N19° 46.1’
D value/Corr	0.5/0.2’	
Trend	Decreasing	
Final Answer	N19° 45.9’	

2.2.20.
S16° 35.4’ per table

2.2.21.
Increasing per table

2.2.22.

Declination	1900	S16° 35.4’
D value/Corr	0.7/0.5’	
Trend	Increasing	
Final Answer	S16° 35.9’	

2.2.23.
1800 is 2000 GMT
Declination at GMT S16° 08.3’

2.2.24.
75° 10.8’ per table

2.2.25.
0° 15.0’ using increments and corrections table

2.2.26.

GHA at 1700	75° 10.8’
Increments	15.0'
Final Answer	75° 25.8'

2.2.27.
0° 27.5’ using increments and corrections table

2.2.28.

GHA at 1700	75° 10.8’
Increments	27.5'
Final Answer	75° 38.3'

2.2.29.

GHA at 1700	75° 10.8’
Increments	2° 21.5’
Final Answer	77° 32.3’

2.2.30.

GHA at 0200	210° 52.8’
Increments	7° 45.5’
Final Answer	218° 38.3’

2.2.31.

GHA at 1800	92° 45.8’
Increments	12° 28.8’
Final Answer	105° 14.6’

2.2.32.

10:29:12 local is	13:29:12 GMT
Declination 1300	S8° 37.6’
D value/Corr	0.9/0.4’
Trend	Decreasing
Final Declination	S8° 37.2’
GHA at 1300	11° 46.2’
Increments	7° 18.0’
Final GHA	19° 04.2’

2.2.33.

18:02:03 Local is	17:02:03 GMT
Declination 1700	N11° 18.6’
D value/Corr	0.9/0.0’
Trend	Increasing
Final Declination	N11° 18.6’
GHA at 1700	75° 14.1’
Increments	00° 30.8’
Final GHA	75° 44.9’

2.2.34.

09:23:49 Local is	18:23:49 GMT
Declination Hour	N 3° 14.7’
D Value/Corr	1.0/0.4’
Trend	Decreasing
Final Declination	N 3° 14.3’
GHA Hour	91° 08.3’
Increments	5° 57.3’
Final GHA	97° 05.6’

2.2.35.

Time zone change	NA
Declination Hour	N3° 53.1’
D Value/Corr	1.0/0.4’
Trend	Decreasing
Final Declination	N3° 52.7’
GHA Hour	210° 59.4’
Increments	5° 35.5’
Final GHA	216° 34.9’

2.2.36.
GHA 005° 09.3’
Dec S12° 07.9’
Hourly values per table

2.2.37.
GHA 240° 47.4’
Dec S0° 53.8’
Hourly values per table

2.2.38.
GHA 316° 19.2’
Dec S1° 54.4’
Hourly values per table

2.2.39.
158° 42.6’ per table

2.2.40.
80° 55.8’ per table

2.2.41.
SHA 158° 57.0’
Dec S11° 03.8’
Hourly values per table

2.2.42.

	Degrees	Minutes	Hemisphere	
Dec	38	45.7	North	
GHA (Aries)	158	42.6		
SHA (Vega)	80	55.8		
(Math)	238	98.4		Calculated
GHA (Vega)	239	38.4		

2.2.43.

	Degrees	Minutes	Hemisphere	
Dec	11	3.8	South	
GHA (Aries)	69	27.0		
SHA (Spica)	158	57.0		
(Math)	227	84.0		Calculated
GHA (Vega)	228	24.0		

2.2.44.

	Degrees	Minutes	Notes
Dec	19	17.0	North
GHA Aries	12	38.1	Tabulated for 0400 hours
Increments	03	18.5	To account for the 13:12 of time
GHA Aries Total	15	56.6	Math
SHA Arcturus	146	17.9	Tabulated
Intermediate step	161	74.5	Convert 60 minutes to 1 degree
GHA Arcturus	162	14.5	

2.2.45.

	Degrees	Minutes	Notes
Dec	08	13.2	South
GHA Aries	259	10.0	Tabulated for 0400 hours
Increments	11	02.3	To account for the 13:12 of time
GHA Aries Total	270	12.3	Math
SHA Rigel	281	35.2	Tabulated
Intermediate step	551	47.5	Subtract 360 degrees
GHA Rigel	191	47.5	

2.2.46.

	Degrees	Minutes	Notes
Dec Hour	00	38.0	South
Inc	00	00.0	d = 0.1 from table
Dec Total	00	38.0	South
GHA Hour	250	13.2	
Inc	03	00.0	for the 12 minutes of time
V Corr		00.5	v=2.6 from table
GHA Total	253	13.7	

2.2.47.

	Degrees	Minutes	Notes
Dec Hour	17	29.2	North, decreasing trend
Inc		00.4	d=0.4, subtract
Dec Total	17	28.8	North, decreasing trend
GHA Hour	256	57.1	
Inc	13	55.0	for the 55:40 of time
V Corr		00.8	v=0.9
GHA Total	270	52.9	subtract 60 minutes and add one degree

2.2.48.

	Degrees	Minutes	Notes
Dec Hour	20	21.6	North, decreasing trend
Inc		0.2	d=0.3, subtract
Dec Total	20	21.4	
GHA Hour	132	53.4	
Inc	7	55.3	for the 33:12 of time
V Corr		4.1	v=7.3
GHA Total	140	52.8	

2.2.49.

	Degrees	Minutes	Notes
Dec Hour	4	32.7	South, decreasing trend
Inc		8.4	d=12.5, subtract
Dec Total	4	24.3	
GHA Hour	58	10.8	
Inc	9	35.1	for the 40:10 of time
V Corr		7.8	v=11.6
GHA Total	67	53.7	

2.3.1.

Latitude:	Interval = 10 Difference = 16min		
	Table 1 correction = 7 minutes		
	Latitude corrected time = 0529		
Longitude:	65° W to standard meridian (60° W) is 5W		
	Convert 5 degrees of arc to time = 20 minutes West (Add)		
	Longitude corrected time = 0549		

2.3.2.

Latitude:	Interval = 10 Difference = 16min		
	Table 1 correction = 7 minutes		
	Latitude corrected time = 1843		
Longitude:	65° W to standard meridian (60° W) is 5W		
	Convert 5 degrees of arc to time = 20 minutes West (Add)		
	Longitude corrected time = 1903		

2.3.3.

Latitude:	Interval	5°	
	Difference	17m	
	Correction	13	
	Latitude corrected time	1943	(1930 +13)
Longitude:	Dist. to Stand Meridian	3° 14'	
	Arc to time	12m56s	added
	Longitude corrected time	1956	

2.3.4.

Latitude:	Interval	2°	
	Difference	6m	
	Correction	4	
	Latitude corrected time	738	(0734+4)
Longitude:	Dist. to Stand Meridian	2° 00'	
	Arc to time	8m	subtracted
	Longitude corrected time	730	

2.3.5.

Latitude:	Interval	5°	
	Difference	8m	
	Correction	5	
	Latitude corrected time	1758	(1803-5)
Longitude:	Dist. to Stand Meridian	2° 20'	
	Arc to time	9m20s	added
	Longitude corrected time	1807	

2.3.6.

Latitude:	Interval	10°	
	Difference	21m	
	Correction	6	
	Latitude corrected time	637	(0631+6)
Longitude:	Dist. to Stand Meridian	1° 40'	
	Arc to time	6m 40s	
	Longitude corrected time	0630	subtract east of ZD

2.3.7.

Base value from almanac	1159	
Standard meridian	60° W	
Distance from standard meridian	4° 30’	west/later
Arc to time	18 min	
Correction	1217	(1159+18)

2.3.8.

Base value from almanac	1159	
Standard Meridian	75° W	
Distance from standard meridian	0° 30’	east/earlier
Arc to time	2 min	
Correction	1157	(1159-2)

2.3.9.

Base value from almanac	1159	
Standard Meridian	15° E	
Distance from standard meridian	3° 30’	east/earlier
Arc to time	14 min	
Correction	1145	(1159-14)

2.3.10.

Base value from almanac	1206	
Standard Meridian	120° E	
Distance from standard meridian	1° 40’	west/later
Arc to time	6m 40s	
Correction	1213	(1206+7)

2.3.11.

		Degree	Minutes	Hemisphere	Notes
Determine GMT	Not necessary for this problem				
Declination (hourly)					
Declination (increment)					
Declination		15	15	S	Given
90-00 for math purpose		89	60		Provided for easy mental subtraction of HO
Ho		30	30		
Zenth Distance (90-Ho)		59	30		
Formula					Lat = ZD - Dec
Latitude		44	15		North

2.3.12.

		Degree	Minutes	Hemisphere	Notes
Determine GMT	Not necessary for this problem				
Declination (hourly)					
Declination (increment)					
Declination		20	20	North	Given
90-00 for math purpose		89	60		Provided for easy mental subtraction of HO
Ho		73	15		
Zenth Distance (90-Ho)		16	45		
Formula					Lat = ZD + Dec
Latitude		36	65	North	
Math adjustment		37	5	North	

2.3.13.

		Degree	Minutes	Hemisphere	Notes
Determine GMT	Not necessary for this problem				
Declination (hourly)					
Declination (increment)					
Declination		15	40	South	Given
90-00 for math purpose		89	60		Provided for easy mental subtraction of HO
Ho		85	12		
Zenth Distance (90-Ho)		4	48		
Formula					Lat = Dec-ZD
Latitude		11	-8	South	
Math adjustment		10	52	South	

2.3.14.

		Degree	Minutes	Hemisphere	Notes
Determine GMT	1447	22-Feb			
Declination (hourly)		10	5.4		South decreasing
Declination (increment)		-	0.7		d=0.9
Declination of Sun		10	4.7	South	
90-00 for math purpose		89	60		
Ho		73	33.3		
Zenith Distance 90-HO		16	26.7		
Formula					Lat = ZD + Dec
Latitude		26	31.4	South	

2.3.15.

		Degree	Minutes	Hemisphere	Notes
Determine GMT	1627	16-Sep			
Declination (hourly)		2	30.4	North decreasing	
Declination (increment)		-	0.5		d=1.0
Declination of Sun		2	29.9	North	
90-00 for math purpose		89	60		
Ho		63	25.3		
Zenith Distance 90-HO		26	34.7		
Formula					Lat = ZD + Dec
Latitude		29	4.6	North	

2.3.16.

		Degree	Minutes	Hemisphere	Notes
Determine GMT	1138 Local + 11 is 2238 GMT				
Declination (hourly)		16	27.4	South decreasing	
Declination (increment)		+	0.4		d=0.7
Declination of Sun		16	27.8	South	
90-00 for math purpose		89	60		
Ho		45	35		
Zenith Distance 90-HO		44	25		
Formula					Lat = ZD - Dec
Latitude		27	57.2	North	

2.3.17.

		Degree	Minutes	Hemisphere	Notes
Determine GMT	1215 local + 10 = 2215 GMT.				
Declination (hourly)		23	4	North decreasing	
Declination (increment)		-	0.1		Mentally interpolate or d=0.2
Declination of Sun		23	3.9	North	
90-00 for math purpose		89	60		
Ho		42	55		
Zenith Distance 90-HO		47	5	S	
Formula					Lat = ZD - Dec
Latitude		24	1.1	South	

Appendix 3

3.1.1.
23° N is nearest whole latitude

3.1.2.
24° N is the nearest whole latitude

3.1.3.
36° S is the nearest whole latitude

3.1.4.
37° S is the nearest whole latitude

3.1.5.

	Deg	Min	Notes
GHA	125	30	
Longitude W	45	30	subtract west
Longitude E			add east
LHA	80	0	Must result in whole value

3.1.6.

	Deg	Min	Notes
GHA	125	30	
Longitude W			subtract west
Longitude E	45	30	add east
LHA	170	60	Must result in whole value
Math (60m = 1 deg)	171	0	

3.1.7.

	Deg	Min	Notes
GHA	95	20	
Longitude W	42	20	subtract west
Longitude E			add east
LHA	53	0	Must result in whole value

3.1.8.

	Deg	Min	Notes
GHA	136	2.1	
Longitude W	43	2.1	subtract west
Longitude E			add east
LHA	93	0	Must result in whole value

3.1.9.

	Deg	Min	Notes
GHA	215	22.6	
Longitude W	125	22.6	subtract west
Longitude E			add east
LHA	90	0	Must result in whole value

3.1.10.

	Deg	Min	Notes
GHA	330	26.9	
Longitude W	36	26.9	subtract west
Longitude E			add east
LHA	294	0	Must result in whole value

3.1.11.

	Deg	Min	Notes
GHA	15	16.3	
GHA (+360)	375	16.3	
Longitude W	136	16.3	subtract west
Longitude E			add east
LHA	239	0	Must result in whole value

3.1.12.

	Deg	Min	Notes
GHA	125	30	
Longitude W		subtract west	
Longitude E	65	30	add east
LHA	190	60	Must result in whole value
Math	191	0	

3.1.13.

	Deg	Min	Notes
GHA	346	12.2	
Longitude W			subtract west
Longitude E	74	47.8	add east
LHA	420	60	Must result in whole value
Math (60 m = 1 deg)	421	0	
Math (-360)	61	0	

3.1.14.

	Deg	Min	Notes
GHA	225	15.3	
Longitude W			subtract west
Longitude E	104	44.7	add east
LHA	329	60	Must result in whole value
Math (60 m = 1 deg)	330	0	

3.1.15.

Dec	23	North/same
Lat	55	
GHA	144	44.6
Ass Long	32	44.6 W
LHA	112	

3.1.16.

Dec	12	South/contrary
Lat	43	
GHA	49	13.2
GHA + 360	409	
Ass Long	113	13.2 W
LHA	296	

3.1.17.

Dec	12	South/same
Lat	63	
GHA	132	14
Ass Long	174	46 E
LHA	307	

3.1.18.

Dec	12	South/contrary
Lat	12	
GHA	330	12.7
Ass Long	33	47.3 E
LHA	364	
LHA	4	

3.2.1.

	Deg	Min
GHA Aries	156	53.7
DR Longitude E		
DR Longitude W	66	48
LHA Aries	90	5.7

Object Mirfak by observation

3.2.2.
Spica by observation

3.2.3.
Antares by observation

3.2.4.
Canopus by observation

3.2.5.
Antares by observation

3.2.6.

	Deg	Min
GHA Aries	145	13
DR Longitude E		
DR Longitude W	45	13
LHA Aries	100	0

Object Sirius, 38° altitude

3.2.7.

	Deg	Min
GHA Aries (hourly)	107	2.4
GHA increment	9	3
GHA Aries (total)	116	5.4
DR Longitude E	162	36
DR Longitude W		
LHA Aries	278	41.1

Object Antares

3.2.8.

	Deg	Min
Convert to GMT	08:20:30 GMT	
GHA Aries (hourly)	294	48.6
GHA increment	5	8.3
GHA Aries (total)	299	56.9
DR Longitude E		
DR Longitude W	39	42
LHA Aries	260	14.9

Object Deneb

3.2.9.
47.9’ directly from table

3.2.10.
0.9’ for March, 0.8’ for latitude, directly from table

3.2.11.
0° 52.6’ using tables

3.2.12.

	Deg	Min
LHA Aries	89	27.2
Ho	29	48.3
Deg +1	1	
A0 correction		31.7
A1 correction		0.5
A2 correction		0.8
Latitude	28	81.3
Math (60m = 1 deg)	29	21.3

3.2.13.

	Deg	Min
LHA Aries	3	27.1
Ho	23	50.1
Deg +1	1	
A0 correction		16.2
A1 correction		0.5
A2 correction		1
Latitude	22	67.8
Math (60m = 1 deg)	23	7.8

3.2.14.

Convert to GMT	1:32:40		12-Feb
		Deg	Min
GHA Aries Tabulated		156	59.7
GHA Increment		8	11.3
GHA Aries Total		165	11
DR Longitude W		110	52.6
LHA Aries		54	18.4
Hs		26	19.8
Index Error	+		2.7
Dip	-		7.5
Altitude Corr (Star)	-		2
Ho		26	13
Deg +1		1	
A0 correction			12.9
A1 correction			0.6
A2 correction			0.7
Latitude		25	27.2

Appendix 4

4.1.1.
Hc 60° 18.8’
d 12.4
Z 96.1°

4.1.2.
Hc = 39° 34.9’
d = -47.9
Z = 142.4°

4.1.3.
Hc = 36° 22.2’
d = -49.7
Z = 144.3°

4.1.4.
Hc = 62° 34.5’
d = 1.3* (this dot indicates an additional correction that may be required)
Z = 79.7°

4.1.5.
Hc = 38° 10.6’
d = 15.1
Z = 85.2°

4.1.6.
Hc = 10° 02.9’
d = 33.1
Z = 27.4°

4.1.7.
Hc = 35° 36.4
d = -41.5
Z = 127.6°

4.1.8.
Hc = 66° 46.7’
d = -59.2
Z = 6.9°

4.1.9.

360-Z = 233° T

4.1.10.

Z=Zn = 46.3° T

4.1.11.

180-Z = 127.2° T

4.1.12.

360-Z = 286.3° T

4.1.13.

180 + Z = 193° T

4.1.14.

180-Z = 130.3° T

4.1.15.

Dec inc	12.5	
d value	+49.9	
correction	+10.4	d x (dec inc / 60)

4.1.16.

Dec inc	42.5	
d value	-22.2	
correction	-15.7	d x (dec inc / 60)

4.1.17.

Dec inc	26.6	
d value	-30.3	
correction	-13.4	d x (dec inc / 60)

4.1.18.

Dec inc	34.9	
d value	+44.2	
correction	+25.7	d x (dec inc / 60)

4.1.19.

tens	20	=8
units	0.5	=0.2
total	8.2	

4.1.20.

Double second difference needed.

4.1.21. (Solved in text)

4.1.22. (Solved in text)

4.1.23.

LHA = 35°
Lat = 16° N
Dec =15° 44.2'N

Hc = 56° 17.8'
d = +6.0
Z = 86.8°

Zn = 273.2°
Hc = 56° 17.8' + 4.4' = 56° 22.2'

4.1.24.

LHA = 293°
Lat = 15° S
Dec =25° 30.9'S

Hc = 26° 50.2'
d = +4.7
Z = 69.2°

Zn = 180 – Z = 110.8° T
Hc = 26° 50.2' + 2.4' = 26° 52.6'

4.1.25.

LHA = 336°
Lat = 25° N
Dec =12° 13.2'S

Hc = 46° 13.2'
d = -51.0
Z = 144.9°

Zn = Z = 144.9° T

Hc = 46° 13.2' – 11.1' = 46° 02.1'

Appendix 5

5.1.1. (Solved in text)

5.1.2.

HO	36° 55.8’
HC	36° 41.2’
Intercept	14.6’ Towards

5.1.3.

HO	15° 02.6’
HC	15° 29.6’
Intercept	27.0’ Away

5.1.4.

HO	22° 20.9’
HC	22° 25.6’
Intercept	4.7’ Away

5.1.5.

HO	63° 42.9’
HC	63° 29.4’
Intercept	13.5’ Towards

5.1.6.

HO	13° 55.6’
HC	14° 02.5’
Intercept	6.9’ Away

5.1.7.

HO 15° 13.6’
HC 14° 49.1’
Intercept Answer. 24.5’ Towards

5.1.8. (Solved in text)

5.1.9.

Next incremental Z	103.7°
Correction	0.33°
Z	103.1° (Zn = 76.9° T)

5.1.10.

Next incremental Z is 93.8°. Correction to base Z is -0.57°. Final Z is 94.1° (Zn = 265.9° T)

5.2.1. (Solved in text)
Assumed Latitude 25° N
Assumed Longitude 144° 13'W
Azimuth 210° T
Intercept 10.5' Away
See plot in text

5.2.2.
Assumed Latitude 17° N
Assumed Longitude 26° 27'E
Azimuth 143° T
Intercept 6.8' Towards
See plot in text

5.2.3.
Assumed Latitude 22° N
Assumed Longitude 47° 05.6'W
Azimuth 085° T
Intercept 21.4' Towards
See plot in text

5.2.4.
Assumed Latitude 15° S
Assumed Longitude 126° 13' W
Azimuth 261° T
Intercept 8.4' Away
See plot in text

5.2.5. (Solved in text)

5.2.6.
41° 14.8'N, 072° 04.2'W. Answers within 0.3nm are acceptable.

5.2.7.
41° 13.0'N, 072° 10.5'W. Answers within 0.3nm are acceptable. You should have advanced the 1330 LOP 2.75nm in the direction of 250° T to account for the 30-minute time interval.

5.2.8. (Solved in text)

5.2.9.
12° 10.0'N, 125° 35.0'W. Answers within 5 nm of longitude are acceptable. See plot in text.

5.2.10.
56° 25.0'N, 166° 11.0'E. Answers within 5 nm of longitude are acceptable. See plot in text.

5.2.11.

15° 32’S, 3° 15.0’E. Answers within 5 nm of longitude are acceptable. See plot in text.

5.2.12. (Solved in text)

5.2.13.
23° 05’S, 111° 06’W. Answers within 5 nm are acceptable. See plot in text.

5.2.14.
Answer. 26° 29.3’ N, 134° 37.1’ E. Answers within 5 nm are acceptable. See plot in text.

4.1.26.

LHA = 318°
Lat = 17° S
Dec =50° 58.8'S

Hc = 42° 52.3'
d = -29.6
Z = 36.0°

Zn = 180° - Z = 144° T
Hc = 42° 54.3' – 29.0' = 42° 25.3'

4.1.27.

LHA = 11°
Lat = 22° S
Dec =38° 39.1'

Hc = 29° 06.9'
d = -58.7
Z = 170.1°

Zn = 180° + Z = 350.1° T
Hc = 29° 06.9' – 38.3' = 28° 28.6'

4.1.28. (Solved in text)