Factorise and draw the cubic function.

Example: Factorise and draw $y = f(x) = x^3 - 2x^2 - 5x + 6$.

Step 1. Find first factor.

Assume this polynomial has a linear factor $(x - a)$, where a is an integer. Then we can write

$$x^3 - 2x^2 - 5x + 6 = (x - a)(x^2 + bx + c) = x^3 - (a - b)x^2 - (ab - c)x - ac$$

By considering the constant term, it can be seen that $ac = -6$.

Thus, only the factors of 6 need be considered $(i.e. \pm 1, \pm 2, \pm 3, \pm 6)$

Try these in turn until a value for a makes $P(a) = 0$. This process is completed in the following example.

$$let`s\ guess\ x = 1,$$

$$then\ \ f(1) = 1^3 - 2(1)^2 - 5(1) + 6 = 0$$

if $f(1) = 0$, then $(x - 1)$ is a factor.

$$f(x) = (x - 1)(something)$$

Step 2. Find other factors.

$$\begin{array}{r|l} & x^2 - x - 6 \\ \hline x - 1 & x^3 - 2x^2 - 5x + 6 \\ & x^3 - x^2 \\ \hline & -x^2 - 5x + 6 \\ & -x^2 + x \\ \hline & -6x + 6 \\ & -6x + 6 \\ \hline & 0 \end{array}$$

Step 3. Factorise the result of the division and put it all together.

$$x^2 - x - 6 = (x + 2)(x - 3)$$

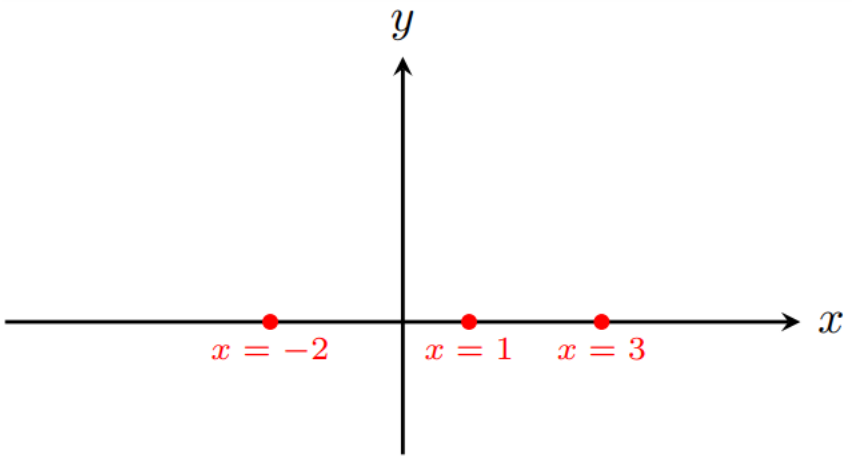
$$f(x) = (x - 1)(x^2 - x - 6) = (x - 1)(x + 2)(x - 3)$$

Factorise and draw the cubic function.

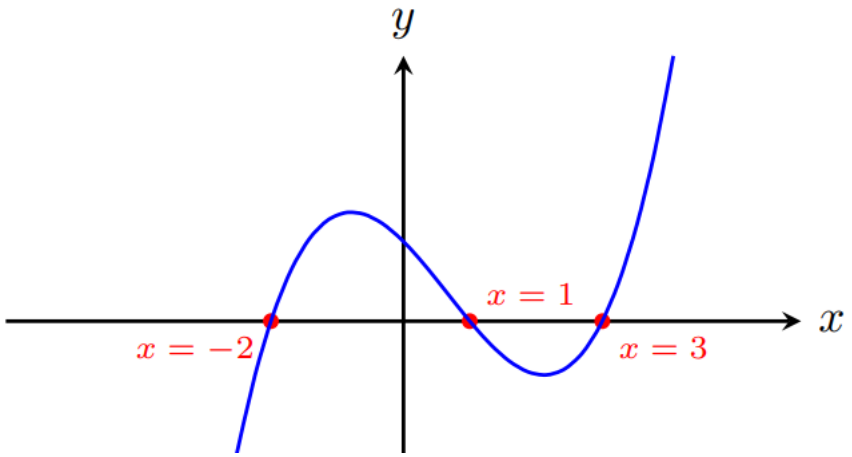
Step 4. Find the x intercept and plot.

$$f(x) = (x-1)(x+2)(x-3) = 0$$

$$so\ \ x\ intercept\ is\ at\ x = 1\ or\ x = -2\ or\ x = 3$$



Step 5. Draw the graph.



Exercise

1. Factorise and draw the each of the following cubic functions.

   a. $y = x^3 - 2x^2 - 5x + 6$

   b. $y = -x^3 + x^2 + 4x - 4$

   c. $y = 2x^3 - x^2 - 5x - 2$

   d. $y = 8x^3 - 2x^2 - 5x - 1$