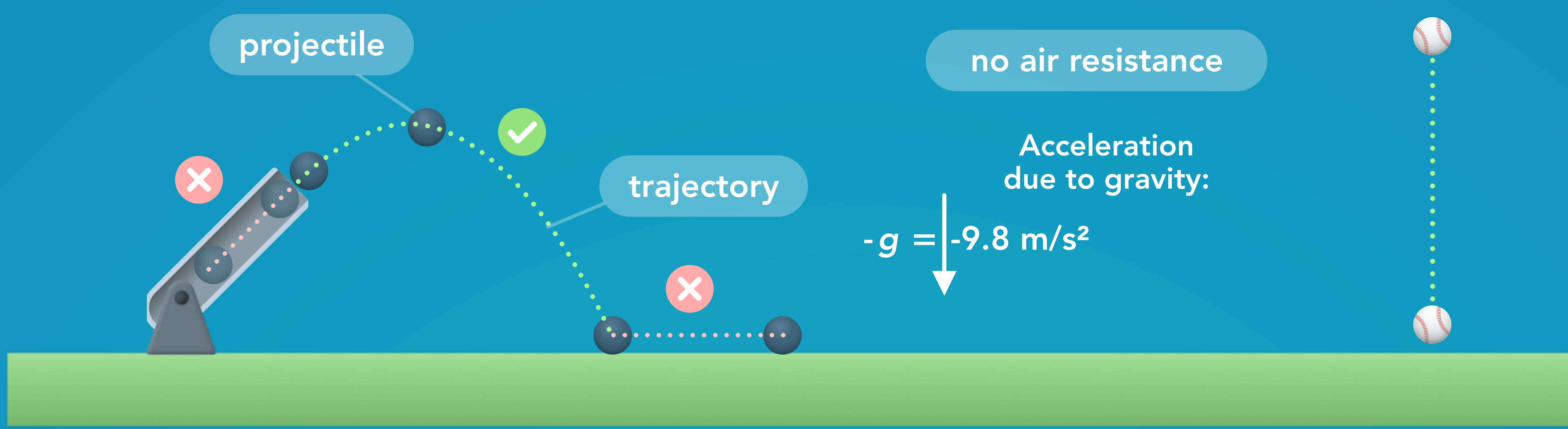
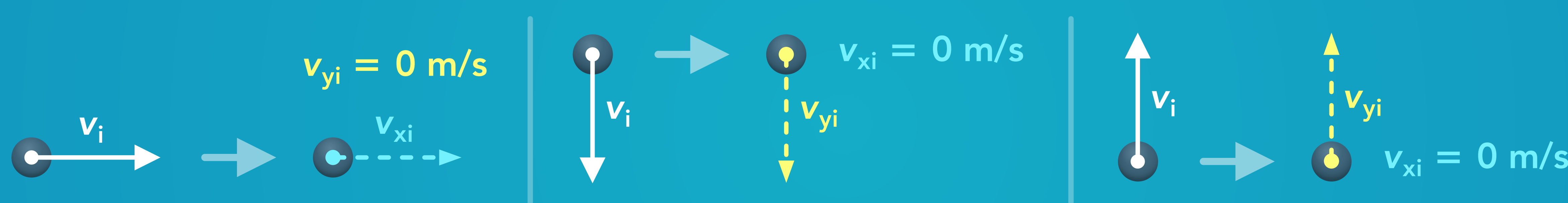
PROJECTILE MOTION

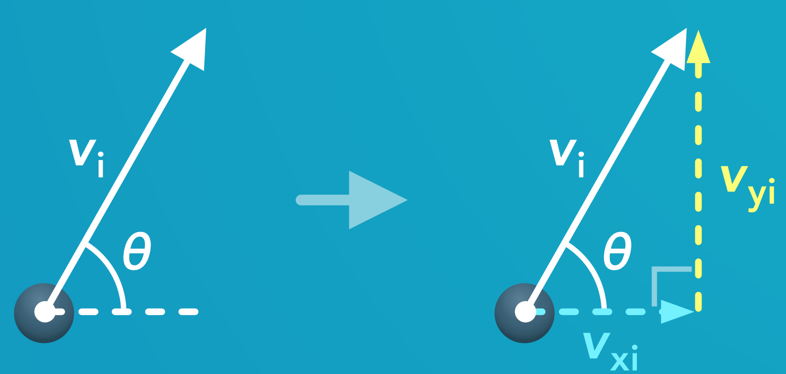
Projectile Motion



- **Projectile motion** is the motion of an object while it's only being affected by gravity and no other forces. An object is only in projectile motion while in the air, not when it's touching the ground or other objects.
- Also referred to as "free fall" (although "free fall" may only refer to 1D projectile motion).
- The vertical acceleration is the acceleration due to gravity, $g = 9.8\ \text{m/s}^2$, which always acts downwards.
- There is no horizontal acceleration so the horizontal velocity is constant.
- The object in projectile motion is called a **projectile** and the path is called the **trajectory**.

Initial Velocity Vector



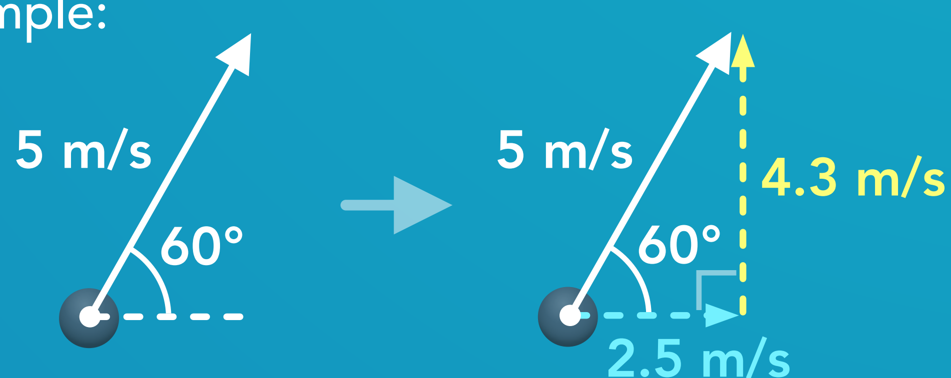


$$\cos(\theta) = \frac{v_{xi}}{v_i} \qquad \sin(\theta) = \frac{v_y}{v_i}$$

$$v_{xi} = v_i \cos(\theta) \qquad v_{yi} = v_i \sin(\theta)$$

*cos and sin are reversed if the other angle is used

Example:



$$v_{xi} = v_i \cos(\theta) \qquad v_{yi} = v_i \sin(\theta)$$

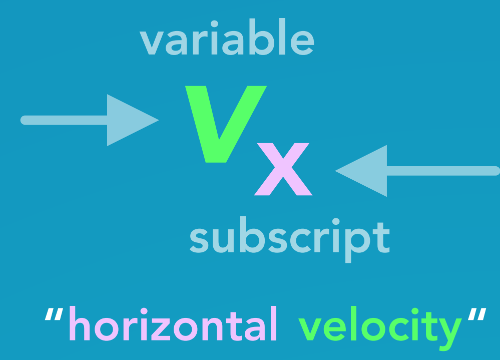
$$v_{xi} = (5\ \text{m/s})\cos(60°) \qquad v_{yi} = (5\ \text{m/s})\sin(60°)$$

$$v_{xi} = 2.5\ \text{m/s} \qquad v_{yi} = 4.3\ \text{m/s}$$

- The components of the initial velocity vector are the initial horizontal velocity and the initial vertical velocity.
- The angle is usually between the vector and the horizontal component but double check which angle is given before using the trig functions (cosine for the adjacent component and sine for the opposite component).
- If the vector is vertical or horizontal then the parallel component is equal to the vector (and the other is zero).
- The initial horizontal velocity will be the horizontal velocity for the entire motion.
- The initial vertical velocity can be used with the vertical motion kinematic equations.

Kinematic Equations and Variables

Variables		SI Unit
t	time	s
x	horizontal position	m
y	vertical position	m
v	velocity	$\frac{m}{s}$
a	acceleration	$\frac{m}{s^2}$



Subscripts		
i	0	initial
f	_	final
x		horizontal
y		vertical

delta

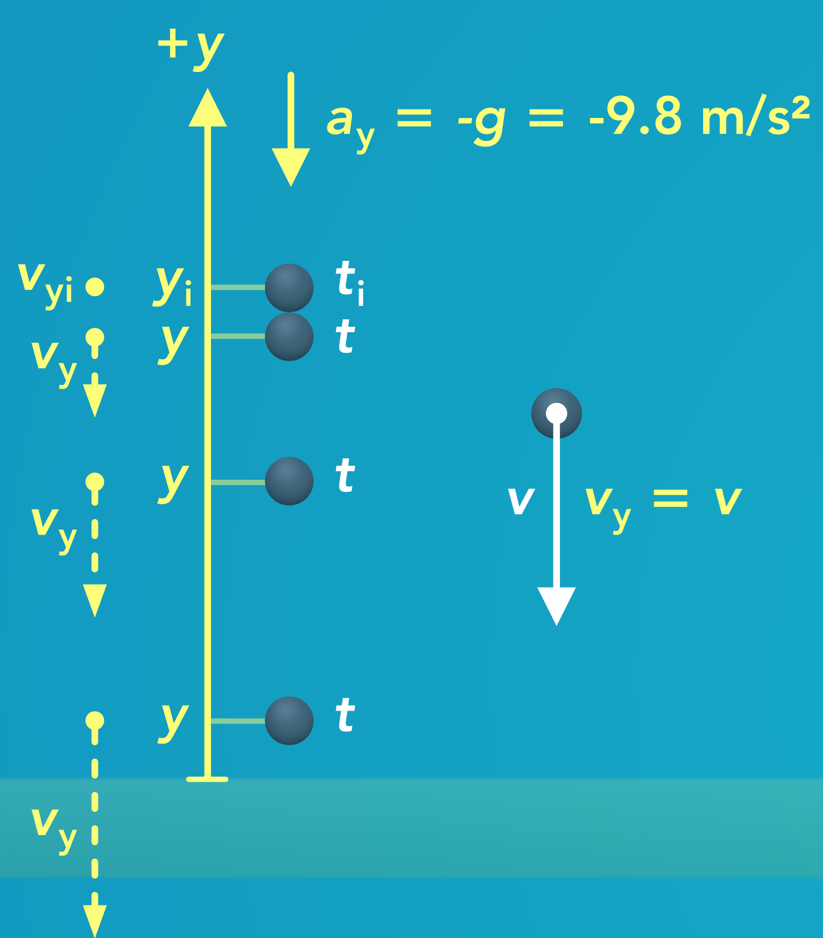
Δ = final − initial

$$\Delta x = x_f - x_i$$

or

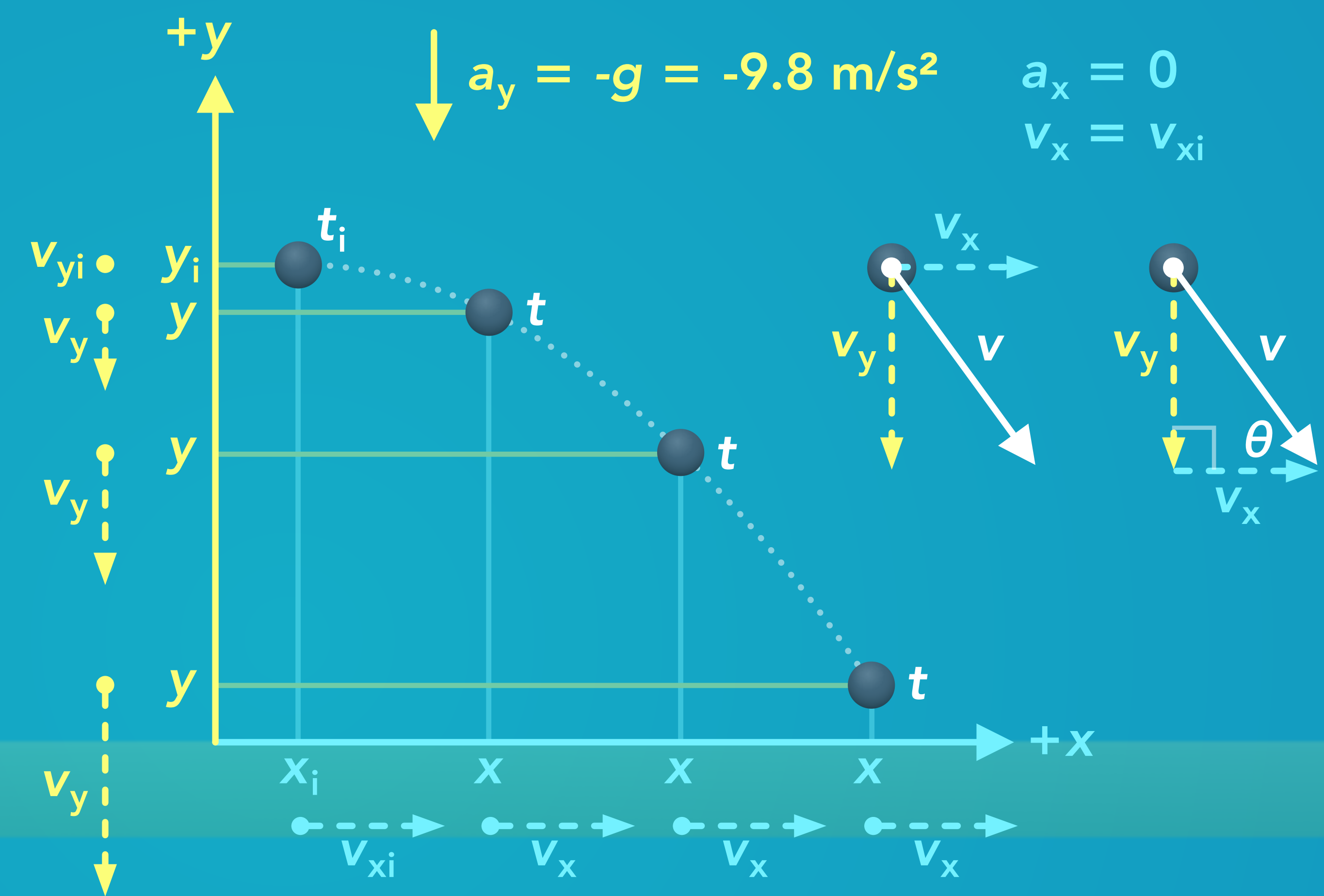
$$\Delta x = x - x_0$$

1D projectile motion



2D projectile motion

*The vertical and horizontal motion are **independent**



Vertical motion:

Displacement: $\Delta y = y_f - y_i$

Acceleration: $a_y = \frac{\Delta v_y}{\Delta t}$

Acceleration (rearranged): $v_{yf} = v_{yi} + a_y \Delta t$

Kinematic equations for constant acceleration: $y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$

$v_{yf}^2 = v_{yi}^2 + 2 a_y (y_f - y_i)$

Horizontal motion:

Displacement: $\Delta x = x_f - x_i$

Velocity: $v_x = \frac{\Delta x}{\Delta t}$

Velocity (rearranged): $x_f = x_i + v_x \Delta t$

- 1D projectile motion only includes motion in the vertical (*y*) direction and 2D projectile motion includes motion in the vertical (*y*) direction and the horizontal (*x*) direction.
- Like with any 2D motion, the horizontal and vertical motions (*x* and *y* motions) are independent from each other and we use separate variables and equations for each direction.

Range

Steps for finding the range:

1. Find the initial horizontal and vertical velocity components, $\boldsymbol{v}_{xi}$ and $\boldsymbol{v}_{yi}$

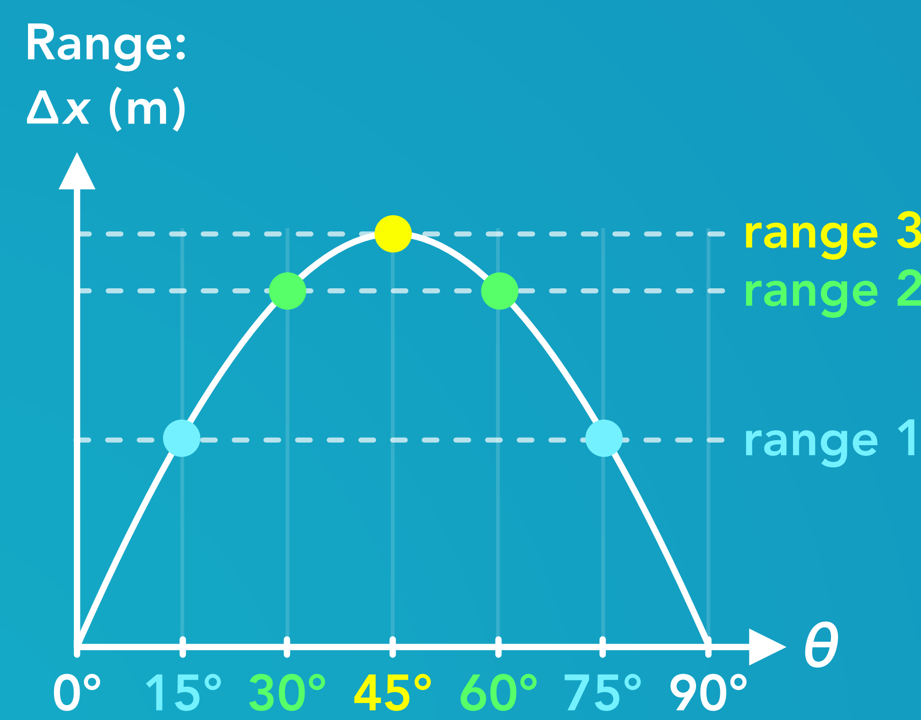
2. Find the time in the air from the y motion using this equation: $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

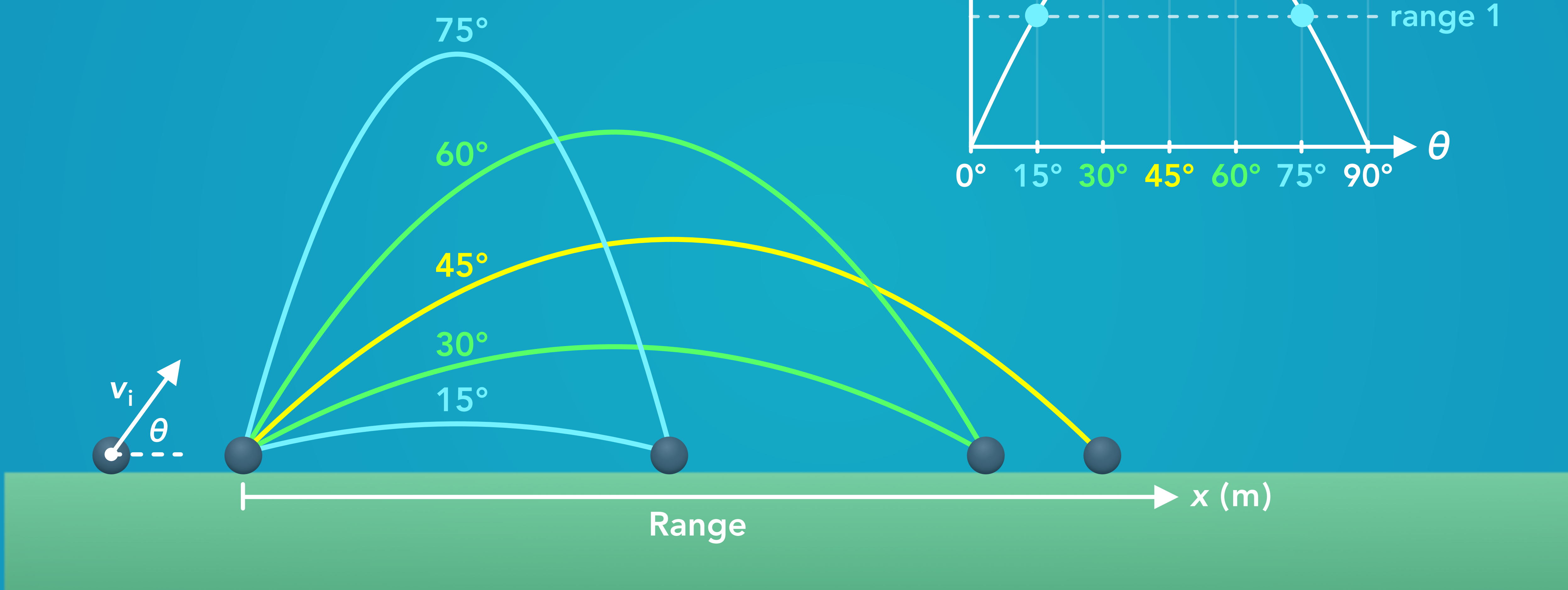
3. Use that time to find the range (horizontal displacement) using this equation: $\Delta x = v_x \Delta t$

Range:
(if $y_i = y_f$)

$$\Delta x = \frac{v_i^2 \sin(2\theta)}{g}$$

$\theta_{max\ range} = 45°$
(if $y_i = y_f$)





- The **range** of a projectile motion is the horizontal distance (Δx) traveled by the projectile.
- The range depends on the initial speed, initial angle, initial height and final height.
- If the initial and final heights are the same (like if a projectile starts and ends on the ground) then an initial launch angle of 45° will result in the maximum range (for any given initial speed). The range decreases as the angle moves farther from 45°. Two angles that are the same amount greater than and less than 45° (such as 30° and 60°) will result in the same range as each other.
- If the initial height is greater than the final height, the angle corresponding to the maximum range is less than 45°.

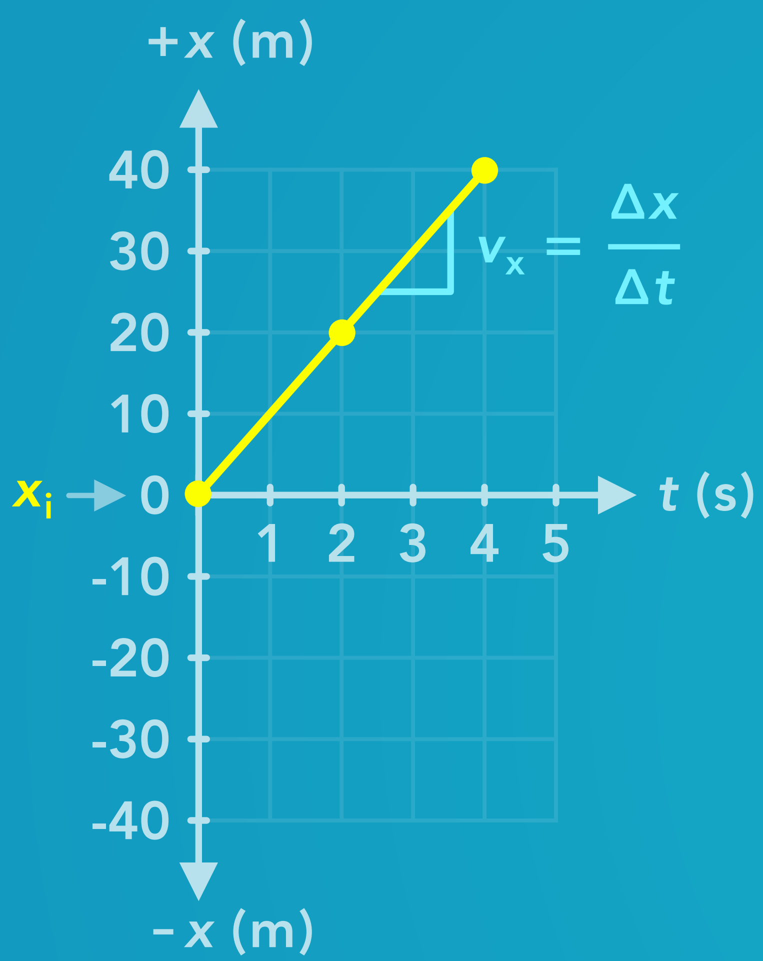
Here are some example graphs for the projectile motion shown below. The graphs for each projectile motion are different but there are some common things for every motion:

- There is no horizontal acceleration so a_x is always 0 m/s².
- The slope of the velocity graph is the acceleration so v_x is a flat line and is always the same as v_{xi} .
- The slope of the position graph is the velocity so x is a straight line with a constant slope.

x

$x_f = x_i + v_x \Delta t$

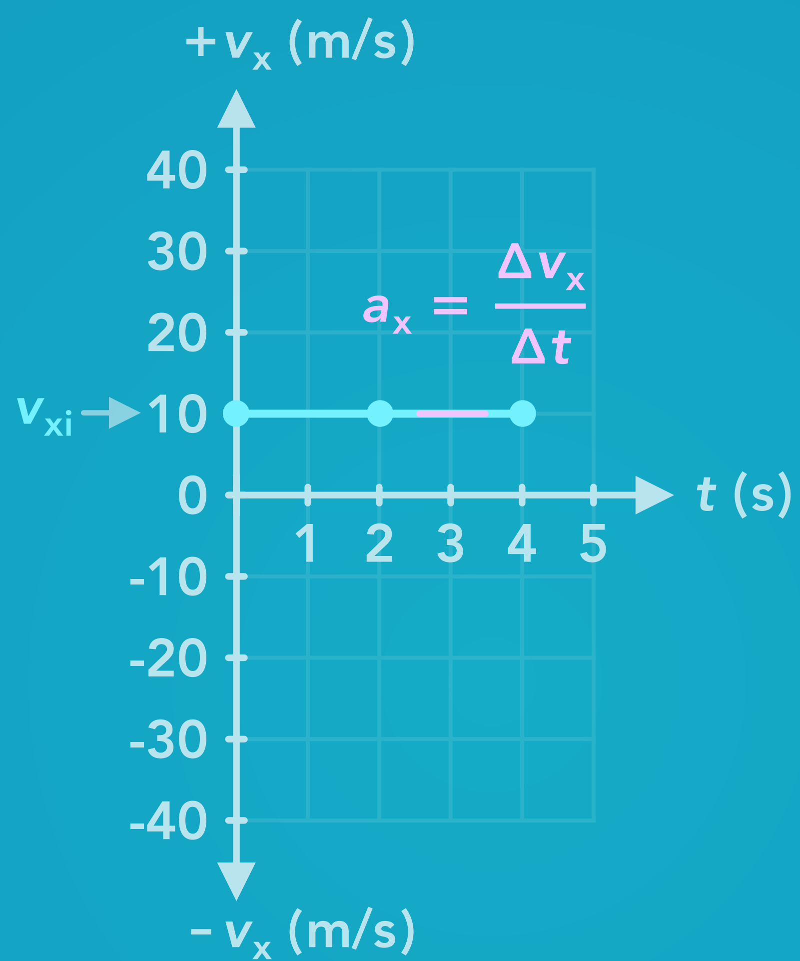
$x(t) = 10t$



v_x

$v_x = v_{xi}$

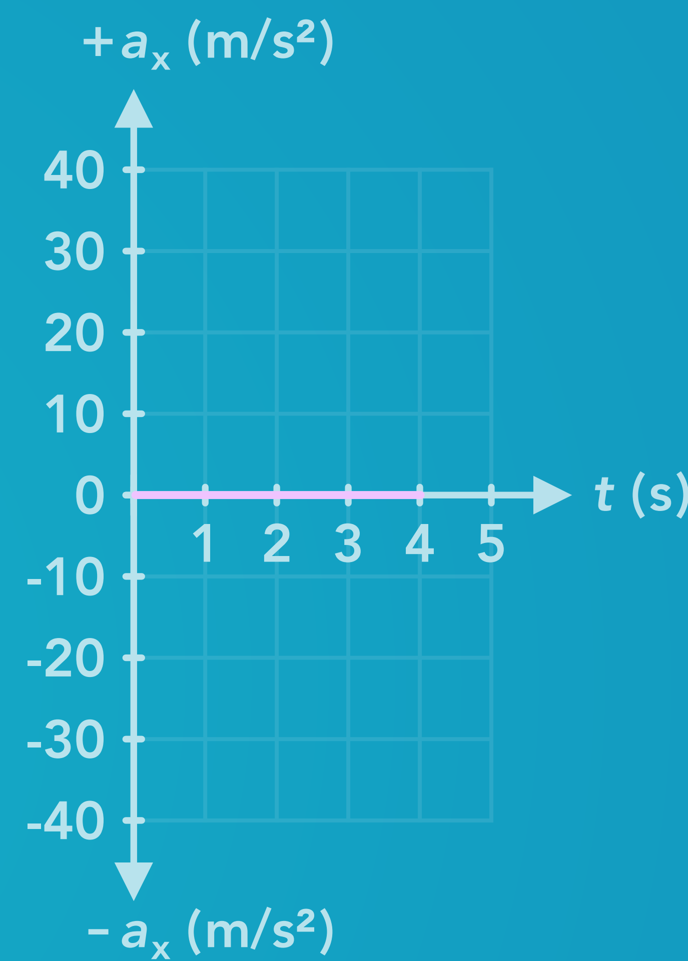
$v_x(t) = 10$

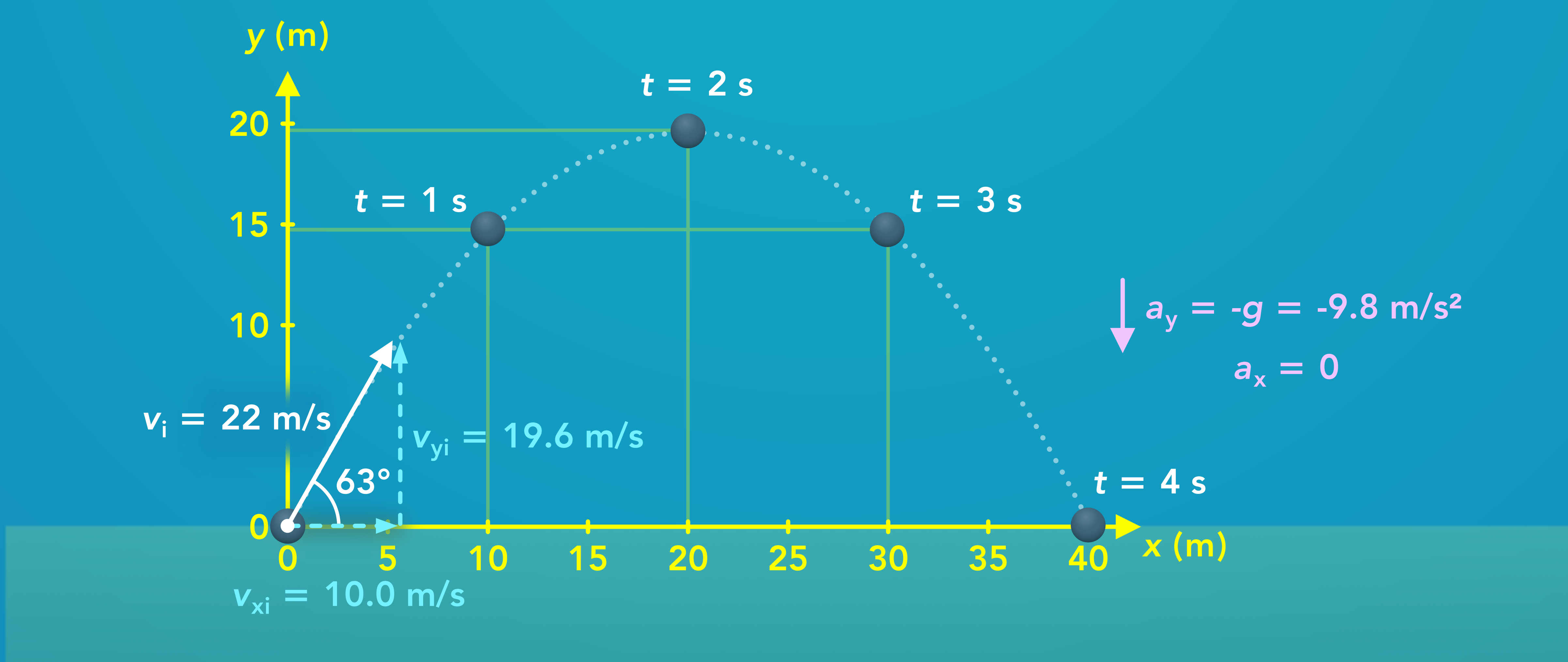


a_x

$a_x = 0$

$a_x(t) = 0$





- The vertical acceleration is always 9.8 m/s² downwards so a_y is -9.8 m/s² (if up is the positive direction).
- The slope of the velocity graph is the acceleration so v_y is a straight line with a constant slope. A projectile is at the maximum height in the trajectory when v_y is 0 m/s.
- The slope of the position graph is the velocity so the y graph is a curved line (parabola) because the velocity is changing (there is acceleration). y is at its maximum when the slope (velocity) is zero.