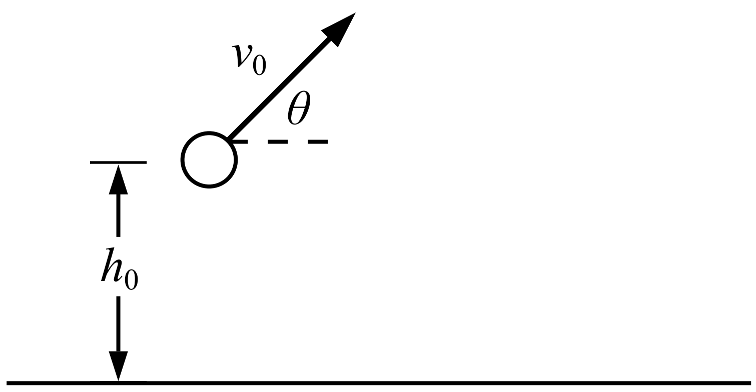
PROJECTILE MOTION



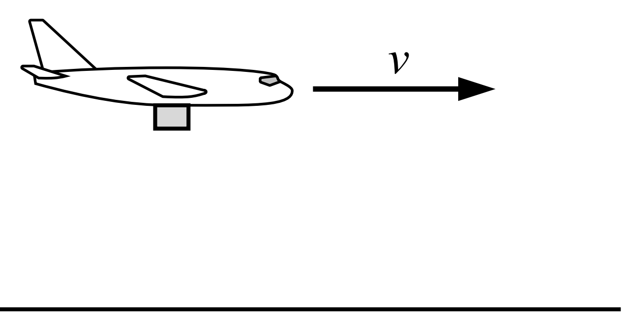
1. A ball at a height h_0 above the ground is given initial velocity v_0 as shown in the figure above. The magnitude of the velocity at the moment before the ball lands on the ground is

   (A) $\sqrt{(v_0\cos\theta)^2 + (v_0\sin\theta)^2}$

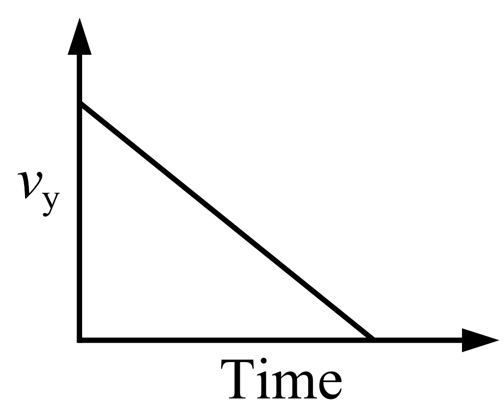
   (B) $\sqrt{(v_0\cos\theta)^2 + (v_0\sin\theta)^2 - 2gh_0}$

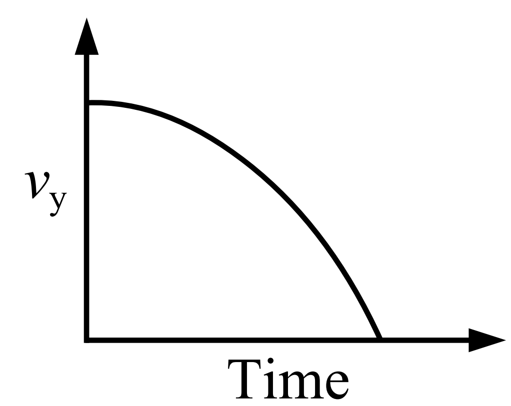
   (C) $v_0\sin\theta$

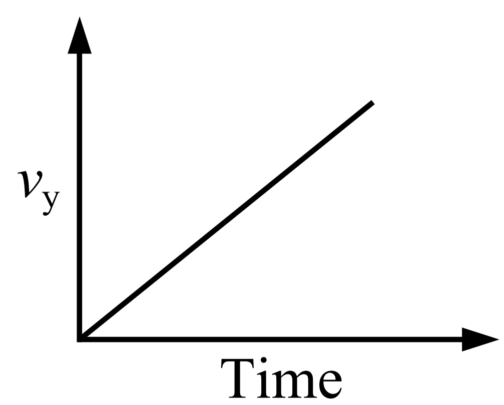
   (D) $\sqrt{(v_0\cos\theta)^2 + (v_0\sin\theta)^2 + 2gh_0}$

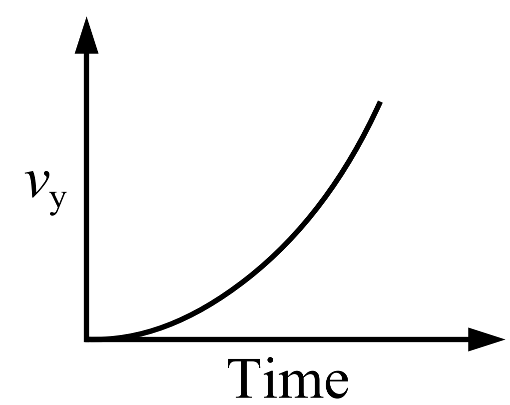


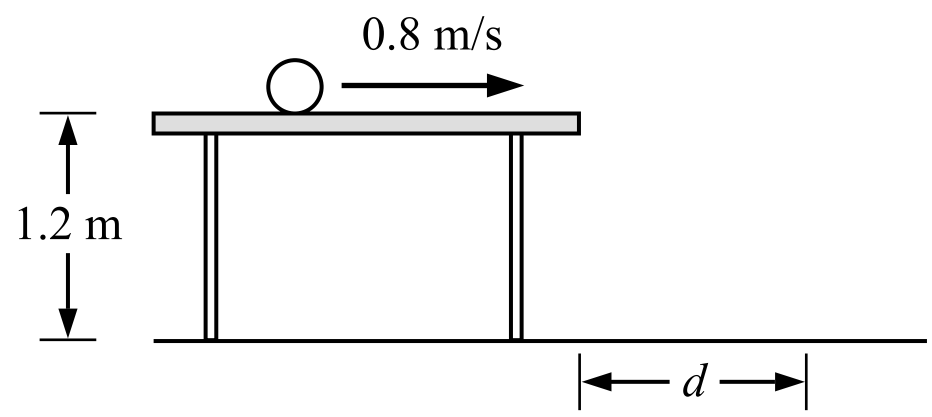
2. A plane is flying at a constant horizontal speed and is carrying a box as shown in the figure above. It then releases the box which falls through the air and lands on the ground. Ignoring air resistance, which of the following graphs shows the magnitude of the vertical velocity of the box before it lands on the ground?

   (A)
   

   (B)
   

   (C)
   

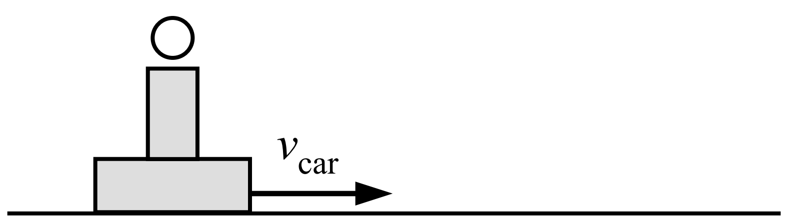
   (D)
   



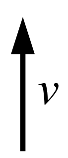
Note: Figure not drawn to scale.

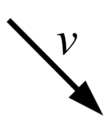
3. A ball is rolling on a table with a constant speed as shown in the figure above. The ball rolls off the table and lands on the ground. What distance d from the edge of the table does the ball first hit the ground?

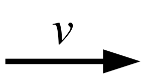
   (A) 0.4 m
   (B) 0.5 m
   (C) 0.8 m
   (D) 1.2 m

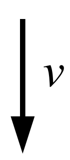


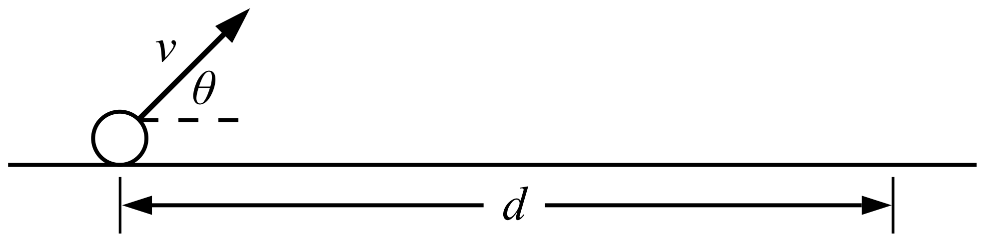
4. A ball is in a spring-loaded gun mounted on the top of a small car which is moving on a frictionless track as shown in the figure above. While the car is moving with a constant speed the ball is launched directly upwards relative to the car. The ball lands back in the gun 3 seconds later. The direction of the ball's velocity vector 2 seconds after being launched (relative to the ground) is most nearly

   (A) 

   (B) 

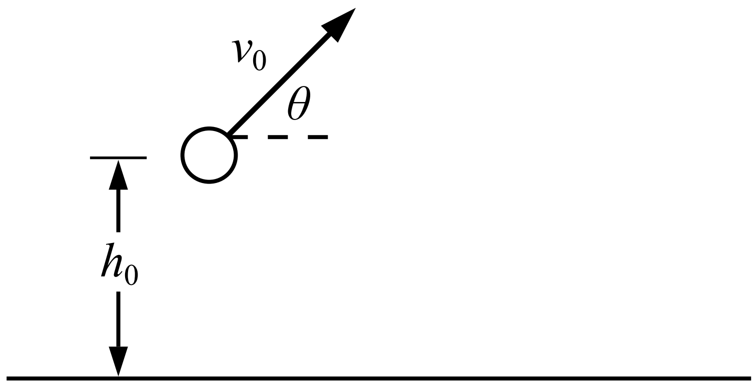
   (C) 

   (D) 



5. A projectile is launched from the ground with an initial speed and angle as shown in the figure above. It travels a distance d and lands on the ground. The projectile is launched five times with the same speed but five different angles: $\theta_1 = 52°$, $\theta_2 = 38°$, $\theta_3 = 45°$, $\theta_4 = 32°$, $\theta_5 = 60°$. It lands at distances d_1, d_2, d_3, d_4 and d_5 respectively. How do the distances compare to each other?

(A) $d_4 < d_2 < d_3 < d_1 < d_5$
(B) $d_5 < d_1 < d_3 < d_2 < d_4$
(C) $d_5 < d_4 < d_1 = d_2 < d_3$
(D) $d_5 < d_1 = d_4 < d_2 < d_3$



1. A ball at a height h_0 above the ground is given initial velocity v_0 as shown in the figure above. The magnitude of the velocity at the moment before the ball lands on the ground is

(A) $\sqrt{(v_0 \cos\theta)^2 + (v_0 \sin\theta)^2}$

(B) $\sqrt{(v_0 \cos\theta)^2 + (v_0 \sin\theta)^2 - 2gh_0}$

(C) $v_0 \sin\theta$

(D) $\sqrt{(v_0 \cos\theta)^2 + (v_0 \sin\theta)^2 + 2gh_0}$

A Incorrect
This answer is the magnitude of the initial velocity v_0.

B Incorrect
This answer is similar to the correct answer but either g or h_0 have the opposite sign.

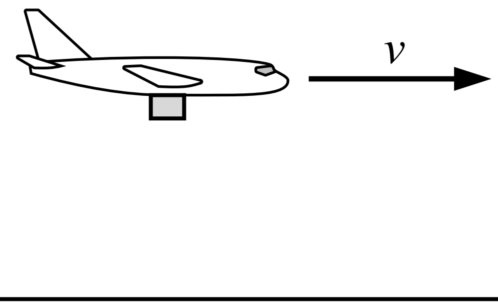
C Incorrect
This answer is the vertical component of the initial velocity v_0.

D Correct
The initial horizontal and vertical velocity components are $v_{xi} = v_0 \cos\theta$ and $v_{yi} = v_0 \sin\theta$. The final horizontal velocity component is the same as the initial component because the horizontal velocity is constant in projectile motion: $v_{xf} = v_{xi} = v_0 \cos\theta$. The final vertical velocity component can be found using this kinematic equation below. The acceleration and the vertical displacement must have the same sign.

$v_{yf}^2 = v_{yi}^2 + 2a_y \Delta y \qquad v_{yf}^2 = (v_0 \sin\theta)^2 + 2(-g)(-h_0)$

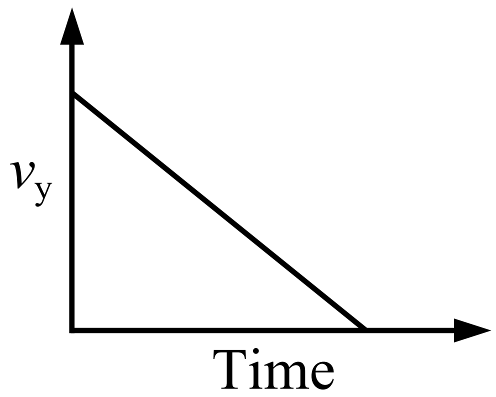
The magnitude of the final velocity can be found using the Pythagorean theorem and the final components:

$v_f^2 = v_{xf}^2 + v_{yf}^2 \qquad v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(v_0 \cos\theta)^2 + (v_0 \sin\theta)^2 + 2gh_0}$

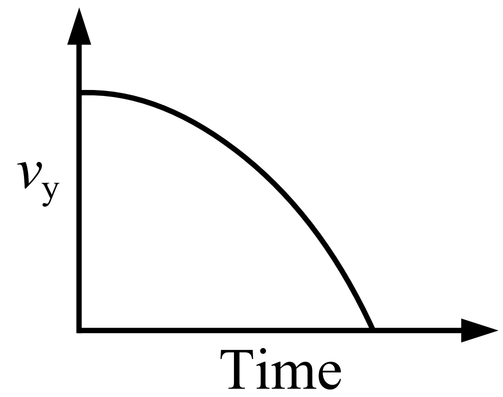


2. A plane is flying at a constant horizontal speed and is carrying a box as shown in the figure above. It then releases the box which falls through the air and lands on the ground. Ignoring air resistance, which of the following graphs shows the magnitude of the vertical velocity of the box before it lands on the ground?

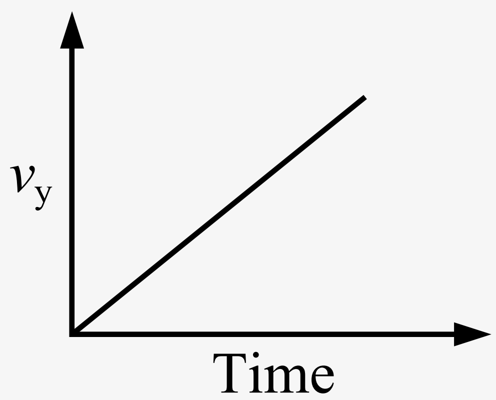
(A)



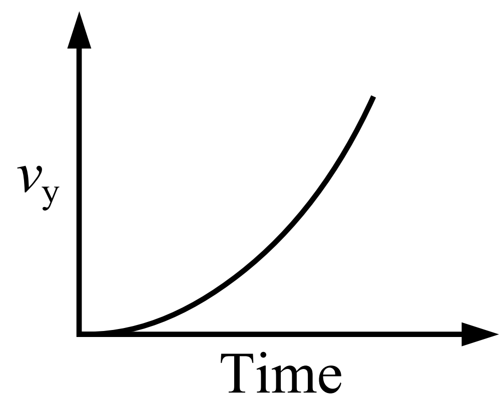
(B)



(C)



(D)

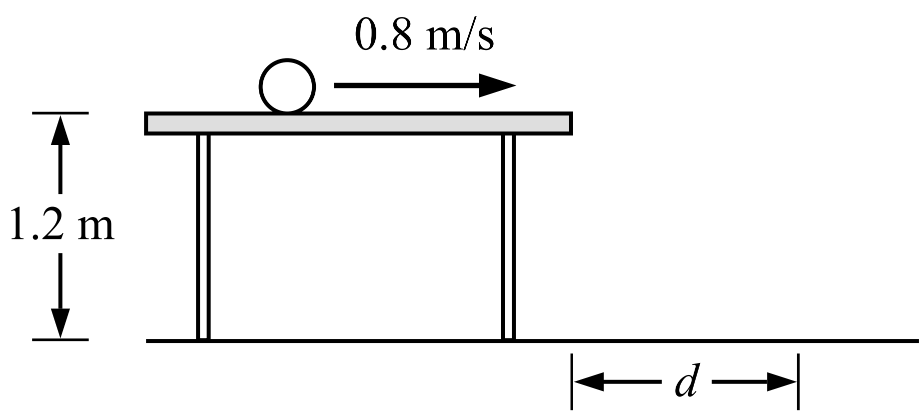


A Incorrect
This is a graph for an object with a positive initial vertical velocity which slows down over time with a constant acceleration.

B Incorrect
This is a graph for an object with a positive initial vertical velocity which slows down over time with a changing acceleration. This curve shows the height of the box over time if the vertical axis represented height.

C Correct
Before being released the box has the same velocity as the plane which is horizontal, so the initial vertical velocity of the box is zero. The box accelerates as it falls due to gravity, so the magnitude of the vertical velocity increases linearly with a constant slope (which is the acceleration).

D Incorrect
This is similar to the correct graph but the slope increases over time which represents an increasing acceleration, but the acceleration due to gravity is constant and air resistance is ignored.



Note: Figure not drawn to scale.

3. A ball is rolling on a table with a constant speed as shown in the figure above. The ball rolls off the table and lands on the ground. What distance d from the edge of the table does the ball first hit the ground?

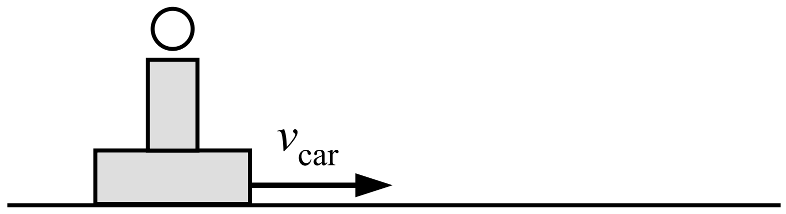
(A) 0.4 m
(B) 0.5 m
(C) 0.8 m
(D) 1.2 m

A Correct
The amount of time the ball takes to hit the ground depends on the vertical motion:
$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \qquad (0\text{ m}) = (1.2\text{ m}) + (0\text{ m/s})t + \frac{1}{2}(-10\text{ m/s}^2)t^2 \qquad t = 0.49\text{ s}$
The horizontal velocity is constant so the horizontal displacement is:
$v_x = \frac{\Delta x}{\Delta t} \qquad \Delta x = v_x \Delta t = (0.8\text{ m/s})(0.49\text{ s}) = 0.4\text{ m}$

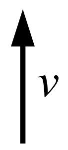
B Incorrect
This answer incorrectly switches the values for the table height and the velocity.

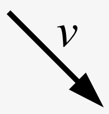
C Incorrect
This answer incorrectly assumes the ball takes 1 second to fall.

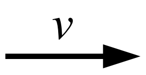
D Incorrect
This answer incorrectly assumes the ball will travel the same distance as the height.

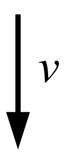


4. A ball is in a spring-loaded gun mounted on the top of a small car which is moving on a frictionless track as shown in the figure above. While the car is moving with a constant speed the ball is launched directly upwards relative to the car. The ball lands back in the gun 3 seconds later. The direction of the ball's velocity vector 2 seconds after being launched (relative to the ground) is most nearly

(A) 

(B) 

(C) 

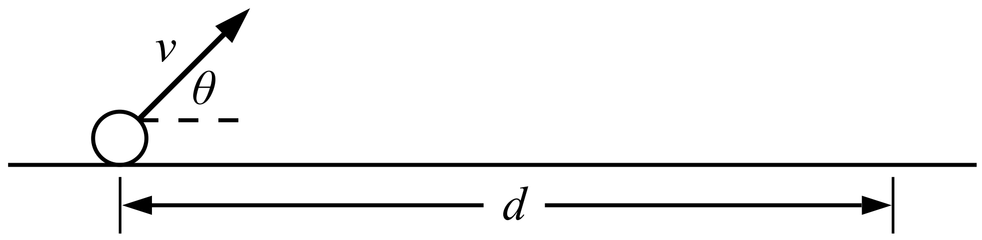
(D) 

A Incorrect
This answer is the direction of the ball's initial velocity vector relative to the car.

B **Correct**
The initial velocity vector of the ball (relative to the ground) has the same horizontal velocity component as the car's velocity and it has a vertical velocity component from being launched upwards. The ball follows a parabolic trajectory through the air (similar to a projectile launched at an angle) and ends at the same height after 3 seconds. The ball is at the maximum height in the trajectory in half of that time, 1.5 seconds. At 2 seconds the ball is falling down so it has a downwards vertical component and the initial horizontal component.

C Incorrect
This answer is the direction of the car's velocity, the ball's horizontal velocity component at any time, and the ball's velocity when it's at the maximum height in the trajectory (at 1.5 seconds).

D Incorrect
This answer is the direction of the vertical component of the ball's velocity between 1.5 seconds and 3 seconds but the ball's velocity also has a horizontal component.



5. A projectile is launched from the ground with an initial speed and angle as shown in the figure above. It travels a distance d and lands on the ground. The projectile is launched five times with the same speed but five different angles: $\theta_1 = 52°$, $\theta_2 = 38°$, $\theta_3 = 45°$, $\theta_4 = 32°$, $\theta_5 = 60°$. It lands at distances d_1, d_2, d_3, d_4 and d_5 respectively. How do the distances compare to each other?

(A) $d_4 < d_2 < d_3 < d_1 < d_5$
(B) $d_5 < d_1 < d_3 < d_2 < d_4$
(C) $d_5 < d_4 < d_1 = d_2 < d_3$
(D) $d_5 < d_1 = d_4 < d_2 < d_3$

A Incorrect

B Incorrect

C **Correct**
For a projectile that starts and ends at the same height, the maximum range corresponds to a launch angle of 45° (θ_3). Two angles that are the same amount greater than and less than 45° result in the same range (θ_1 and θ_2 are both 7° away from 45°). Angles farther from 45° result in a shorter range.

D Incorrect