

Example 2: In DR Lat. $33^{\circ}24.0'S$, the azimuth of the Sun is observed as 042.5° pgc. At the time of the observation, the declination of the Sun is $20^{\circ}13.8'N$; the local hour angle of the Sun is $316^{\circ}41.2'$.

Example 3: The gyro error.

Solution: By Pub 229. The error of the gyrocompass is found as shown in Table 2016b.

AMPLITUDES

2017. Amplitudes

For checking the compass, an azimuth observation of a celestial body at low altitude is desirable because it can be measured easiest and most accurately. If the body is observed when its center is on the celestial horizon, the **amplitude** (A), which is the arc of the horizon between the prime vertical and the body, can be taken directly from Volume II, Table 22.

The amplitude is given the prefix E (east) if the body is rising and W (west) if setting. It is given the suffix N if the body rises or sets north of the prime vertical (which it does if it has northerly declination) and S if it rises or sets south of the prime vertical (having southerly declination). The suffix is given to agree with the declination of the body. Interconversion of amplitude and azimuth is similar to that of azimuth angle and azimuth. Thus, if $A = E15^{\circ}S$, the body is 15° south of east or $90^{\circ} + 15^{\circ} = 105^{\circ}$. For any given body, the numerical value of amplitude would be the same at rising and setting if the declination did not change.

When the center of the Sun is on the celestial horizon, its lower limb is about two-thirds of a diameter above the visible horizon, they are a little more than one sun diameter above the visible horizon. In high latitudes, amplitudes should be observed on the visible horizon.

If the body is observed when its center is on the visible horizon the observed value should be corrected by the value from the Volume II, Table 23, using the rules given with the table, before comparison with the value taken from Table 22. If preferred, the correction can be applied with reversed sign to the value taken from Table 22 and compared with the uncorrected observed value. This is the procedure used if amplitude or azimuth is desired when the celestial body is on the visible horizon.

A celestial body's **amplitude angle** is the complement of its azimuth angle. At the moment that a body rises or sets, the amplitude angle is the arc of the horizon between the body and the East/West point of the horizon where the observer's prime vertical intersects the horizon (at 90°), which is also the point where the plane of the equator intersects the horizon (at an angle numerically equal to the observer's colatitude), see Figure 2017.

In practical navigation, a bearing (psc or pgc) of a body can be observed when it is on either the celestial or the visible horizon. To determine compass error, simply convert the computed amplitude angle to true degrees and compare it with the observed compass bearing.

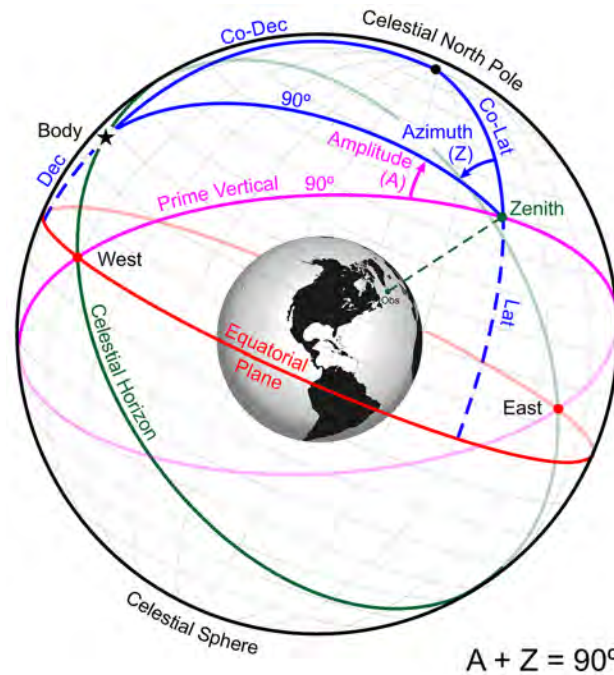


Figure 2017. The amplitude angle (A) subtends the arc of the horizon between the body and the point where the prime vertical and the equator intersect the horizon. Note that it is the complement of the azimuth angle (Z).

The angle is computed by the formula:

$$\sin A = \sin \text{Dec} / \cos \text{Lat.}$$

This formula gives the angle at the instant the body is on the celestial horizon. It does not contain an altitude term because the body's computed altitude is zero at this instant.

The angle is prefixed E if the body is rising and W if it is setting. This is the only angle in celestial navigation referenced **FROM** East or West, i.e. from the prime vertical. A body with northerly declination will rise and set North of the prime vertical. Likewise, a body with southerly declination will rise and set South of the prime vertical. Therefore, the angle is suffixed N or S to agree with the name of the body's declination. A body whose declination is zero rises and sets exactly on the prime vertical.

Due largely to refraction, dip, and its disk size, the Sun is on the celestial horizon when its lower limb is approximately two thirds of a diameter above the visible horizon. The Moon is on the celestial horizon when its upper limb is

on the visible horizon. Stars and planets are on the celestial horizon when they are approximately one Sun diameter above the visible horizon.

2018. Amplitude of a Body Observed on the Celestial Horizon

Mariners may use Volume II, Table 22 (Amplitudes) to determine the Sun's computed amplitude. The procedure is similar to that done in Section 2017. Comparing the computed amplitude to the amplitude measured with the gyro-compass determines the gyro error. In computing an amplitude, interpolate the tabular amplitude angle for the difference between the table arguments and the actual values of declination and latitude.

Do this double interpolation of the amplitude angle as follows:

- Enter Volume II, Table 22 (Amplitudes) with the nearest integral values of declination and latitude. Extract a base amplitude angle.
- Reenter the table with the same declination argument but with the latitude to the next tabulated value (greater or less than the base latitude argument, depending upon whether the actual latitude is greater or less than the base argument). Record the amplitude and the difference between it and the base amplitude angle and label it Diff.
- Reenter the table with the base latitude argument but with the declination to the next tabulated value (greater or less than the base declination argument, depending upon whether the actual declination is

greater or less than the base argument). Record the amplitude and the difference between it and the base amplitude angle and label it Diff.

- Compute the corrections due to latitude and declination not being exactly at a tabular value. Apply these corrections to obtain a final amplitude. The final amplitude is then converted to a true bearing. The difference between the true bearing and the gyro bearing gives the gyro error.

Example: The DR latitude of a ship is 51° 24.6' N. The navigator observes the setting Sun on the celestial horizon. Its declination is N 19° 40.4'. Its observed bearing is 303° pgc.

Required: Gyro error.

Solution: Interpolate in Table 22 for the Sun's calculated amplitude as follows. See Figure 2018.

Find the tabulated values of latitude and declination closest to these actual values. In this case, these tabulated values are L = 51° and dec. = 19.5°. Record the amplitude corresponding to these base values, 32.0°, as the base amplitude.

Next, holding the base declination value constant at 19.5°, increase the value of latitude to the next tabulated value: N 52°. Note that this value of latitude was increased because the actual latitude value was greater than the base value of latitude. Record the tabulated amplitude for L = 52° and dec. = 19.5°: 32.8°. Then, holding the base latitude value constant at 51°, increase the declination value to the next tabulated value: 20°. Record the tabulated amplitude for L = 51° and dec. = 20°: 32.9°.

Latitude	Declination												Latitude	
	18.0°	18.5°	19.0°	19.5°	20.0°	20.5°	21.0°	21.5°	22.0°	22.5°	23.0°	23.5°		24.0°
0	18.0	18.5	19.0	19.5	20.0	20.5	21.0	21.5	22.0	22.5	23.0	23.5	24.0	0
10	18.3	18.8	19.3	19.8	20.3	20.8	21.3	21.8	22.4	22.9	23.4	23.9	24.4	10
15	18.7	19.2	19.7	20.2	20.7	21.3	21.8	22.3	22.8	23.3	23.9	24.4	24.9	15
20	19.2	19.7	20.3	20.8	21.3	21.9	22.4	23.0	23.5	24.0	24.6	25.1	25.6	20
25	19.9	20.5	21.1	21.6	22.2	22.7	23.3	23.9	24.4	25.0	25.5	26.1	26.7	25
49	28.1	28.9	29.8	30.6	31.4	32.3	33.1	34.0	34.8	35.7	36.6	37.4	38.3	49
50	28.7	29.6	30.4	31.3	32.1	33.0	33.9	34.8	35.6	36.5	37.4	38.3	39.3	50
51	29.4	30.3	31.2	32.0	32.9	33.8	34.7	35.6	36.5	37.5	38.4	39.3	40.3	51
52	30.1	31.0	31.9	32.8	33.7	34.7	35.6	36.5	37.5	38.4	39.4	40.4	41.3	52
53	30.9	31.8	32.8	33.7	34.6	35.6	36.5	37.5	38.5	39.5	40.5	41.5	42.5	53
54	31.7	32.7	33.6	34.6	35.6	36.6	37.6	38.6	39.6	40.6	41.7	42.7	43.8	54
55	32.6	33.6	34.6	35.6	36.6	37.6	38.7	39.7	40.8	41.9	42.9	44.0	45.2	55
56				36.7	37.7	38.8	39.9	41.0	42.1	43.2	44.3	45.5	46.7	56

Figure 2018. Extracts from Table 22.

The latitude's actual value (51.4°) is 0.4 of the way between the base value (51°) and the value used to determine the tabulated amplitude (52°). The declination's actual value (19.67°) is 0.3 of the way between the base value (19.5°) and the value used to determine the tabulated amplitude (20.0°). To determine the total correction to base amplitude, multiply these increments (0.4 and 0.3) by the respective difference between the base and tabulated values (+0.8 and +0.9, respectively) and sum the products. The total correction is +0.6°. Add the total correction (+0.6°) to the base amplitude (32.0°) to determine the final amplitude (32.6°) which will be converted to a true bearing.

Because of its northerly declination (in this case), the Sun was 32.6° north of west when it was on the celestial horizon. Therefore its true bearing was 302.6° (270° + 32.6°) at this moment. Comparing this with the gyro bearing of 303° gives an error of 0.4°W, which can be rounded to 1/2°W.

$L = 51^{\circ} 24.6' N = 51.4^{\circ} N$
 $dec. = N 19^{\circ} 40.4' = N 19.67^{\circ}$

	19.5°	19.67°	20.0°
51°	32.0	32.3	32.9
51.41°		32.6	
52°	32.8	33.1	33.7

 $pgc = 303.0^{\circ}$

$W 32.6^{\circ} N = 302.6^{\circ} T$
 Gyro Error = 0.4° W.

2019. Amplitude of a Body Observed on the Visible Horizon

In higher latitudes, amplitude observations should be made when the body is on the visible horizon because the value of the correction is large enough to cause significant error if the observer misjudges the exact position of the celestial horizon. The observation will yield precise results whenever the visible horizon is clearly defined.

When observing a body on the visible horizon, a correction from Volume II, Table 23 - *Correction of Amplitude as Observed on the Visible Horizon* must be applied. This correction accounts for the slight change in bearing as the body moves between the visible and celestial horizons. It reduces the bearing on the visible horizon to the celestial horizon, from which the table is computed.

For the Sun, stars, and planets, apply this correction to the observed bearing in the direction away from the elevated pole. For the Moon, apply one half of the correction toward the elevated pole. Note that the algebraic sign of the correction does not depend upon the body's declination, but only on the observer's latitude.

TABLE 23 Correction of Amplitude as Observed on the Visible Horizon														
Latitude	Declination													Latitude
	0°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°	
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
42	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.7	0.7	0.7	0.7	0.7	0.7	10
44	0.7	0.7	0.7	0.6	0.6	0.7	0.7	0.7	0.8	0.8	0.8	0.8	0.8	15
46	0.7	0.7	0.7	0.7	0.7	0.8	0.8	0.8	0.8	0.8	0.8	0.9	0.9	46
48	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.8	0.9	0.9	1.0	1.0	1.0	48
50	0.8	0.8	0.8	0.8	0.9	0.9	0.9	0.9	0.9	1.0	1.0	1.1	1.0	50
51	0.8	0.8	0.8	0.8	0.8	0.9	0.9	0.9	0.9	1.0	1.1	1.1	1.1	51
52	0.9	0.9	0.9	0.9	0.9	0.9	1.0	1.0	1.0	1.1	1.1	1.1	1.3	52
53	0.9	0.9	0.9	0.9	0.9	0.9	1.0	1.0	1.0	1.1	1.2	1.2	1.3	53
54	1.0	1.0	1.0	1.0	1.0	1.0	1.1	1.1	1.1	1.2	1.2	1.3	1.3	54
55	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.1	1.2	1.3	1.3	1.4	55
75.0	2.6	2.7	2.8	2.9	3.2	3.7	4.7	9.3						75.0
75.5	2.7	2.8	2.8	3.0	3.3	3.9	5.3							75.5
76.0	2.8	2.8	2.9	3.2	3.5	4.2	5.6							76.0
76.5	2.9	3.0	3.1	3.3	3.7	4.5	7.3							76.5
77.0	3.0	3.1	3.2	3.5	4.0	5.1	10.2							77.0

19.67°
51.41°

For the sun, a planet, or a star, apply the correction to the observed amplitude in the direction away from the elevated pole. For the moon apply half the correction toward the elevated pole.

Figure 2019a. Extracts from Table 23 for Example 1.

TABLE 22
Amplitudes

Latitude	Declination												Latitude	
	18.0°	18.5°	19.0°	19.5°	20.0°	20.5°	21.0°	21.5°	22.0°	22.5°	23.0°	23.5°		24.0°
0	18.0	18.5	19.0	19.5	20.0	20.5	21.0	21.5	22.0	22.5	23.0	23.5	24.0	0
10	18.3	18.8	19.3	19.8	20.3	20.8	21.3	21.8	22.4	22.9	23.4	23.9	24.4	10
15	18.7	19.2	19.7	20.2	20.7	21.3	21.8	22.3	22.8	23.3	23.9	24.4	24.9	15
20	19.2	19.7	20.3	20.8	21.3	21.9	22.4	23.0	23.5	24.0	24.6	25.1	25.6	20
25	19.9	20.5	21.1	21.6	22.2	22.7	23.3	23.9	24.4	25.0	25.5	26.1	26.7	25
30	20.9	21.5	22.1	22.7	23.3	23.9	24.4	25.0	25.6	26.2	26.8	27.4	28.0	30
32	21.4	22.0	22.6	23.2	23.8	24.4	25.0	25.6	26.2	26.8	27.4	28.0	28.7	32
34	21.9	22.5	23.1	23.7	24.4	25.0	25.6	26.2	26.9	27.5	28.1	28.7	29.4	34
36	22.5	23.1	23.7	24.4	25.0	25.7	26.3	26.9	27.6	28.2	28.8	29.4	30.1	36
38	23.1	23.7	24.4	25.1	25.7	26.4	27.1	27.7	28.4	29.0	29.6	30.2	30.8	38

Annotations: 18.4° (Declination), 33.41° (Latitude)

Figure 2019b. Extracts from Table 22 for Example 2.

TABLE 23
Correction of Amplitude as Observed on the Visible Horizon

Latitude	Declination												Latitude	
	0°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°		24°
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0
10	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	10
15	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	15
20	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	20
25	0.3	0.3	0.3	0.3	0.3	0.4	0.3	0.3	0.3	0.3	0.3	0.3	0.3	25
30	0.4	0.4	0.4	0.4	0.5	0.4	0.4	0.4	0.4	0.4	0.4	0.5	0.5	30
32	0.4	0.4	0.4	0.4	0.5	0.4	0.4	0.4	0.4	0.4	0.4	0.5	0.5	32
34	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	34
36	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.6	0.5	0.6	0.6	0.6	36
38	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	38

Annotations: 18.4° (Declination), 33.41° (Latitude)

74.0	2.4	2.4	2.5	2.7	2.9	3.3	3.8	5.6						74.0
74.5	2.5	2.6	2.7	2.8	3.0	3.4	4.2	6.8						74.5
75.0	2.6	2.7	2.8	2.9	3.2	3.7	4.7	9.3						75.0
75.5	2.7	2.8	2.8	3.0	3.3	3.9	5.3							75.5
76.0	2.8	2.8	2.9	3.2	3.5	4.2	5.6							76.0
76.5	2.9	3.0	3.1	3.3	3.7	4.5	7.3							76.5
77.0	3.0	3.1	3.2	3.5	4.0	5.1	10.2							77.0

For the sun, a planet, or a star, apply the correction to the observed amplitude in the direction away from the elevated pole. For the moon apply half the correction toward the elevated pole.

Figure 2019c. Extracts from Table 23 for Example 2.

Example 1: The DR latitude of a ship is 51°24'.6N, at a time when the declination of the Sun is 19°40'.4N.

Required: (1) The amplitude (A) when the center of the setting Sun is on the celestial horizon. (2) The amplitude when the center of the setting Sun is on the visible horizon. (3) The azimuth when the center of the setting Sun is on the visible horizon.

Solutions:

(1) A W32.6°N (Table 22, see Figure 2018.)

T 23 1.1°S (away from elevated pole, see Figure 2019a.)

(2) A W33.7°N

(3) Zn 303.7°

Example 2: The DR latitude of a ship is 33°24.6' S, at a time when the declination of the Moon 18° 24' S. An amplitude of the Moon is observed with the center of the Moon is on the visible horizon bearing 108.0° psc. The variation is 2° E.