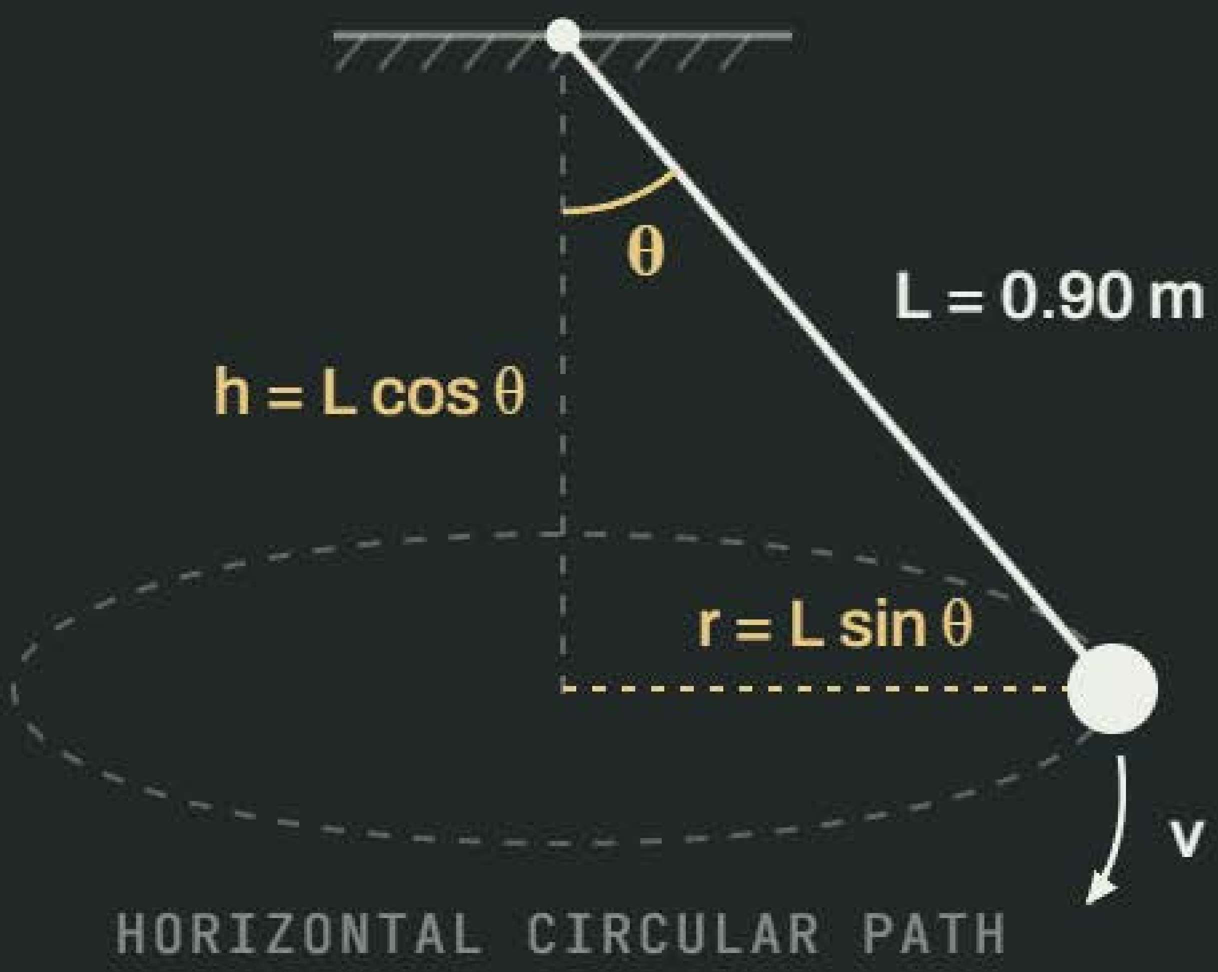
Can a conical pendulum keep time?

A ball on a light string of length $L = 0.90$ m is driven in a horizontal circle at constant speed. The string sweeps a cone of half-angle θ.



CONICAL

$$T = 2\pi\sqrt{L\cos\theta / g}$$

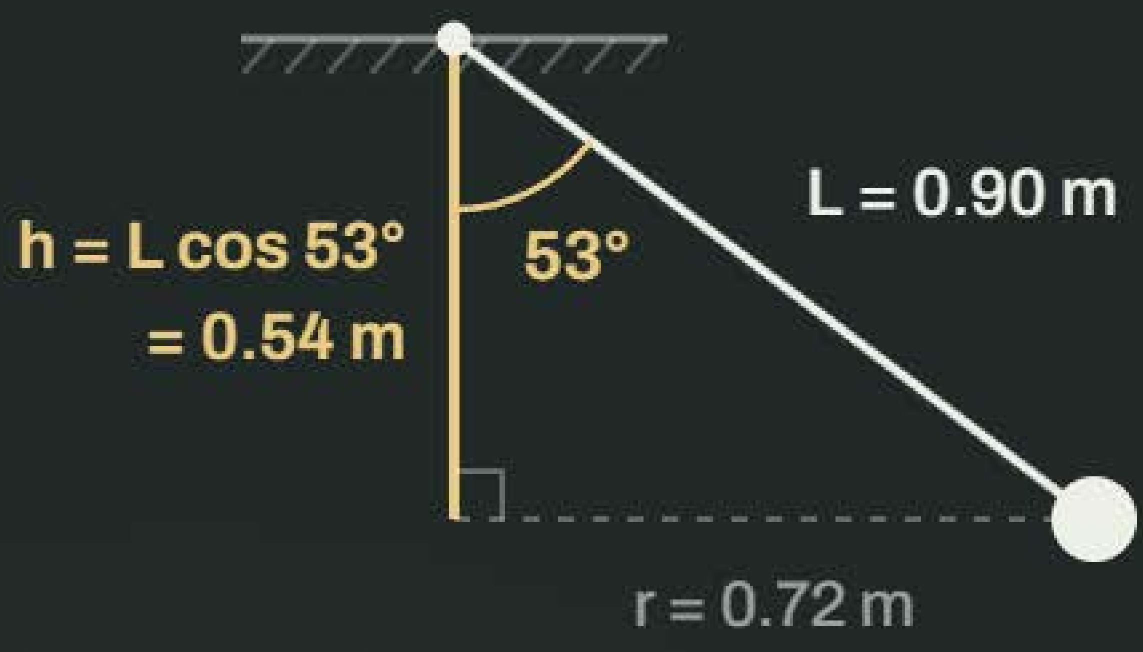
SIMPLE, SMALL AMPLITUDE

$$T_s = 2\pi\sqrt{L / g}$$

Notice what is missing: no mass, no speed. Only L and θ set the period.

$g = 10\ \text{m/s}^2$

Set θ = 53°. Find the period.



$$T = 2\pi\sqrt{L\cos\theta / g}$$
$$= 2\pi\sqrt{(0.90)(0.60) / 10}$$
$$= 2\pi\sqrt{0.054} = 2\pi(0.232)$$

ANSWER

$T \approx 1.46$ s

THE IDEA WORTH KEEPING

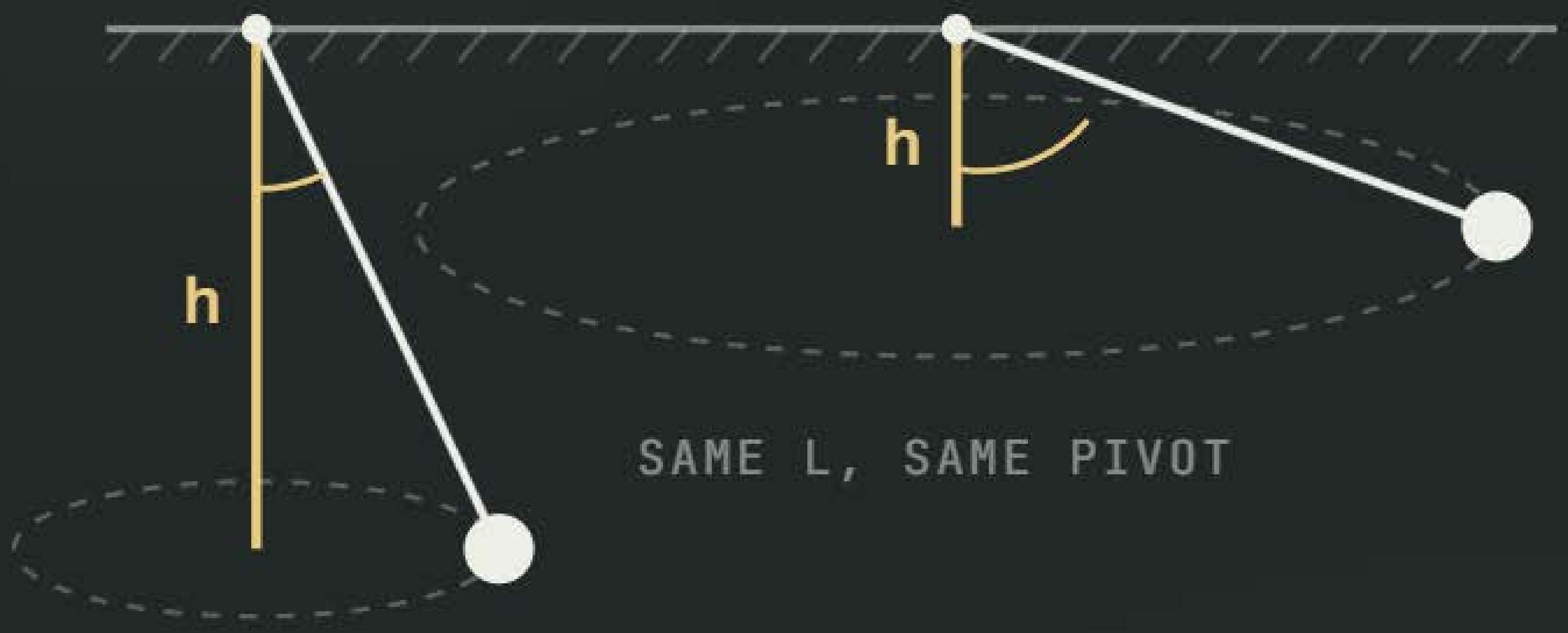
Only the cone's vertical height $h = L\cos\theta$ sets the period — so $T = 2\pi\sqrt{h / g}$. The cone is a pendulum of length h.

TRAP

cos 53° = 0.60, not 0.80. Reaching for sin gives 1.69 s.

Open the cone wider. What happens to T?

The period gets shorter.



$\theta = 25°$
$h = 0.82$ m → $T = 1.79$ s

$\theta = 70°$
$h = 0.31$ m → $T = 1.10$ s

THE CHAIN

$\theta\uparrow \rightarrow \cos\theta\downarrow \rightarrow h = L\cos\theta\downarrow \rightarrow T \propto \sqrt{h}\downarrow$

TRAP

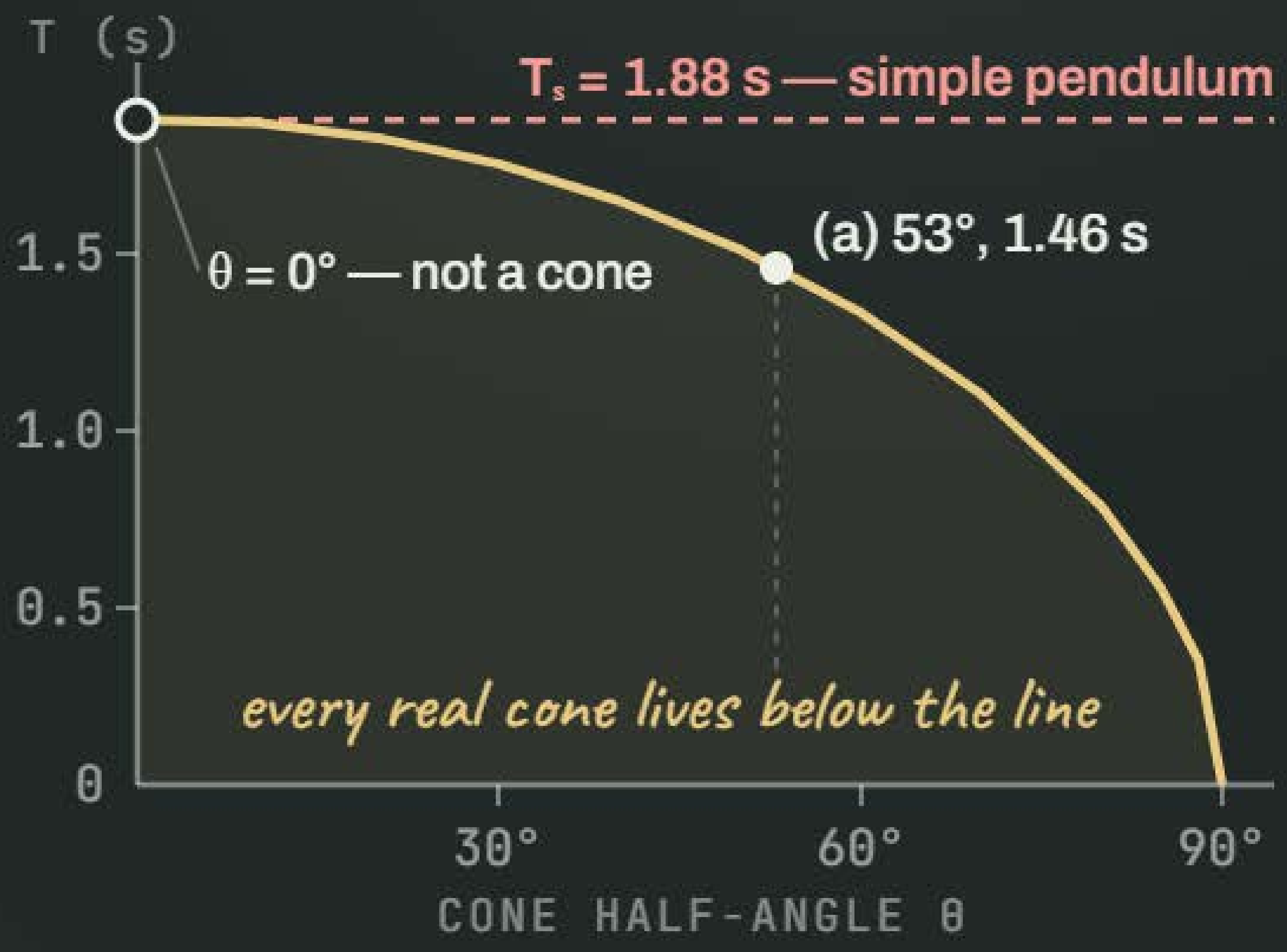
"Bigger circle, longer trip." No — the speed climbs faster than the circumference does.

Can T ever equal the simple pendulum's T_s?

$$2\pi\sqrt{L\cos\theta / g} = 2\pi\sqrt{L / g}$$

$$\cos\theta = 1 \Rightarrow \theta = 0°$$

The algebra allows exactly one angle — and at $\theta = 0°$ the string hangs straight down. No circle, no cone, no conical pendulum.



So $T < T_s$ for every real cone. It only approaches T_s in the limit $\theta \to 0°$.

TRAP

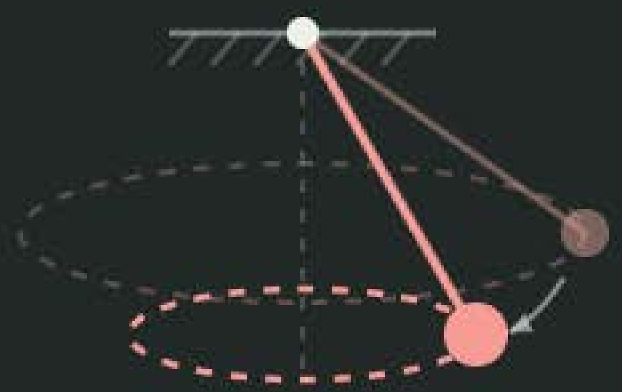
Answering just "no, it can't" leaves points on the table. Show the $\theta = 0°$ solution first, then rule it out physically.

"Its period depends on how fast you launch it."

Half right — but not for the reason the student thinks. Launch speed never appears in T. It matters only because it **chooses the angle.**

$v = \sqrt{g L \sin\theta \tan\theta}$ so $v\uparrow \rightarrow \theta\uparrow \rightarrow h\downarrow \rightarrow T\downarrow$

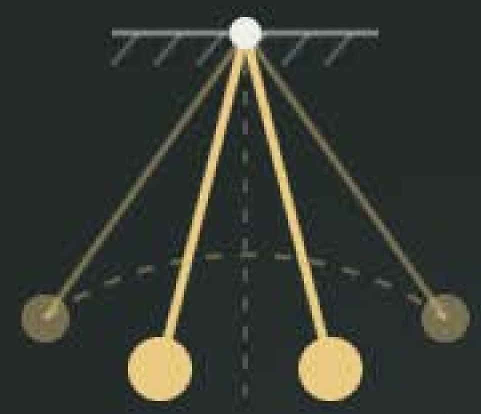
The real reason it fails: a clock must be isochronous — its period cannot care how much energy is left.



θ shrinks → h grows → T grows

CONICAL — FAILS

Energy drains, v drops, the cone narrows, h grows — **T drifts longer every minute.**



amplitude shrinks → T unchanged

SIMPLE — PASSES

T_s holds no amplitude at all — **the swing dies, the beat doesn't.**

$$T = 2\pi\sqrt{L \cos\theta / g} = 2\pi\sqrt{h / g}$$

the cone's vertical height is the whole clock

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01 Mass and speed never enter T. Only L and θ do.

02 Wider cone → shorter period, always. And $T < T_s$ for every angle a real cone can have.

03 Isochronism is what makes a timekeeper. Lose energy: the cone's period drifts, the simple pendulum's does not.