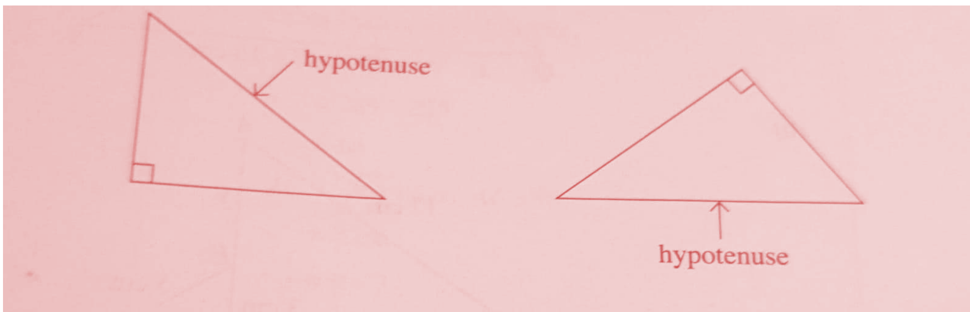
NOTES:

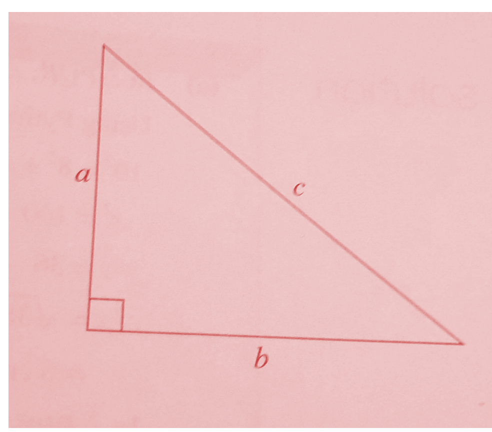
9: Pythagoras' Theorem

Pythagoras' Theorem:
1, In a right-angled triangle, the side that is opposite to the right angle, which is the longest side of the triangle, is called the **hypotenuse**.



2. The **Pythagoras' Theorem** states that, for any **right-angled** triangle, the **square** of the length of the **hypotenuse** is equal to the **sum of the squares** of the length of the **other two sides**.

Pythagoras' Theorem:

For any right-angled triangle, $c^2 = a^2 + b^2$ where c is the length of the hypotenuse. Note that if the length of any two sides of a right-angled triangle are given, the length of the third side can be found using the Pythagoras' Theorem	

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Finding Lengths of A Right-Angled Triangle:
3. The Pythagoras' Theorem can be used to find out **any length** of the triangle if the **other two lengths** of the triangle are **given** and that the triangle is a **right-angled** triangle.

Let there be a right-angled triangle such that c is the hypotenuse, a & b are the other two sides.

To find c (the hypotenuse), $c = \sqrt{a^2 + b^2}$

To find a (side length), $a = \sqrt{c^2 - b^2}$

To find b (side length), $b = \sqrt{c^2 - a^2}$

All these are derived from the Pythagoras' Theorem.

Commonly Seen Right-Angled Triangles:
4. There are many right-angled triangles with **all sides being of integer length**. Here are some **common** ones that may come out in questions.

Side length, a	**Side length,** b	**Hypotenuse,** c
3	4	5
5	12	13
6	8	10
7	24	25

Converse of Pythagoras' Theorem:
5. Of course, we cannot memorize the exhaustive lists of these right-angled triangles. **The converse of Pythagoras' Theorem** is also true, and thus can be used to **prove** that a particular triangle is a right-angled triangle.

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Converse of Pythagoras' Theorem:

In $\Delta ABC,\ if\ c^2 = a^2 + b^2$

then we can conclude
(i) ΔABC is a right-angled triangle, and
(ii) $\angle ACB = 90°$.

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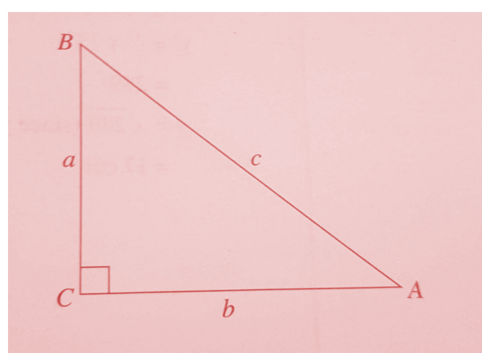

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Solving Questions:

1. Find the length of the unknown sides in each of the following triangles.
(a) $In\ \Delta PRS,\ \angle PRS = 90°,\ PR = 8,\ RS = 15,\ find\ PS$
(b) $In\ \Delta PRQ,\ \angle PRQ = 90°,\ PR = 8,\ PQ = 10,\ find\ QR$

Solution:

(a) $In\ \Delta PRS,\ \angle R = 90°$
Using Pythagoras' Theorem,
$PS^2 = PR^2 + RS^2$
$PS^2 = 8^2 + 15^2$
$= 289$

$\therefore PS = \sqrt{289}\ \ (since\ PS > 0)$
$= 17$

(b) $In\ \Delta PRQ, \angle R = 90°$
Using Pythagoras' Theorem,
$PQ^2 = PR^2 + QR^2$
$10^2 = 8^2 + QR^2$

$QR^2 = 100 - 64$
$= 36$
$\therefore QR = \sqrt{36}\ \ (since\ QR > 0)$
$= 6$

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2. $Determine\ if\ \Delta ABC,\ where\ AB = \sqrt{5},\ BC = 1,\ AC = 2\ \ is\ a\ right\text{-}angled\ triangle.$

Solution:

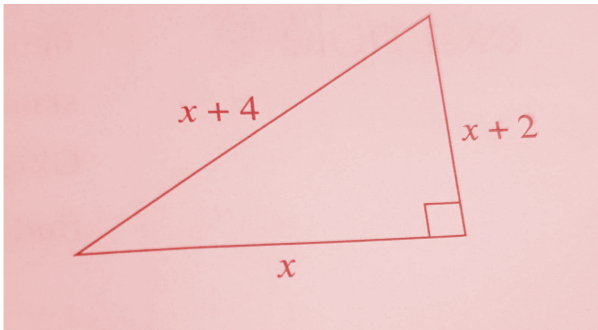
$AB^2 = (\sqrt{5})^2 \quad (AB\ is\ the\ longest\ side\ of\ \Delta ABC)$
$= 5$

$BC^2 + AC^2 = 1^2 + 2^2$
$= 5$

$\therefore$ Since $AB^2 = BC^2 + AC^2$, using the converse of Pythagoras' Theorem,
$\Delta ABC\ is\ a\ right\text{-}angled\ triangle\ where\ \angle C = 90°$.

3. The figure shows a cell structure in a human cell that is also a right-angled triangle. The dimensions are given in μm.

(a) Form an equation in x and solve it.
(b) Hence, find the perimeter of the cell structure.



Solution:

(a) Using Pythagoras' Theorem,

$(x + 4)^2 = x^2 + (x + 2)^2$

$x^2 + 8x + 16 = x^2 + x^2 + 4x + 4$

$x^2 - 4x - 12 = 0$

Using cross method or multiplication frame, we get

$(x - 6)(x + 2) = 0$

$x - 6 = 0 \quad or \quad x + 2 = 0$

$\therefore x = 6 \quad or \quad x =$ -2 (rejected since $x > 0$)

(b) Dimensions of the triangle are x, $(x + 2)$, $(x + 4)$.

$x = 6$

$x + 2 = 8$

$x + 4 = 10$

$\therefore$ Perimeter of the cell structure $= 6 + 8 + 10$

$= 24\ \mu m$

Version 3