MOMENTUM & IMPULSE

Linear Momentum

- The **linear momentum** of an object or system is its mass times its velocity. The word "momentum" by itself usually refers to linear momentum.
- Momentum is a **vector** so it has a magnitude and a direction.
- The momentum vector is in the same direction as the velocity vector.
- Linear momentum may seem similar to linear kinetic energy but momentum is a vector while kinetic energy is a scalar quantity, and the equations are different.

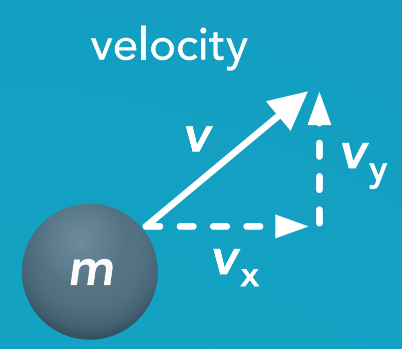
Variables		SI Unit
p	momentum	$\frac{kg \cdot m}{s}$
m	mass	kg
v	velocity	$\frac{m}{s}$

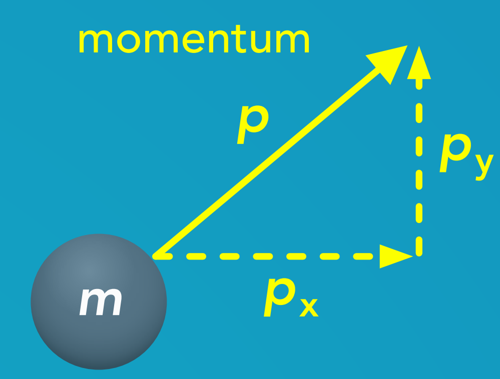
Momentum

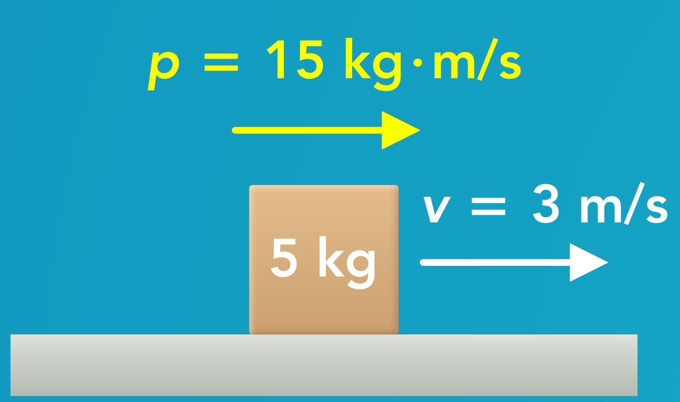
$$\vec{p} = m\vec{v}$$

Momentum vector components

$$p_x = mv_x \qquad p_y = mv_y$$

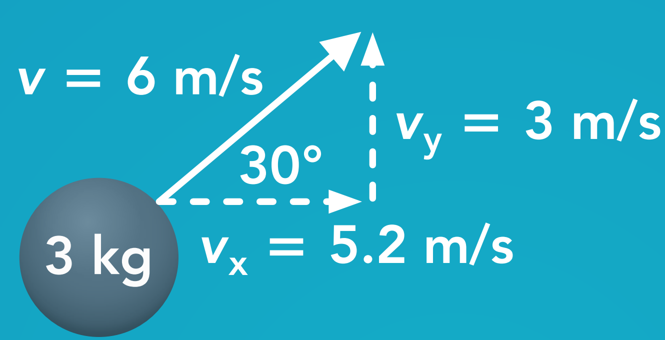


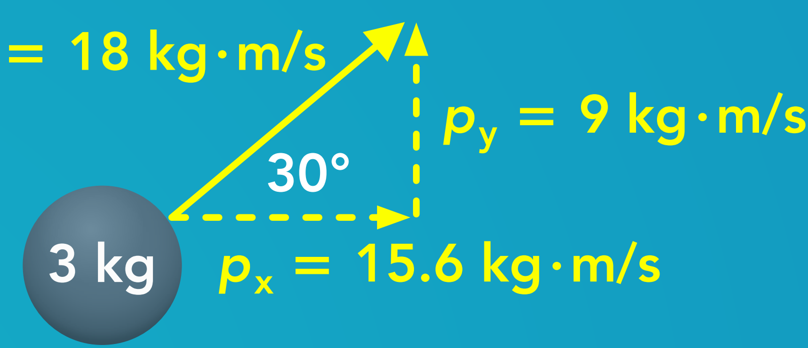




$p = (5 \text{ kg})(3 \text{ m/s})$

$p = 15 \text{ kg} \cdot \text{m/s}$





$$p = (3 \text{ kg})(6 \text{ m/s}) = 18 \text{ kg} \cdot \text{m/s}$$

$v_x = (6 \text{ m/s})\cos(30°)$ $\qquad$ $p_x = (18 \text{ kg} \cdot \text{m/s})\cos(30°)$

$v_y = (6 \text{ m/s})\sin(30°)$ $\qquad$ $p_y = (18 \text{ kg} \cdot \text{m/s})\sin(30°)$

Angular Momentum

- The **angular momentum** of a rotating object or system is its rotational inertia multiplied by its angular velocity.
- Angular momentum is a **vector** so it has a magnitude and a direction.
- The angular momentum vector is in the same direction as the angular velocity, either clockwise (CW) or counterclockwise (CCW).
- Angular momentum may seem similar to rotational kinetic energy but angular momentum is a vector while rotational kinetic energy is a scalar, and the equations are different.

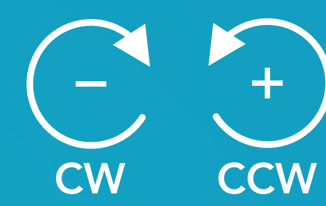
Variables		SI Unit
L	angular momentum	$\frac{kg \cdot m^2}{s}$
I	rotational inertia	$kg \cdot m^2$
ω	angular velocity	$\frac{rad}{s}$

Angular momentum

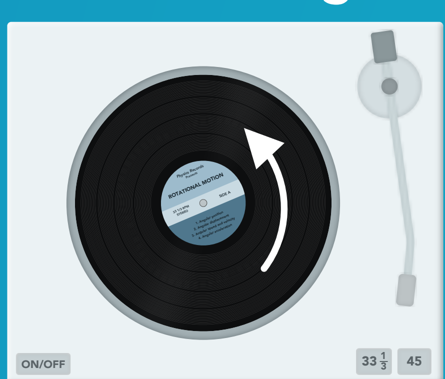
$$L = I\omega$$

angular momentum is in the same direction as the angular velocity

counterclockwise direction is positive
clockwise direction is negative



$\omega = 3.5$ rad/s
$I = 0.001\ kg \cdot m^2$



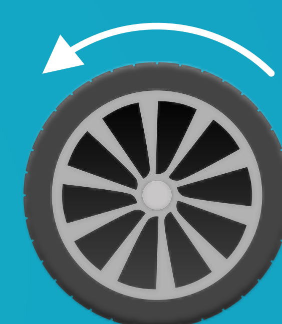
$L = (0.001)(3.5)$
$L = 0.0035\ kg \cdot m^2/s$

$\omega = 60$ rad/s

$I = 0.001\ kg \cdot m^2$

$L = (0.001)(60)$
$L = 0.06\ kg \cdot m^2/s$

$\omega = 100$ rad/s



$I = 0.4\ kg \cdot m^2$

$L = (0.4)(100)$
$L = 40\ kg \cdot m^2/s$

- Although we usually associate angular momentum with things that are rotating, **an object moving with a linear velocity also has angular momentum** about a chosen reference point, even if it's moving in a straight line.
- If we choose a reference point in space and connect the reference point and the moving object with a straight line, that line will rotate over time and the angle between that line and the object's velocity vector will change.
- If we treat the mass as a point mass, we would find that this equation below is equivalent to the one above by substituting the rotational inertia of a point mass and connecting the linear velocity with the angular velocity.

Angular momentum for an object about a reference point

$$L = rmv\sin(\theta)$$

Impulse

Variables		SI Unit
J	impulse	$\frac{\text{kg}\cdot\text{m}}{\text{s}} = \text{N}\cdot\text{s}$
p	momentum	$\frac{\text{kg}\cdot\text{m}}{\text{s}}$
F	force	N
t	time	s

- **Impulse** is the change in momentum of an object or system, which is caused by an external force applied over a period of time.
- When a force is applied to an object or system, Newton's 2nd law of motion ($\vec{F}_{net} = m\vec{a}$) says that force causes an acceleration. Acceleration is a change in velocity over time, so that force changes the object's or system's velocity, which means it also changes the momentum (which depends on the velocity).
- Impulse is a **vector** so it has a magnitude and a direction. The impulse has the same direction as the applied force.

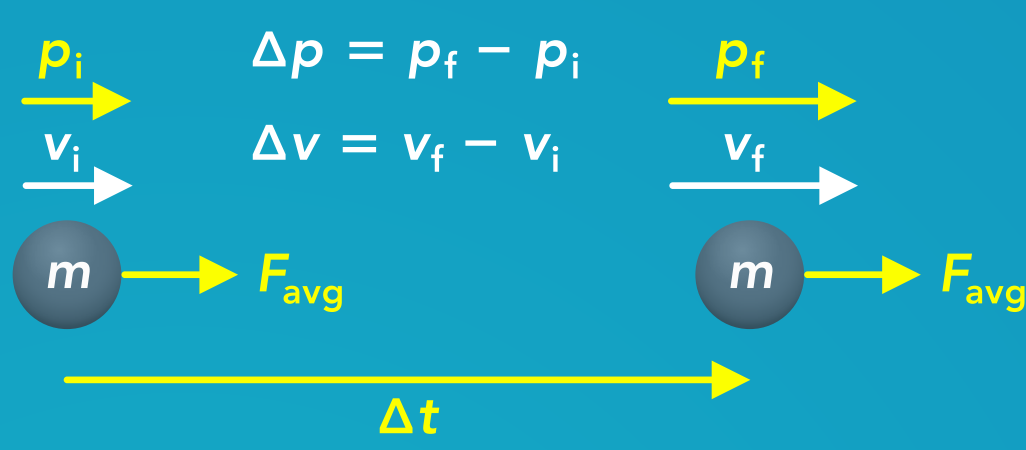
Impulse

$$\vec{J} = \Delta\vec{p} = \vec{F}_{avg}\Delta t$$

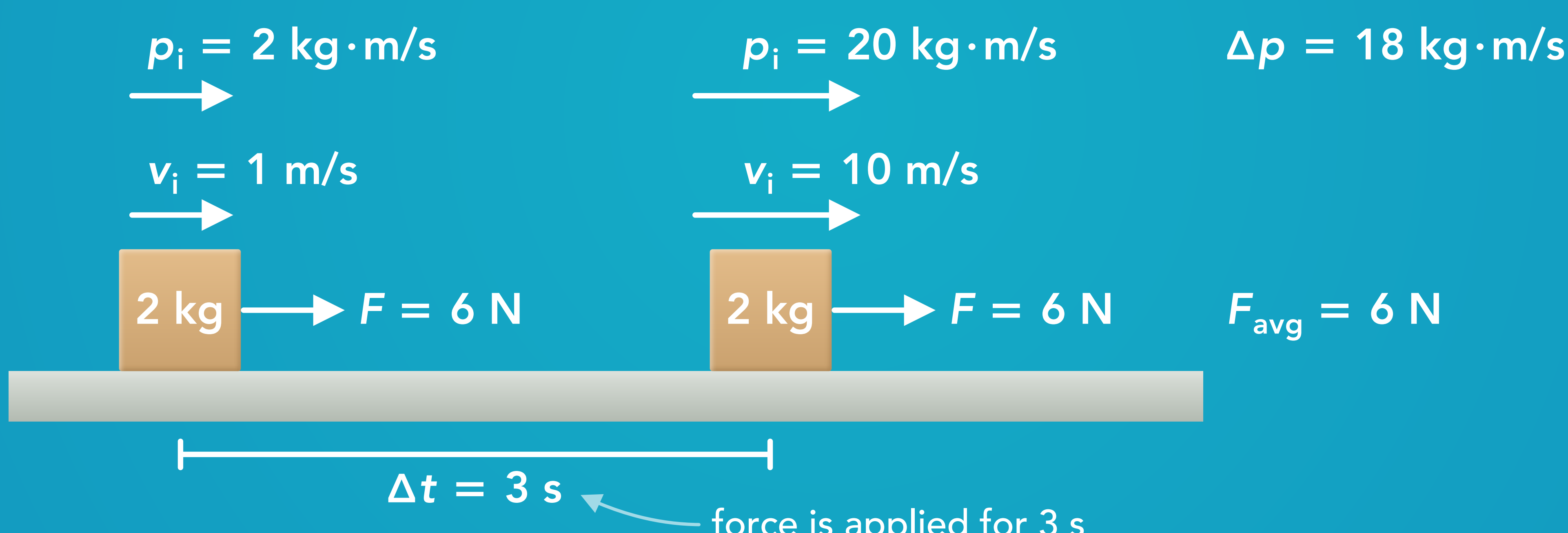
F_{avg} : average force over time

If we assume m is constant:

$$F_{avg}\Delta t = m\Delta v$$



Example: A constant force is applied to a block sliding on a frictionless surface. The force is in the same direction as the initial velocity and momentum so the impulse from the force increases the block's momentum and velocity.



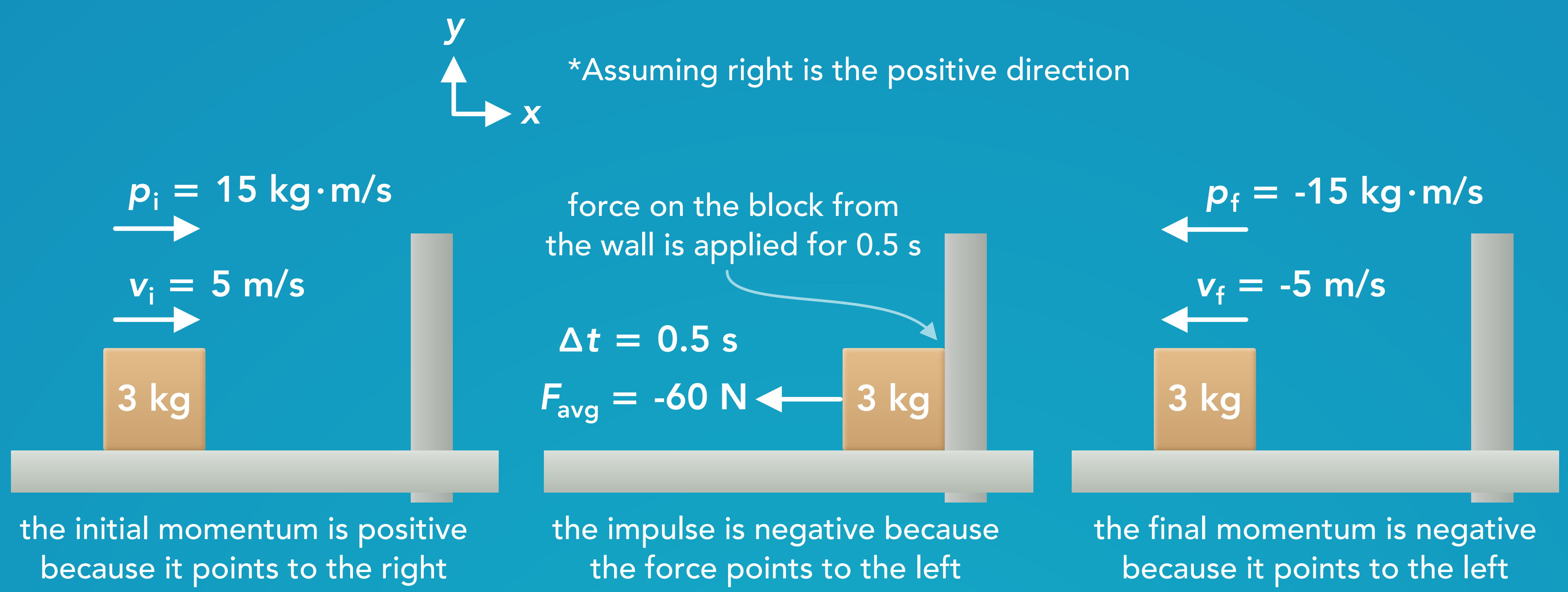
$$J = \Delta p = F_{avg}\Delta t$$

$$J = p_f - p_i = F_{avg}\Delta t$$

$$J = (2\text{ kg})(10\text{ m/s}) - (2\text{ kg})(1\text{ m/s}) = (6\text{ N})(3\text{ s})$$

$$J = 18\text{ kg}\cdot\text{m/s} = 18\text{ N}\cdot\text{s or kg}\cdot\text{m/s}$$

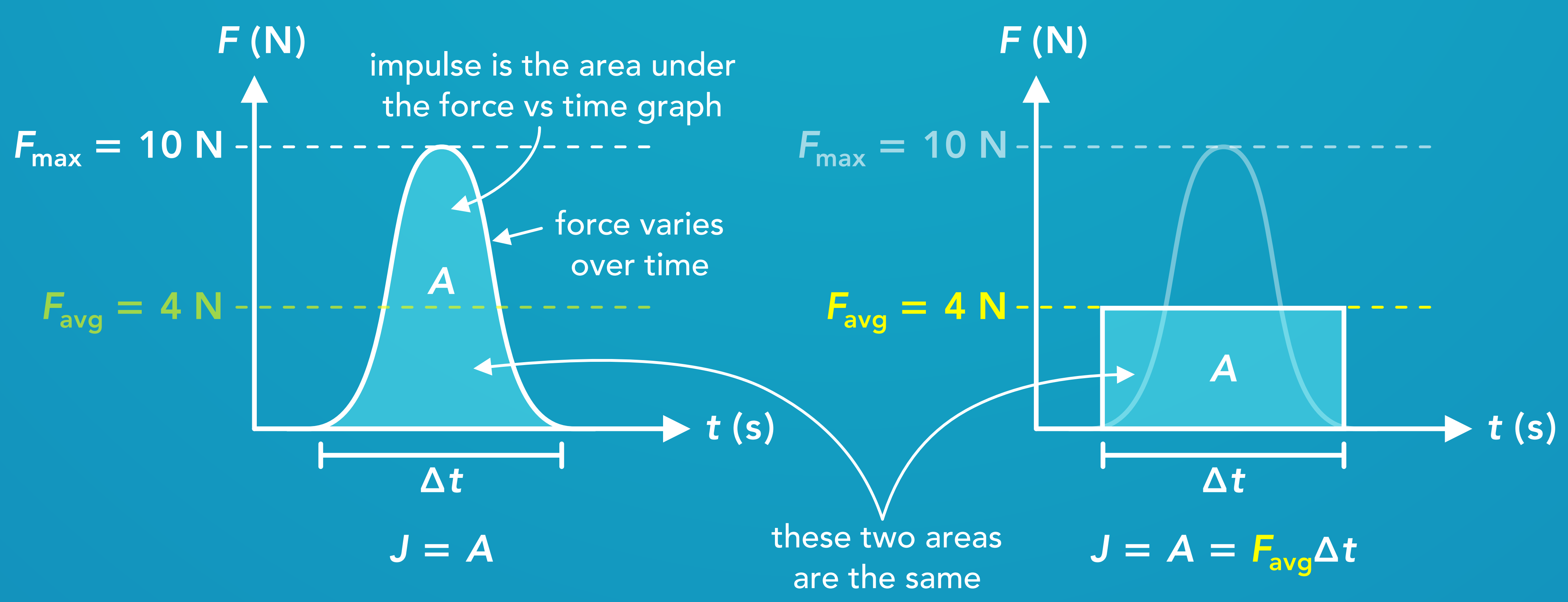
Example: A block slides on a frictionless surface and bounces off a wall, which applies a force to the block for a short period of time. The force is in the opposite direction as the initial velocity and momentum so the impulse from the force decreases and reverses the block's momentum and velocity.



$$J = \Delta p = F_{avg}\Delta t$$
$$J = p_f - p_i = F_{avg}\Delta t$$
$$J = (3\text{ kg})(-5\text{ m/s}) - (3\text{ kg})(5\text{ m/s}) = (-60\text{ N})(0.5\text{ s})$$
$$J = -30\text{ kg·m/s} = -30\text{ N·s or kg·m/s}$$

- The force applied to the object or system can vary in magnitude over time. This is often the case in the real world.
- F_{avg} is the average magnitude of the force over the period of time that it's acting on the object.
- If we have a graph of the force applied vs time, **the impulse is the area under the curve** (the area between the graphed line and the horizontal axis). In calculus, that would be the integral of the force over time. But if we know the value of the average force or if the graph is a rectangle or a triangle, we can find the area using geometry.

The area under the curve of the force vs time graph is equal to the area under the curve of the average force during that same interval, and the areas are equal to the impulse



- The concept of impulse also applies to rotational dynamics, although there's no word for "rotational impulse".
- When a torque is applied to an object or system, that torque causes an angular acceleration so the torque changes the object's angular velocity and its angular momentum (which depends on the angular velocity).

Variables		SI Unit
τ	torque	$N \cdot m$
L	angular momentum	$\frac{kg \cdot m^2}{s}$
I	rotational inertia	$kg \cdot m^2$
ω	angular velocity	$\frac{rad}{s}$

Rotational impulse

$$\Delta L = \tau_{avg} \Delta t$$

τ_{avg} : average torque over time

If we assume I is constant:

$$\tau_{avg} \Delta t = I \Delta \omega$$