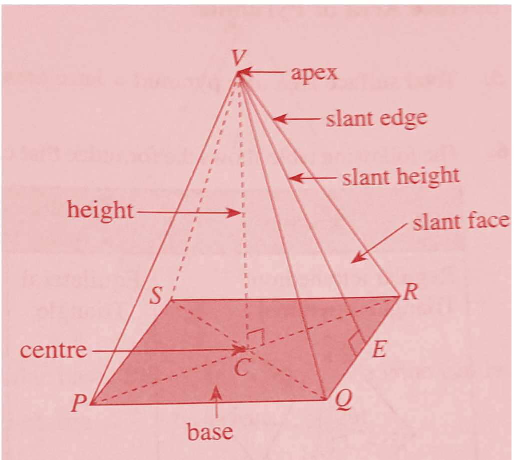
NOTES:

11: Volume and Surface Area of Pyramids, Cones, and Spheres

Pyramids:

1. A **pyramid** is a solid which has a **polygonal base** and its slanted faces are **triangles** joined to the sides of the polygonal base. The corner points of a pyramid are known as vertices and the vertex where all the slanted faces meet is called the **apex**.



2. A pyramid is named for its base. Some special types of pyramids are:

(i) A **right pyramid** is a pyramid with its apex perpendicular above the centre of the base.
A square pyramid and a rectangular pyramid are examples of right pyramids.

(ii) A **regular pyramid** is a right pyramid whose base is a regular polygon.
A square pyramid and an equilateral triangular pyramid are examples of right pyramids.

(iii) A **tetrahedron** is a triangular pyramid.

3. To find the height of a right pyramid, we can use Pythagoras' Theorem.
For example, consider a square pyramid as shown in the diagram with known values of the slant height, e.g. l, and the length of the side of the base, e.g. $2x$.

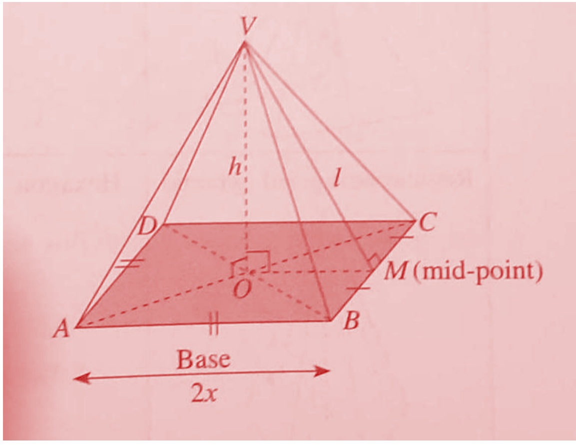
Using Pythagoras' Theorem:

In ΔVOM,

$VM^2 = VO^2 + OM^2$

$l = h^2 + x^2$

$\therefore h = \sqrt{l^2 - x^2}$



Volume of Pyramids:

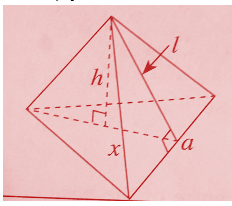
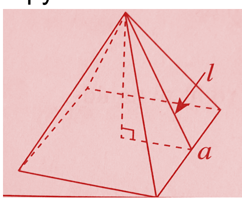
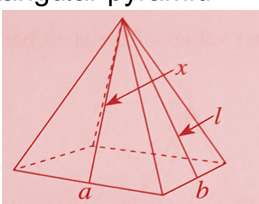
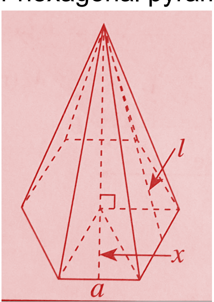
4. Volume of pyramid = $\frac{1}{3} \times base\ area \times height$

For example:
Volume of a square pyramid with a base of side x and a height h
= $\frac{1}{3} \times area\ of\ square \times height$
= $\frac{1}{3}x^2h$

Surface Area of Pyramids:

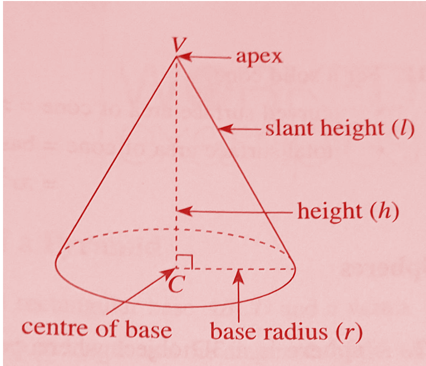
5. Total surface area of a pyramid = $base\ area + total\ area\ of\ all\ slant\ faces$

6. The following table shows the formulae that can be used to find the total surface area of some common pyramids. (SA = surface area)

Pyramid	Shape of Base	Number of Slanted Faces	Total Surface Area (square units)
Regular tetrahedron/ Triangular pyramid  	Equilateral Triangle	3	Base area = $\frac{1}{2}ax$ Area of all slant faces = $3 \times \frac{1}{2}al = \frac{3}{2}al$ ∴ Total SA = $\frac{1}{2}ax + \frac{3}{2}al$
Square pyramid  	Square	4	Base area = a^2 Area of all slant faces = $4 \times \frac{1}{2}al = 2al$ ∴ Total SA = $a^2 + 2al$
Rectangular pyramid  	Rectangle	4	Base area = ab Area of all slant faces = $2 \times \frac{1}{2}ax + 2 \times \frac{1}{2}bl = ax + bl$ ∴ Total SA = $ab + ax + bl$
Regular hexagonal pyramid  	Hexagon	6	Base area = $6 \times \frac{1}{2}ax = 3ax$ Area of all slant faces = $6 \times \frac{1}{2}al = 3al$ ∴ Total SA = $3ax + 3al$

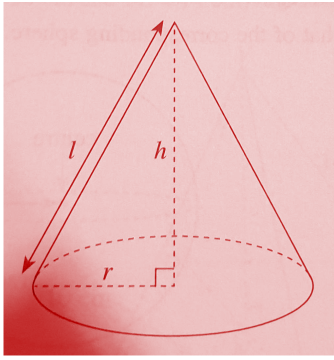
Cones:

7. A cone is a solid which has a circular base and an apex that is vertically above the centre of the base.
The height of the cone is the perpendicular distance from the apex to the centre of the base and its slant height is the distance from the apex to the circumference of the base.



Volume of Cones:

8. Volume of a cone = $\frac{1}{3} \times corresponding\ cylinder$
= $\frac{1}{3} \times base\ area \times height$
= $\frac{1}{3}\pi r^2 h$



A cone is not a pyramid but it is like a pyramid with a circular base. Hence, the volume of a cone can be calculated in the same way as a volume of a pyramid, i.e. volume = $\frac{1}{3} \times base\ area \times height$.

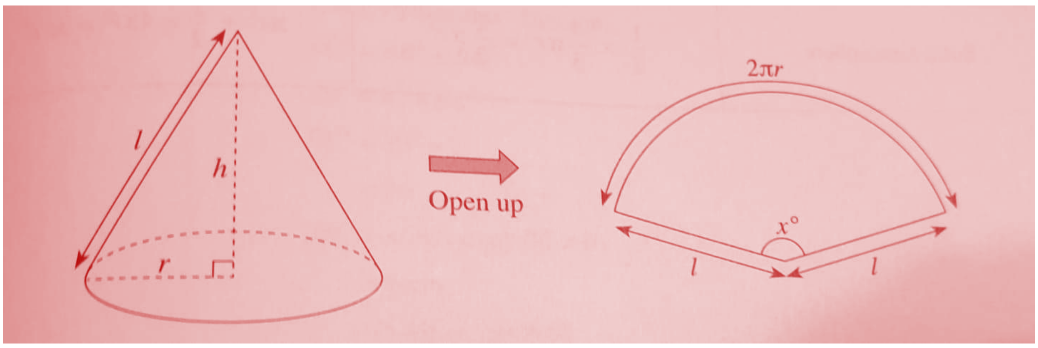
9. To find the slant height, l, of a cone, we can use the Pythagoras' Theorem as follows:

$$l^2 = h^2 + r^2$$

$$\therefore l = \sqrt{h^2 + r^2}$$

Surface Area of Cones:

10.



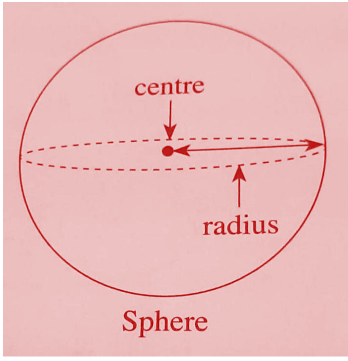
When a hollow cone is unfolded, the curved surface of the cone will form a sector as shown above and we will observe the following,
(a) slant height of the cone = radius of the sector,
(b) circumference of the base of the cone = arc length of the sector,
(c) curved surface area of the cone = area of the sector.

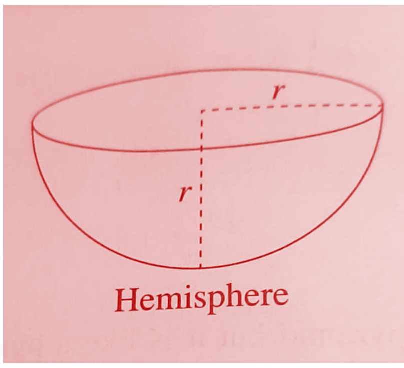
11. For a solid cone,
- curved surface area of cone = $\pi r l$
- total surface area of cone = base area + curved surface area

$$= \pi r^2 + \pi r l$$

Spheres:

12. A **sphere** is a 3D object where every point on its surface is **equidistant** from the centre of the sphere. A half sphere is known as a hemisphere which consists of a circular base and a curved surface that is half of that of the corresponding sphere.





Volume and Surface Area of Spheres:

13.

Object	Volume (cubic units)	Surface Area (square units)
Solid sphere	$\frac{4}{3}\pi r^3$	Total SA = $4\pi r^2$
Solid hemisphere	$\frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$	Curved SA = $\frac{1}{2} \times 4\pi r^2 = 2\pi r^2$ Total SA = $2\pi r^2 + \pi r^2 = 3\pi r^2$

Version 1