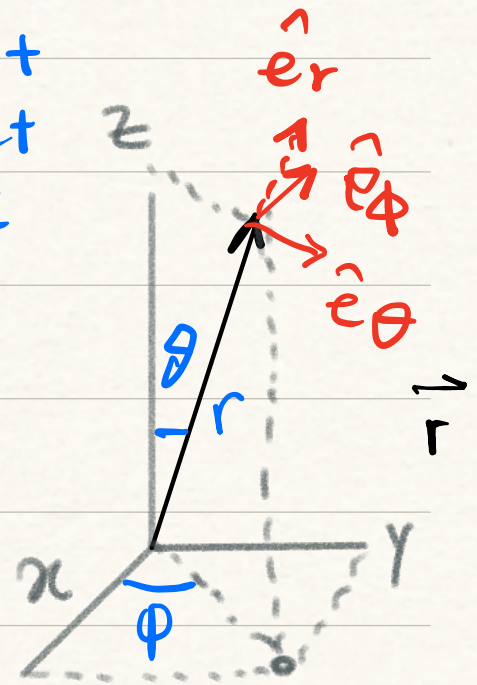
$$\vec{r} = r \cdot \hat{e}_r$$

$$\vec{v} = \frac{d\vec{r}}{dt} = r \cdot \left[\frac{d\hat{e}_r}{dt}\right] + \hat{e}_r \dot{r}$$

$$\frac{d}{dt}abc = \dot{a}bc + a\dot{b}c + ab\dot{c}$$




$$\hat{e}_r = \sin\theta\cos\varphi\,\hat{i} + \sin\theta\sin\varphi\,\hat{j} + \cos\theta\,\hat{k}$$

$$\hat{e}_\theta = \cos\varphi\cos\theta\,\hat{i} + \sin\varphi\cos\theta\,\hat{j} - \sin\theta\,\hat{k}$$

$$\hat{e}_\varphi = -\sin\varphi\,\hat{i} + \cos\varphi\,\hat{j}$$

$$\frac{d\hat{e}_r}{dt} = \frac{d}{dt}\left[\sin\theta\cos\varphi\,\hat{i} + \sin\theta\sin\varphi\,\hat{j} + \cos\theta\,\hat{k}\right]$$

$$\dot{\theta}\cos\theta\cos\varphi\,\hat{i} - \dot{\varphi}\sin\theta\sin\varphi\,\hat{i}$$

$$\dot{\theta}\cos\theta\sin\varphi\,\hat{j} + \dot{\varphi}\sin\theta\cos\varphi\,\hat{j}$$

$$-\dot{\theta}\sin\theta\,\hat{k}$$

$$\frac{d\hat{e}_r}{dt} = \dot{\theta}\cos\theta\cos\varphi\,\hat{i} - \dot{\varphi}\sin\theta\sin\varphi\,\hat{i} + \dot{\theta}\cos\theta\sin\varphi\,\hat{j} + \dot{\varphi}\sin\theta\cos\varphi\,\hat{j} - \dot{\theta}\sin\theta\,\hat{k}$$

$$\frac{d\hat{e}_r}{dt} = \dot{\theta}\underbrace{[\cos\theta\cos\varphi\,\hat{i} + \cos\theta\sin\varphi\,\hat{j} - \sin\theta\,\hat{k}]}_{\hat{e}_\theta} + \dot{\varphi}\sin\theta\underbrace{[-\sin\varphi\,\hat{i} + \cos\varphi\,\hat{j}]}_{\hat{e}_\varphi}$$

$$\frac{d\hat{e}_r}{dt} = \dot{\theta}\,\hat{e}_\theta + \dot{\varphi}\sin\theta\,\hat{e}_\varphi$$

$$\vec{v} = \frac{d\vec{r}}{dt} = r\cdot\dot{\theta}\,\hat{e}_\theta + r\,\dot{\varphi}\sin\theta\,\hat{e}_\varphi + \hat{e}_r\,\dot{r}$$