FLOW & BERNOULLI'S EQUATION

Flow & Bernoulli's Equation

1. Which of the following are properties of an ideal fluid? (Select all that apply)
   - A It has no viscosity
   - B It moves with laminar flow
   - C It is completely incompressible
   - D It has the same pressure throughout the fluid

2. In which direction does fluid always flow?
   - A From an area of higher pressure to an area of lower pressure
   - B From an area of lower pressure to an area of higher pressure
   - C From a greater height to a lower height
   - D None of the above

3. For an ideal fluid flowing through a section of a tube, which of the following is true? (Select all that apply)
   - A The volume of fluid that flows into the section must equal the volume of fluid that flows out of the section
   - B The mass of fluid that flows into the section must equal the mass of fluid that flows out of the section
   - C The flow rate into the section must equal the flow rate out of the section
   - D The density of the fluid is the same everywhere in the tube

4. Which of the following is equal to the flow rate of a fluid through an area? (Select all that apply)
   - A $\frac{v}{\Delta t}$
   - B $\frac{V}{\Delta t}$
   - C Av
   - D AV

5. Bernoulli's equation is derived from which of the following?
   - A The law of conservation of mass
   - B The law of conservation of flow rate
   - C Torricelli's theorem
   - D The law of conservation of energy

6. A large tank of oil is open to the atmosphere at the top. A cap is removed from a hole in the side of the tank and a stream of oil exits the hole at a velocity of v. If the vertical distance between the hole and the top surface of the oil in tank were doubled, what would be the new velocity of the oil stream in terms of v?
   - A $\frac{v}{\sqrt{2}}$
   - B $\sqrt{2}\,v$
   - C $\frac{v}{2}$
   - D $2v$

7. In what direction does the fluid flow inside the tube shown in Figure 1?
   - A Right
   - B Left
   - C It would not flow
   - D Cannot be determined

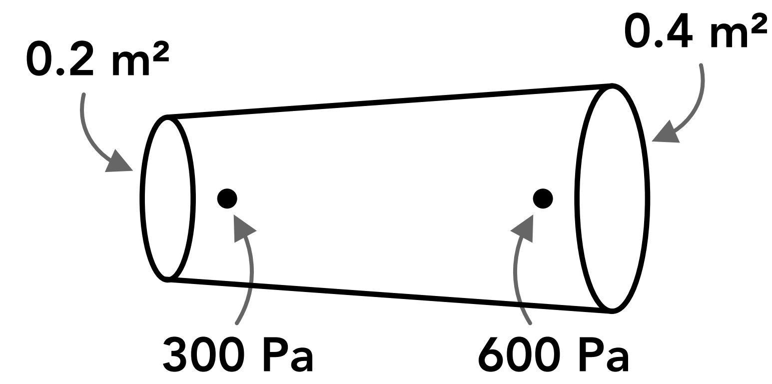

Figure 1

8. A fluid is flowing through the tube shown in Figure 2. The area of the outlet is half of the area of the inlet. If the flow rate into the tube is 10 cm³/s what is the flow rate out of the tube?
   - A 5 cm³/s
   - B 10 cm³/s
   - C 20 cm³/s
   - D Cannot be determined

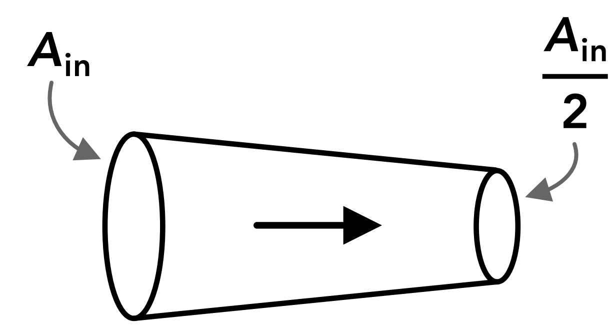

Figure 2

9. The areas of the inlet and outlet of a tube and the flow velocities at the inlet and outlet are shown in Figure 3. Which of the following is equal to the outlet flow velocity v_2?
   - A $\frac{A_2}{A_1 v_1}$
   - B $\frac{A_2 v_1}{A_1}$
   - C $\frac{A_1 v_1}{A_2}$
   - D $A_1 v_1$

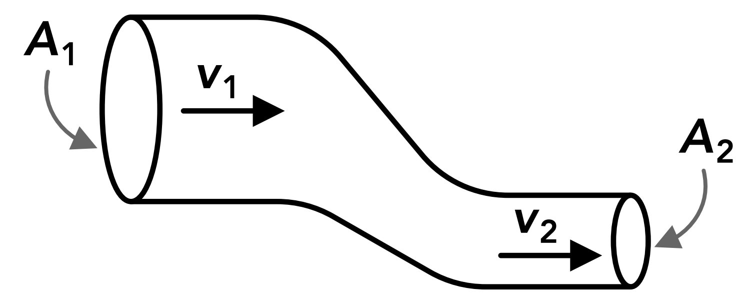

Figure 3

10. Fluid is flowing through the tube shown in Figure 4. How do the pressures at the two points shown compare?
    - A $P_1 < P_2$
    - B $P_1 = P_2$
    - C $P_1 > P_2$
    - D Cannot be determined

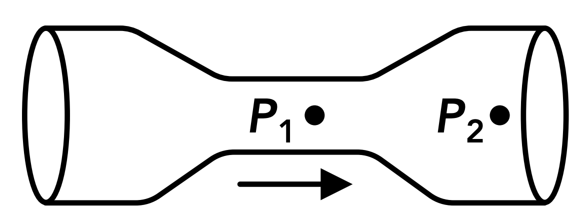

Figure 4

11. Fluid is flowing through the tube shown in Figure 5. How do the pressures at the two points shown compare?
    - A $P_1 < P_2$
    - B $P_1 = P_2$
    - C $P_1 > P_2$
    - D Cannot be determined

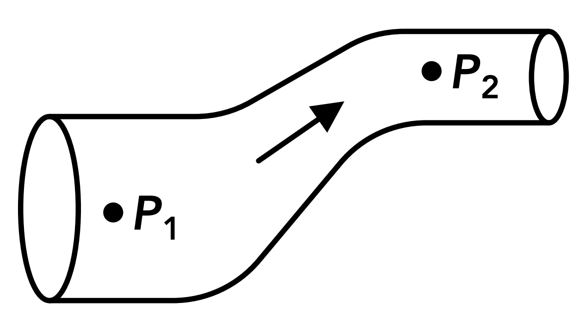

Figure 5

12. A tank of water is shown in Figure 6. There are two holes in the side of the tank which allow water to exit freely. How does the velocity of the water exiting each hole compare?

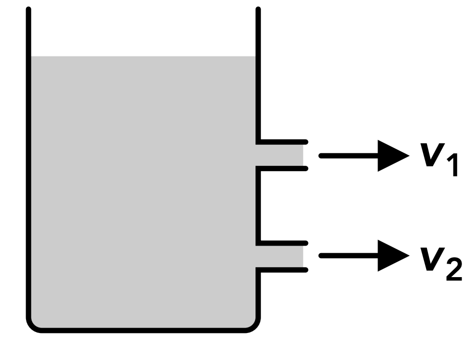

Figure 6

- A $v_1 > v_2$
- B $v_1 = v_2$
- C $v_1 < v_2$
- D Cannot be determined

13. Water is flowing through a garden hose at a rate of 600 cm³/s. If the hose has an internal diameter of 3 cm what is the velocity of the water moving through the hose?

14. A liquid is flowing through a section of a tube and the inlet and outlet of the section have different areas. If the liquid is flowing into the section at 40 cm/s and is flowing out of the section at 50 cm/s, what is the ratio of the inlet area to the outlet area A_{in} / A_{out} ?

15. A liquid is flowing into the tube shown in Figure 7 with a velocity of 0.8 m/s. What is the volume of liquid that flows out of the right end of the tube during a period of 5 seconds?

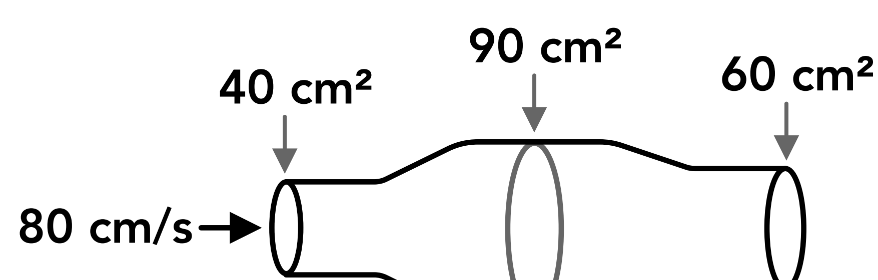

Figure 7

16. Water is flowing through the tube shown in Figure 8. What is the velocity of the flow at the inlet v?

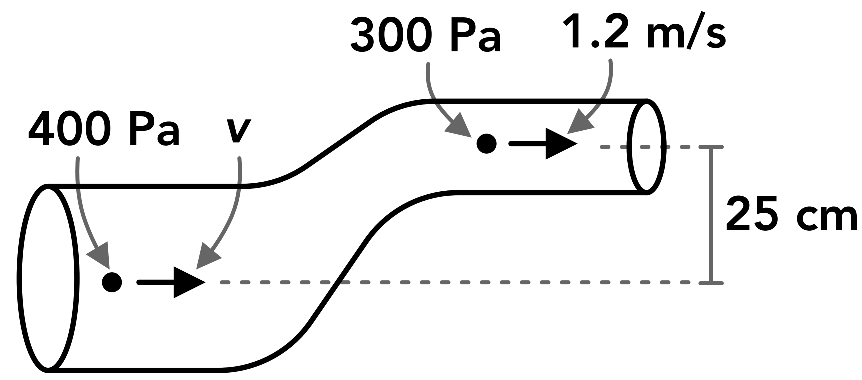

Figure 8

17. Water is flowing through the tube shown in Figure 9. What is the pressure near the outlet P?

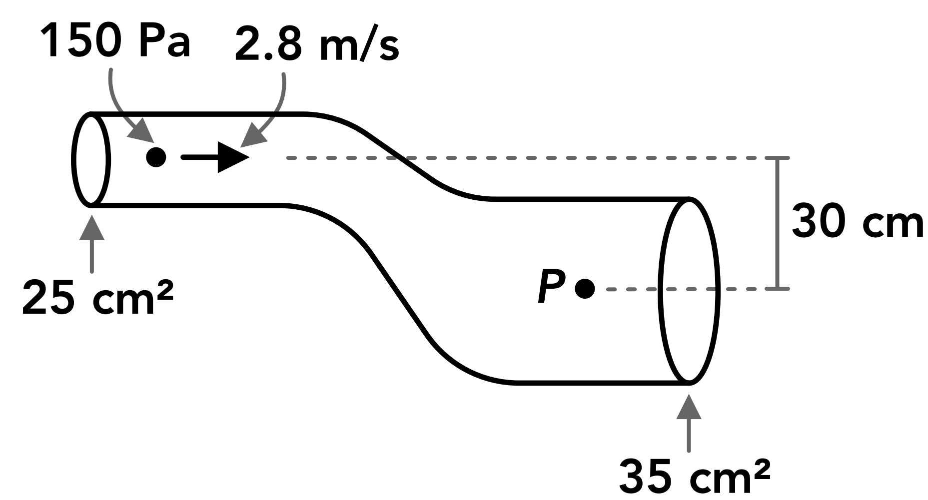

Figure 9

18. A stream of oil is shooting out of a small horizontal tube on the side of a large tank as shown in Figure 10. How far from the base of the tank does the stream of oil hit the ground?

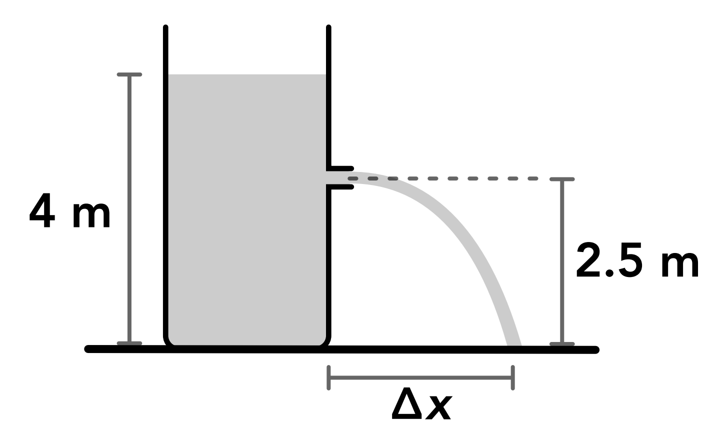

Figure 10

Answers

1. A, B, C
2. A
3. A, B, C, D
4. B, C
5. D
6. B
7. B
8. B
9. C
10. A
11. C
12. C
13. 84.9 cm/s
14. 5/4
15. 16,000 cm^3
16. 2.5 m/s
17. 5,010 Pa
18. 3.8 m

Answers - Flow & Bernoulli's Equation

1. **Answer: A, B, C**
   An ideal fluid has no viscosity (it does not resist flow and it flows without friction), it moves with laminar flow (it flows smoothly in one direction parallel to the tube) and it is completely incompressible (its volume and density is constant).

2. **Answer: A**
   Fluid always flows from an area of higher pressure to an area of lower pressure. A fluid can flow from a lower height to a greater height if the pressure at the lower height is greater.

3. **Answer: A, B, C, D**
   The fluid follows the law of conservation of mass. Because an ideal fluid is incompressible its density is the same everywhere in the tube, the amount of mass that enters the section of the tube must equal the amount of mass that exits the tube (the total mass in the section of the tube does not increase or decrease over time). The same is true for the volume of fluid that enters and exits the section of the tube. The flow rate is the volume of fluid that moves through an area over a period of time, so the flow rate is also the same at the inlet and outlet of that section of the tube.

4. **Answer: B, C**
   The flow rate of a fluid is the volume of fluid V that passes through the area divided by the period of time Δt, which is also equal to the area A multiplied by the velocity of the fluid v.
   $$\frac{V}{\Delta t} = Av$$

5. **Answer: D**
   Bernoulli's equation is derived from the law of conservation of energy. The sum of the work done by the pressure force, the gravitational potential energy and the kinetic energy of the fluid is constant over time.

6. **Answer: B**
   The equation for the velocity of a stream of liquid exiting a hole in a container (Torricelli's theorem) is given below. If the depth Δy is multiplied by 2 then the velocity v must be multiplied by $\sqrt{2}$.
   $$v = \sqrt{2g\Delta y}$$

7. **Answer: B**
   Fluid flows from an area of higher pressure to an area of lower pressure. The pressure at the right end of the tube (600 Pa) is greater than the pressure at the left end of the tube (300 Pa) so the fluid flows to the left.

8. **Answer: B**
   The flow rate into the tube is equal to the flow rate out of the tube. The flow velocity is different at the inlet and outlet because the areas are different.

9. **Answer: C**
The flow rate into the tube is equal to the flow rate out of the tube and we can use the equation below to relate the areas and velocities at two points in the flow.

$$A_1 v_1 = A_2 v_2 \qquad v_2 = \frac{A_1 v_1}{A_2}$$

10. **Answer: A**
We can assume from the figure that the cross-sectional area of the tube in the middle section is less than the cross-sectional area of the outlet section. The flow rate is the same everywhere in the tube so the velocity of the flow through the middle section must be greater than the velocity through the outlet section.
$A_1 v_1 = A_2 v_2 \qquad v_1 > v_2$ if $A_1 < A_2$
Then we can use Bernoulli's equation to relate the pressures at the two points. The two points are at the same height, so if $v_1 > v_2$ then $P_1 < P_2$.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 \qquad P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

11. **Answer: C**
We can assume from the figure that the cross-sectional area of the tube at the outlet section is less than the cross-sectional area of the inlet section. The flow rate is the same everywhere in the tube so the velocity of the flow through the outlet section must be greater than the velocity through the inlet section.
$A_1 v_1 = A_2 v_2 \qquad v_1 < v_2$ if $A_1 > A_2$
Then we can use Bernoulli's equation to relate the pressures at the two points. Point 2 is at a greater height so $y_1 < y_2$, and $v_1 < v_2$ so therefore $P_1 > P_2$.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

12. **Answer: C**
Based on Torricelli's theorem (shown below) a greater depth Δy between the surface of the liquid and the hole will result in a greater velocity of the exit stream.
$v = \sqrt{2g\Delta y}$

13. **Answer: 84.9 cm/s**
The flow rate is equal to the cross sectional area multiplied by the velocity.

$$\frac{V}{\Delta t} = Av \qquad (600\ \text{cm}^3/\text{s}) = \pi(1.5\ \text{cm})^2 v \qquad v = 84.9\ \text{cm/s}$$

14. **Answer: 5/4**
The flow rate is the same into and out of the section of the tube, and the areas and velocities are related using the equation below.

$$A_1 v_1 = A_2 v_2 \qquad A_{in}(40\ \text{cm/s}) = A_{out}(50\ \text{cm/s}) \qquad \frac{A_{in}}{A_{out}} = \frac{50\ \text{cm/s}}{40\ \text{cm/s}} = \frac{5}{4}$$

15. **Answer: 16,000 cm³**
The flow rate into the tube is equal to the flow rate out of the tube. The flow rate is equal to a cross-sectional area multiplied by the velocity of the fluid moving through that area.

$$\frac{V}{\Delta t} = Av \qquad \frac{V}{(5\ \text{s})} = (40\ \text{cm}^2)(80\ \text{cm/s}) \qquad V = 16{,}000\ \text{cm}^3$$

16. **Answer: 2.5 m/s**
We can use Bernoulli's equation for the two points shown in the flow, and set $y = 0$ at the height of the lower first point.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$(400\text{ Pa}) + (1{,}000\text{ kg/m}^3)g(0\text{ m}) + \frac{1}{2}(1{,}000\text{ kg/m}^3)v^2 \ldots$$

$$\ldots = (300\text{ Pa}) + (1{,}000\text{ kg/m}^3)g(0.25\text{ m}) + \frac{1}{2}(1{,}000\text{ kg/m}^3)(1.2\text{ m/s})^2$$

$$v = 2.5\text{ m/s}$$

17. **Answer: 5,010 Pa**
We need to use Bernoulli's equation to find the pressure, but before that we need to find the velocity of the flow near the outlet (at the point where we want to find the pressure P). We can convert the two areas from cm² to m² and use the flow rate equation below.

$$A_1 = \frac{25\text{ cm}^2}{} \times \frac{1\text{ m}^2}{100^2\text{ cm}^2} = 0.0025\text{ m}^2 \qquad A_2 = \frac{35\text{ cm}^2}{} \times \frac{1\text{ m}^2}{100^2\text{ cm}^2} = 0.0035\text{ m}^2$$

$$A_1 v_1 = A_2 v_2 \qquad (0.0025\text{ m}^2)(2.8\text{ m/s}) = (0.0035\text{ m}^2)v_2 \qquad v_2 = 2.0\text{ m/s}$$

Then we can use Bernoulli's equation to find the pressure P. We can set $y = 0$ at the first point or second point. If we set $y = 0$ at the first point (near the inlet) then the height at the second point (near the outlet) is -30 cm.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$(150\text{ Pa}) + (1{,}000\text{ kg/m}^3)g(0\text{ m}) + \frac{1}{2}(1{,}000\text{ kg/m}^3)(2.8\text{ m/s})^2 \ldots$$

$$\ldots = P + (1{,}000\text{ kg/m}^3)g(-0.30\text{ m}) + \frac{1}{2}(1{,}000\text{ kg/m}^3)(2.0\text{ m/s})^2$$

$$P = 5{,}010\text{ Pa}$$

18. **Answer: 3.8 m**
We can treat the stream of oil as an object in projectile motion, but we first need to know the initial velocity of the oil as it exits the tank, which we can find using Torricelli's theorem. The depth of the hole is 1.5 m below the surface of the oil (4 m - 2.5 m).

$$v = \sqrt{2g\Delta y} = \sqrt{2g(4\text{ m} - 2.5\text{ m})} = 5.42\text{ m/s}$$

Then we can use the kinematic equations from projectile motion to find the range of the stream of oil. We first need to find the time it takes the oil to fall and hit the ground, then we can use that time and the horizontal velocity to find the range. The known values are: $y_i = 2.5\text{ m}$, $y_f = 0\text{ m}$, $v_{yi} = 0$, $v_x = 5.42\text{ m/s}$, $a_y = -9.8\text{ m/s}^2$.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \qquad (0\text{ m}) = (2.5\text{ m}) + (0\text{ m/s})t + \frac{1}{2}(-9.8\text{ m/s}^2)t^2 \qquad t = 0.71\text{ s}$$

$$v_x = \frac{\Delta x}{\Delta t} \qquad (5.42\text{ m/s}) = \frac{\Delta x}{(0.71\text{ s})} \qquad \Delta x = 3.8\text{ m}$$