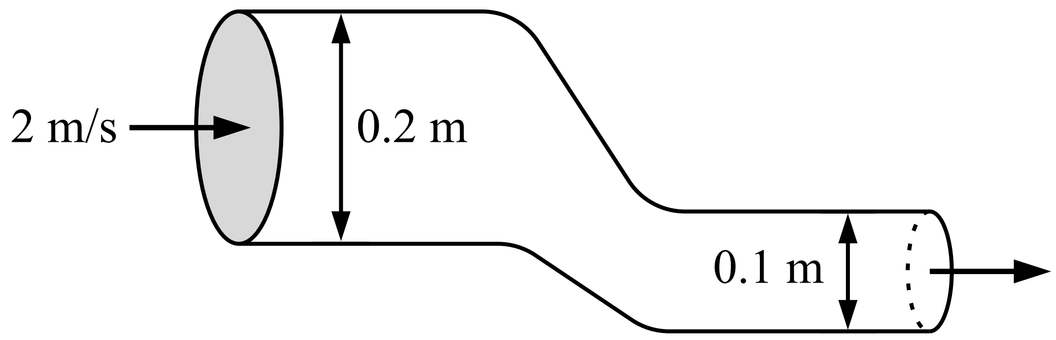
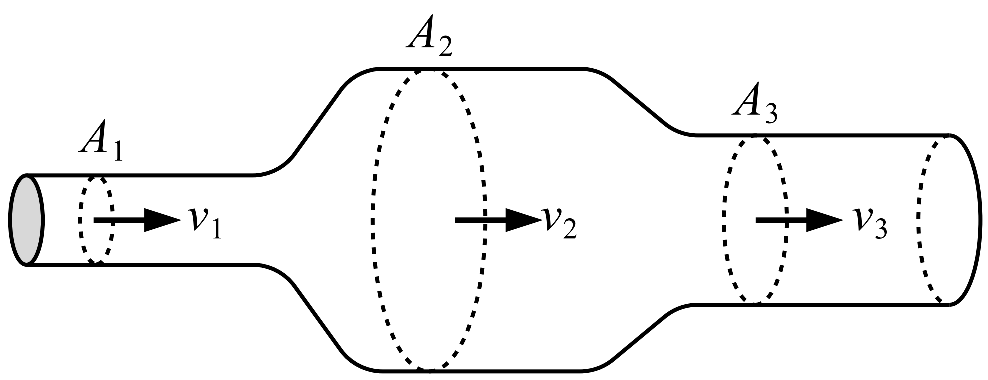
FLOW & BERNOULLI'S EQUATION



1. An ideal fluid is flowing into the tube shown in the figure above with a speed of 2 m/s. The diameter of the inlet is 0.2 m and the diameter of the outlet is 0.1 m. The volume of fluid that exits the tube over a period of 3 seconds is most nearly

   (A) $0.19\ m^3$
   (B) $0.06\ m^3$
   (C) $0.09\ m^3$
   (D) $0.38\ m^3$



2. An ideal fluid is flowing through the tube shown in the figure above. Which of the following correctly ranks the speed of the fluid through the three areas shown above?

   (A) $v_1 < v_3 < v_2$
   (B) $v_1 = v_2 = v_3$
   (C) $v_2 < (v_1 = v_3)$
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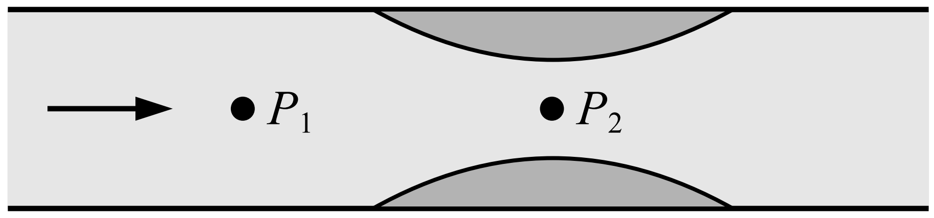
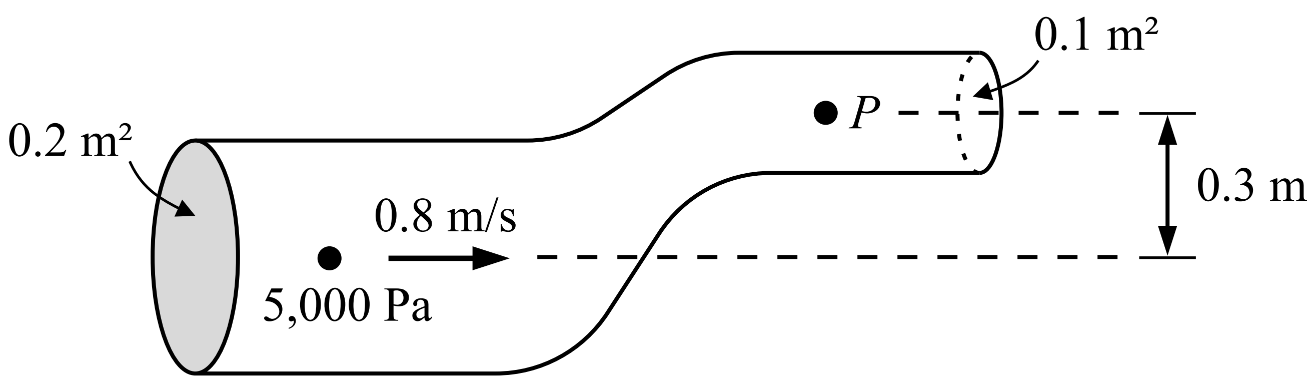


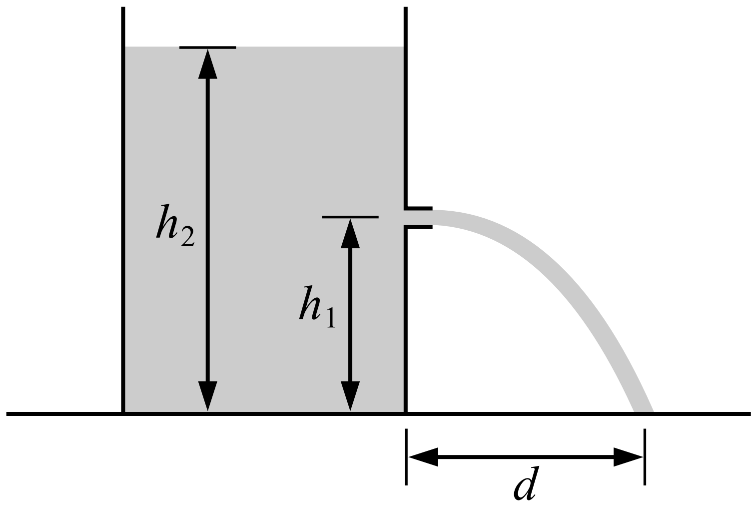
3. Water is flowing through a pipe as shown in the figure above. Some material has collected inside the pipe which has created a narrower opening for the fluid to flow through. How do the pressures at the two points shown compare?

(A) $P_1 = P_2$
(B) $P_1 > P_2$
(C) $P_1 < P_2$
(D) Cannot be determined



4. Water is flowing through the tube shown in the figure above. What is the pressure P at the point shown in the figure? The density of the water is 1,000 kg/m³.

(A) 1,040 Pa
(B) 2,320 Pa
(C) 2,000 Pa
(D) 3,000 Pa



5. A large tank of oil is open at the top and has a hole in the side as shown in the figure above. A stream of oil exits the hole horizontally and then lands on the ground a distance of d from the base of the tank. The hole is at a height of h_1 and the top surface of the oil is at a height of h_2. Which of the following is a correct expression for the distance d that the oil stream lands away from the tank?

(A) $\sqrt{\dfrac{2h_1}{g}}$

(B) $\sqrt{2g(h_2 - h_1)}$

(C) $\sqrt{4h_1(h_2 - h_1)}$

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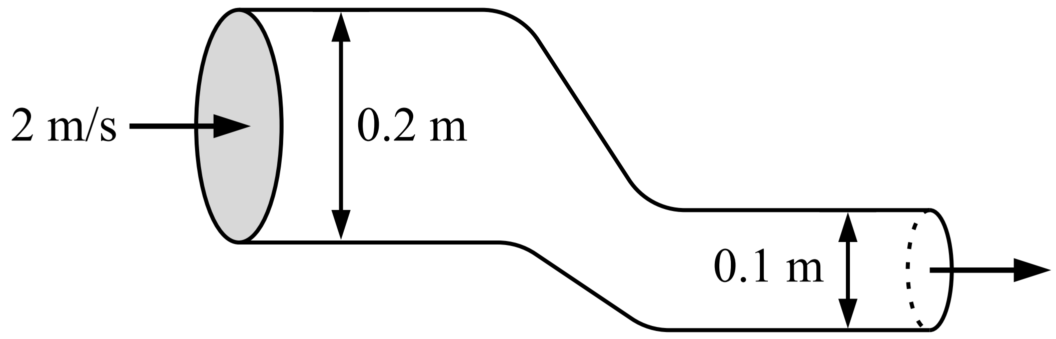


1. An ideal fluid is flowing into the tube shown in the figure above with a speed of 2 m/s. The diameter of the inlet is 0.2 m and the diameter of the outlet is 0.1 m. The volume of fluid that exits the tube over a period of 3 seconds is most nearly

(A) 0.19 m^3
(B) 0.06 m^3
(C) 0.09 m^3
(D) 0.38 m^3

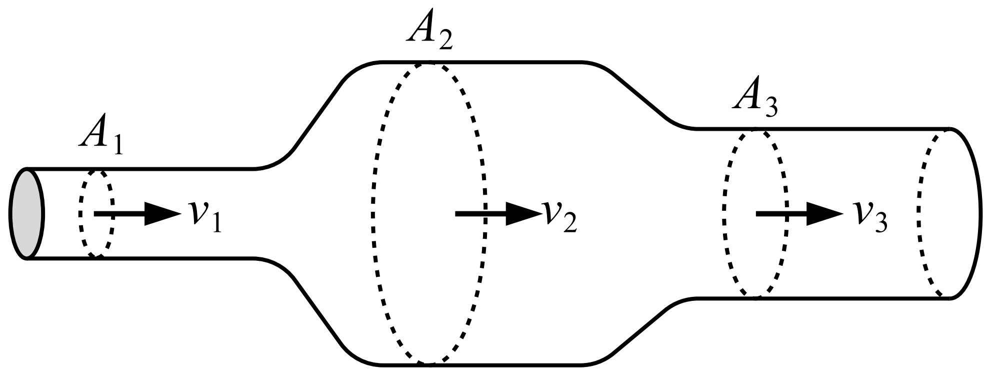
A Correct
An ideal fluid is incompressible so the mass and volume that enters the tube over a period of time must equal the amount of mass and volume that exits the tube. The flow rates into and out of the tube are equal. The flow rate through an area is equal to the area multiplied by the speed of the flow.

$$\frac{V_{in}}{\Delta t} = \frac{V_{out}}{\Delta t} \qquad A_{in} v_{in} = \frac{V_{out}}{\Delta t} \qquad \pi(0.2\text{ m} / 2)^2(2\text{ m/s}) = \frac{V_{out}}{(3\text{ s})} \qquad V_{out} = 0.19\text{ m}^3$$

B Incorrect
This is the flow rate through the tube (0.06 m^3/s) with the unit of m^3.

C Incorrect
This answer may have been found by incorrectly dividing the volume by 2.

D Incorrect
This answer may have been found by incorrectly multiplying the volume by 2.



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(C) $v_2 < (v_1 = v_3)$

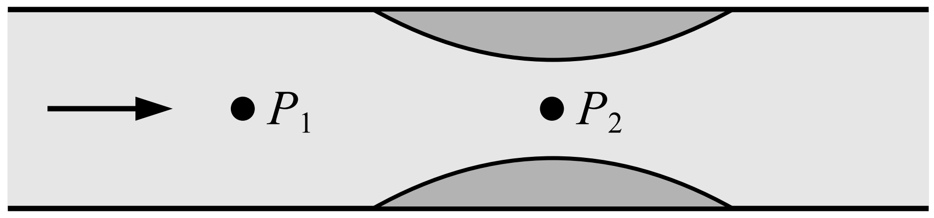
(D) $v_2 < v_3 < v_1$

A Incorrect
This answer is the reverse ranking of the correct answer.

B Incorrect
The flow rate is the same through each area but the speed is not the same.

C Incorrect

D Correct
An ideal fluid is incompressible so the flow rate is the same everywhere in the tube. The flow rate is equal to the cross sectional area multiplied by the speed through that area as given below. The speed will be slower through a larger area and faster through a smaller area.
$A_1 v_1 = A_2 v_2 = A_3 v_3$



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(B) $P_1 > P_2$
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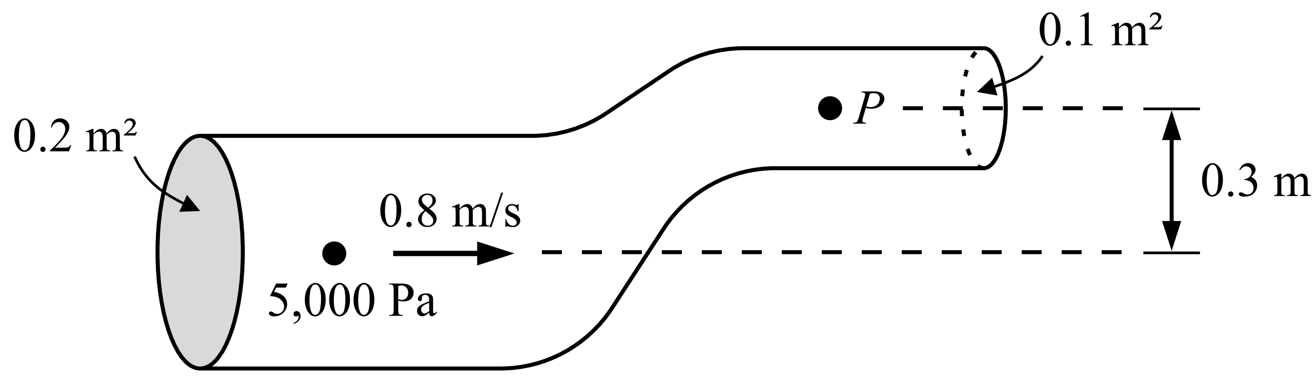
(D) Cannot be determined

A Incorrect
This answer may have been found by incorrectly assuming the pressures are equal because they are at the same level, but this is only true if the fluid is not moving (static).

B **Correct**
The cross-sectional area of the flow is smaller at point 2 (where the blockage is) than it is at point 1. The flow rate is the same at each point (and everywhere in the pipe) so the speed of the flow must be greater at point 2.
$A_1 v_1 = A_2 v_2 \qquad A_2 < A_1$ so $v_2 > v_1$
We can use Bernoulli's equation to compare the pressures at the two points. The points are at the same height so $y_1 = y_2$. If $v_1 < v_2$ then $P_1 > P_2$.
$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2 \qquad P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

C Incorrect

D Incorrect



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A Correct
We need to use Bernoulli's equation to find the pressure P but we first need to find the speed of the flow at that point using the conservation of flow rate equation.
$A_1 v_1 = A_2 v_2$ $(0.2\ \text{m}^2)(0.8\ \text{m/s}) = (0.1\ \text{m}^2) v_2$ $v_2 = 1.6\ \text{m/s}$
Then we can use Bernoulli's equation to find the pressure at point 2 and set $y = 0\ \text{m}$ at the first point.

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$(5{,}000\ \text{Pa}) + (1{,}000\ \text{kg/m}^3) g (0\ \text{m}) + \frac{1}{2}(1{,}000\ \text{kg/m}^3)(0.8\ \text{m/s})^2 = \ldots$$

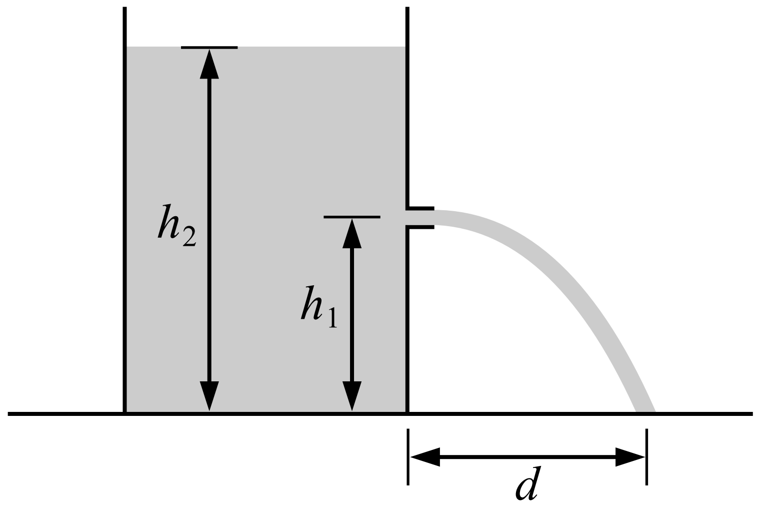
$$\ldots\ P + (1{,}000\ \text{kg/m}^3) g (0.3\ \text{m}) + \frac{1}{2}(1{,}000\ \text{kg/m}^3)(1.6\ \text{m/s})^2$$

$P = 1{,}040\ \text{Pa}$

B Incorrect
This answer incorrectly assumes the speed at the second point is 0 m/s.

C Incorrect
This answer incorrectly finds the pressure at the second point due to the difference in height only, which is only true if the fluid is not moving (static).

D Incorrect
This answer incorrectly finds the pressure at a depth of 0.3 m in water that is not moving (static).



5. A large tank of oil is open at the top and has a hole in the side as shown in the figure above. A stream of oil exits the hole horizontally and then lands on the ground a distance of d from the base of the tank. The hole is at a height of h_1 and the top surface of the oil is at a height of h_2. Which of the following is a correct expression for the distance d that the oil stream lands away from the tank?

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(B) $\sqrt{2g(h_2 - h_1)}$

(C) $\sqrt{4h_1(h_2 - h_1)}$

(D) $\sqrt{4(h_2 - h_1)}$

A Incorrect

This is an expression for the time it takes the oil to hit the ground after exiting the tank.

B Incorrect

This is an expression for the speed of the oil as it exits the hole.

C **Correct**

First we can use Bernoulli's equation to derive an equation for the speed of the oil stream when it exits the hole (known as Torricelli's theorem). Point 1 is at the top surface of the oil in the tank and point 2 is where the oil stream exits the hole. The oil is exposed to the atmosphere at both points so the pressure at both points is atmospheric pressure. The area of the top surface is much greater than the area of the hole so we assume that the speed at the top surface is zero.

$$P_{top} + \rho g y_{top} + \frac{1}{2}\rho v_{top}^2 = P_{hole} + \rho g y_{hole} + \frac{1}{2}\rho v_{hole}^2 \qquad P_{atm} + \rho g h_2 + \frac{1}{2}\rho(0)^2 = P_{atm} + \rho g h_1 + \frac{1}{2}\rho v_{hole}^2$$

$$v_{hole} = \sqrt{2g(h_2 - h_1)}$$

Then we can use the kinematic equations from projectile motion to find the range of the stream of oil. We first need to find the time it takes the oil to fall from the hole and hit the ground, then we can use that time and the horizontal velocity to find the range.

$$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2 \qquad (0\text{ m}) = h_1 + (0\text{ m/s})t + \frac{1}{2}(-g)t^2 \qquad t = \sqrt{\frac{2h_1}{g}}$$

$$v_x = \frac{\Delta x}{t} = \frac{d}{t} \qquad d = v_x t \sqrt{2g(h_2 - h_1)}\sqrt{\frac{2h_1}{g}} = \sqrt{4h_1(h_2 - h_1)}$$

D Incorrect