NOTES:

6: Algebraic Fractions and Formulae

Simplifying Algebraic Fractions:

1, An algebraic fraction is a fraction in the form of $\frac{A}{B}$ and $B \neq 0$, where A and/or B are expressions that contain algebraic terms. Some examples of algebraic fractions are $\frac{2x+1}{3}$, $\frac{6}{x-1}$, $\frac{4a^2b}{3c}$, $\frac{x^2-2x-3}{2x-1}$.

2. The value of the fraction remains the same if both its numerator and denominator are multiplied or divided by the identical non-zero number or expression,
i.e. $\frac{a}{b}$ = $\frac{a \times c}{b \times c}$ and $\frac{a}{b}$ = $\frac{a \div c}{b \div c}$, where $c \neq 0$.

3. To simplify an algebraic fraction, we can divide (or "cancel") both the numerator and the denominator by their common factors.

Multiplication and Division of Algebraic Fractions:

4. Algebraic fractions are multiplied and divided in the same way as numeric fractions.

5. The following table illustrates the steps we can use to **multiply** two algebraic fractions.

Step	Process
1	Factorise both the numerator and the denominator where possible.
2	Perform division of common factors for both the numerator and the denominator (i.e. cancelling common factors)
3	Multiply the remaining terms within the numerator and the denominator respectively, i.e. $\frac{a}{b} \times \frac{c}{d}$ = $\frac{a \times c}{b \times d}$ = $\frac{ac}{bd}$, where $b \neq 0$ and $d \neq 0$.

Continue on the next page.

6. The following table illustrates the steps we can use to **divide** two algebraic fractions.

Step	Process
1	Factorise both the numerator and the denominator where possible.
2	Multiply the first fraction by the reciprocal of the second fraction, i.e. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ Note: The reciprocal of a is $\frac{1}{a}$ whereas the reciprocal of $\frac{c}{d}$ is $\frac{d}{c}$
3	Perform division of common factors for both the numerator and the denominator (i.e. cancelling common factors).
4	Multiply the remaining terms within the numerator and denominator respectively.

Addition and Subtraction of Algebraic Fractions:

7. Algebraic fractions are added and subtracted in the same way as numeric fractions.

8. The following table illustrates the steps we can use to add or subtract two algebraic fractions.

Step	Process
1	Factorise both the numerator and the denominator where possible.
2	Perform division of common factors for both the numerator and the denominator (i.e. cancelling common factors) where possible.
3	Find the lowest common multiple (LCM) of the denominators.
4	Express the algebraic fraction as like fractions so that they have a common denominator (i.e. denominators are the same) as obtained in step 3.
5	Add or subtract the fractions in a similar way as numeric fractions, paying attention to the change of operation signs when expanding negative terms.

Continue on the next page.

Solving Equations involving Algebraic Fractions:

9. To solve equations involving algebraic fractions,

- if there are only two algebraic fractions or numerals within the equation, use the cross-multiplication method,

- if there are 3 or more algebraic fractions or numerals within the equation, use one of the methods shown in the table below.

Step	Method 1	Method 2
	Express as like fractions	**Multiply throughout by the LCM of denominators**
1	Express all fractions, both algebraic and numeric, as like fractions by using the LCM of the denominators.	Eliminate the denominators of all the fractions by multiplying each term in the equation by the LCM of the denominators.
2	Ensure each side of the equation (i.e. LHS and RHS) are expressed as a single fraction.	Solve the equation in the usual manner.
3	Perform cross-multiplication to obtain a new equation without fractions.	Substitute the solutions into the original equation to check for validity (i.e. ensure the denominators of the fractions are non-zero numbers.)
4	Solve the equation in the usual manner.	
5	Substitute the solutions into the original equation to check for validity (i.e. ensure the denominators of the fractions are non-zero numbers.)	

Changing the Subject of a Formula:

10. A **formula** is an equation (identity) that describes the mathematical relationship between one quantity and the other quantities, and the equality will hold true for any arbitrary values of unknown. The **subject** of the formula is a single positive term that appears alone on the LHS of an equation and nowhere else.

For example: In $V = \pi r^2 h$, V is the subject of the formula.

11. To make a particular variable the subject of a formula, we need to perform algebraic manipulations such as expansions, taking out common factors or any other ways so as to make

that particular variable appear on the LHS of the formula. This process is known as **changing the subject of the formula**.

Example question:

Express z in terms of x and y in $x = \sqrt{\frac{y+3z}{2y-z}}$.

Solution: $x = \sqrt{\frac{y+3z}{2y-z}}$

$x^2 = \frac{y+3z}{2y-z}$ (Take the square of both sides)

$x^2(2y - z) = y + 3z$ (Multiply by $2y - z$ on both sides)

$2yx^2 - x^2z = y + 3z$ (Apply Distributive Law / Expand)

$x^2z + 3z = 2yx^2 - y$ (Manipulate it so the like terms of z are on LHS)

$z(x^2 + 3) = 2yx^2 - y$ (Factorise z from the like terms)

$z = \frac{2yx^2 - y}{x^2+3}$ (Divide by $x^2 + 3$ on both sides so LHS is z)

Version 1