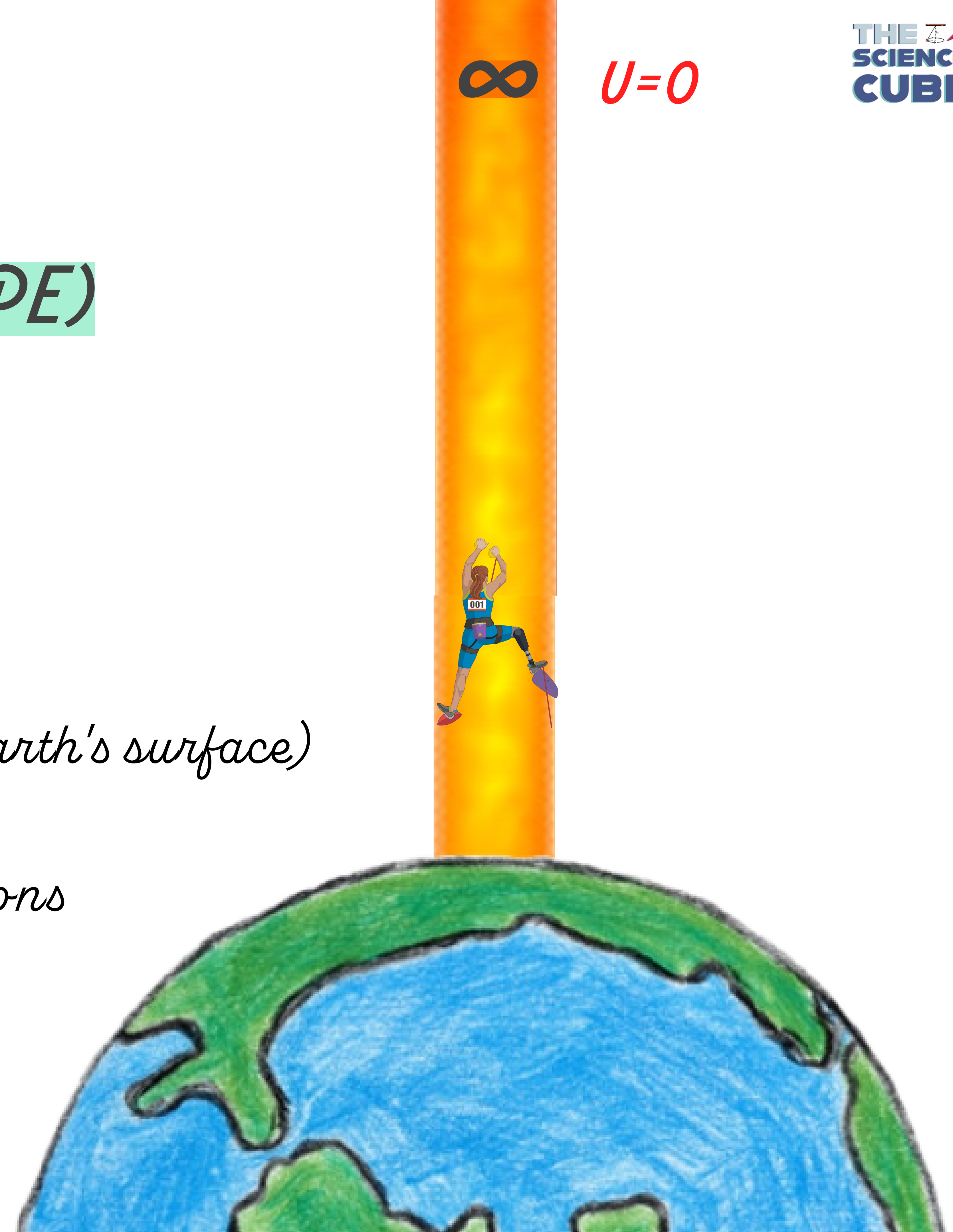
CUBE NOTES

AP PHYSICS . JEE . NEET. GRADE 11/12

Gravitational potential energy (GPE)

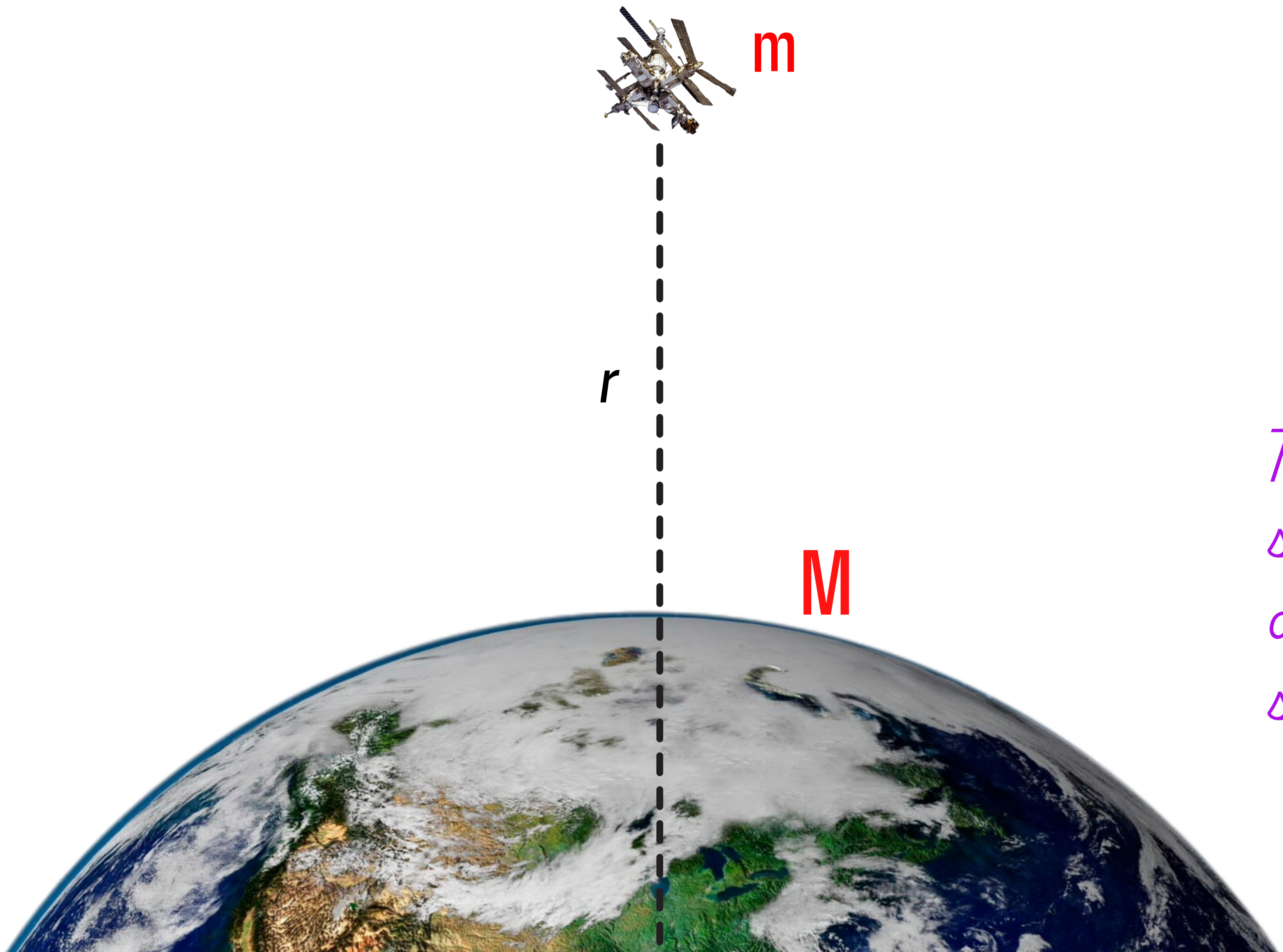
1. What is gravitational potential energy
2. GPE is property of a system
3. Derivation of the formula $U = -GMm/r$
4. Physical meaning of the formula
5. Comparing 2 reference points (∞ and Earth's surface)
6. Formulas and equations
7. Common misconceptions and clarifications



WHAT IS GRAVITATIONAL POTENTIAL ENERGY (GPE)?

Gravitational potential energy of a system of two masses M and m, separated by a distance r (measured from center to center), is given as :

$$U = -GMm / r$$

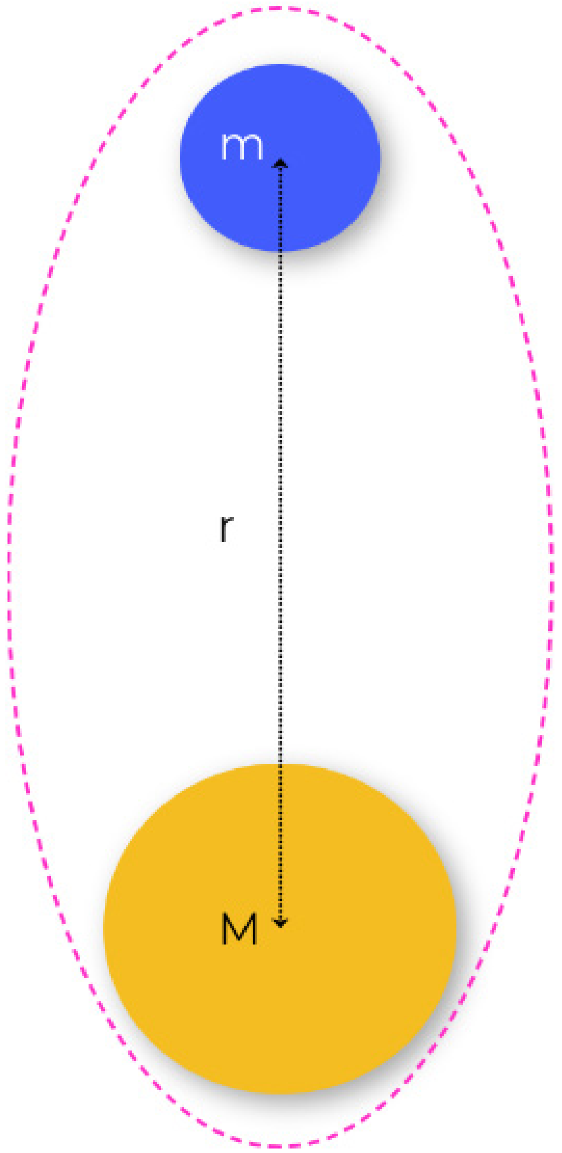


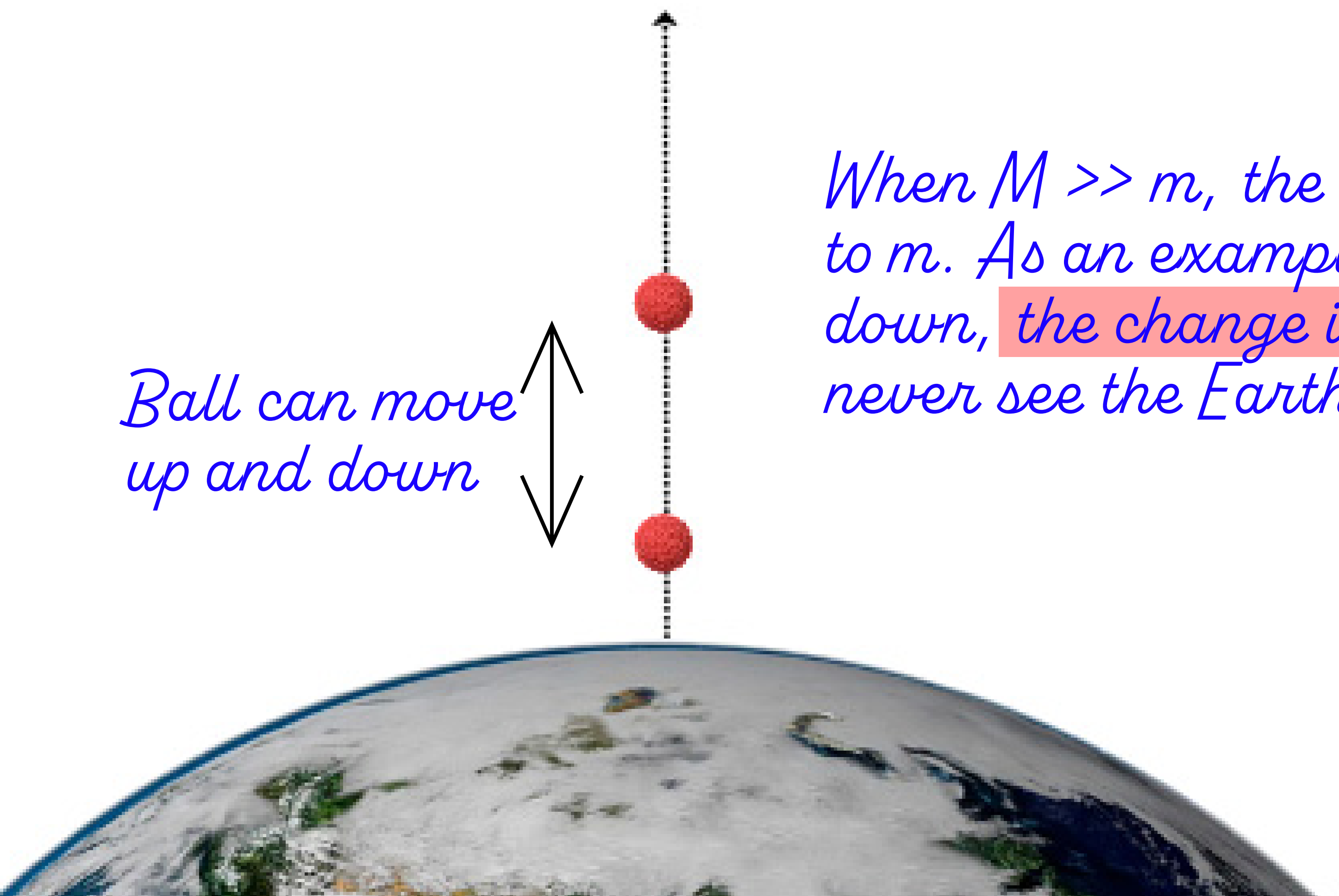
Think of GPE as the 'stored energy' in a system of masses due to their gravitational attraction, similar to how a stretched spring stores potential energy

SYSTEM VS. OBJECT

1. GPE is not a property of a single object – it is a property of the system of two masses.
2. When one mass is much larger (like Earth and a ball), we often attribute the energy to the smaller object for convenience.
3. When both masses are similar, we must treat the energy as shared by the system.

U is a shared property of M and m





When M >> m, the PE or U can be attributed to m. As an example when ball moves up and down, the change in U is largely in m (you never see the Earth moving up or down)

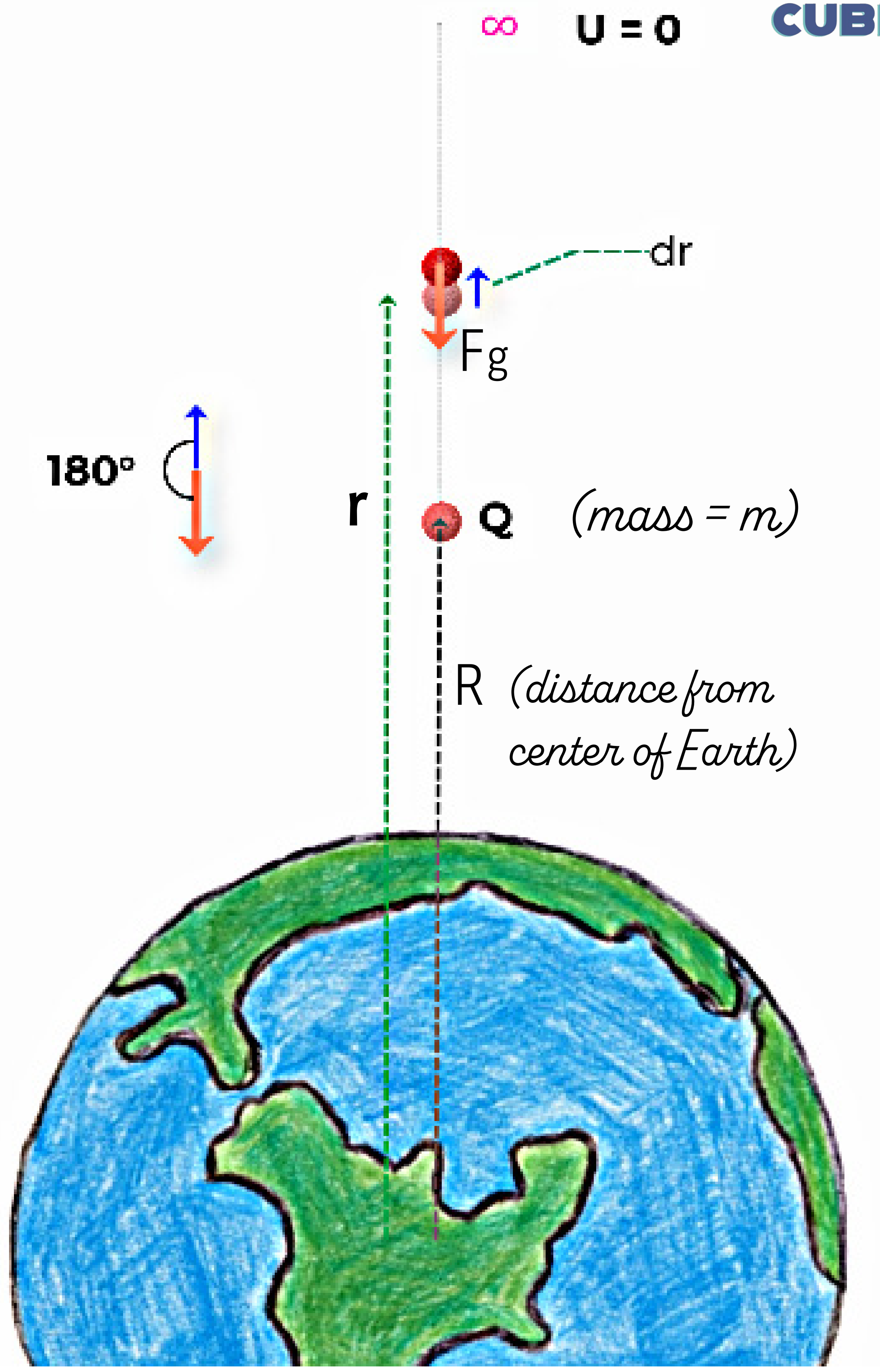
Note: there is no way to divide this energy and say that so much belongs to one particle and so much to the other

DERIVATION OF U = -GMm / R

U at point Q : Step-by-step Overview

1. $F_g = GMm / r^2$ *(Gravitational force)*
2. $dW = F.dr$ *(Work done to move distance*
   $dW = F\, dr \cos(180°)$ *dr - when object is at distance r)*
   $dW = -(GMm / r^2)\ dr$
3. $W_g = -GMm / R$ *(After integrating, r = R to ∞)*
4. $\Delta U = -W$
5. $U_\infty - U = -W$ *(U = Grav. PE at Q, at separation R)*
6. $0 - U = -W$
7. $U = W$
8. $U = -GMm / R$
9. $U = -GMm / r$ *(Using a more general value r)*

By taking U = 0 at infinity, we simplify calculations.
What is of relevance is the change in U or ΔU

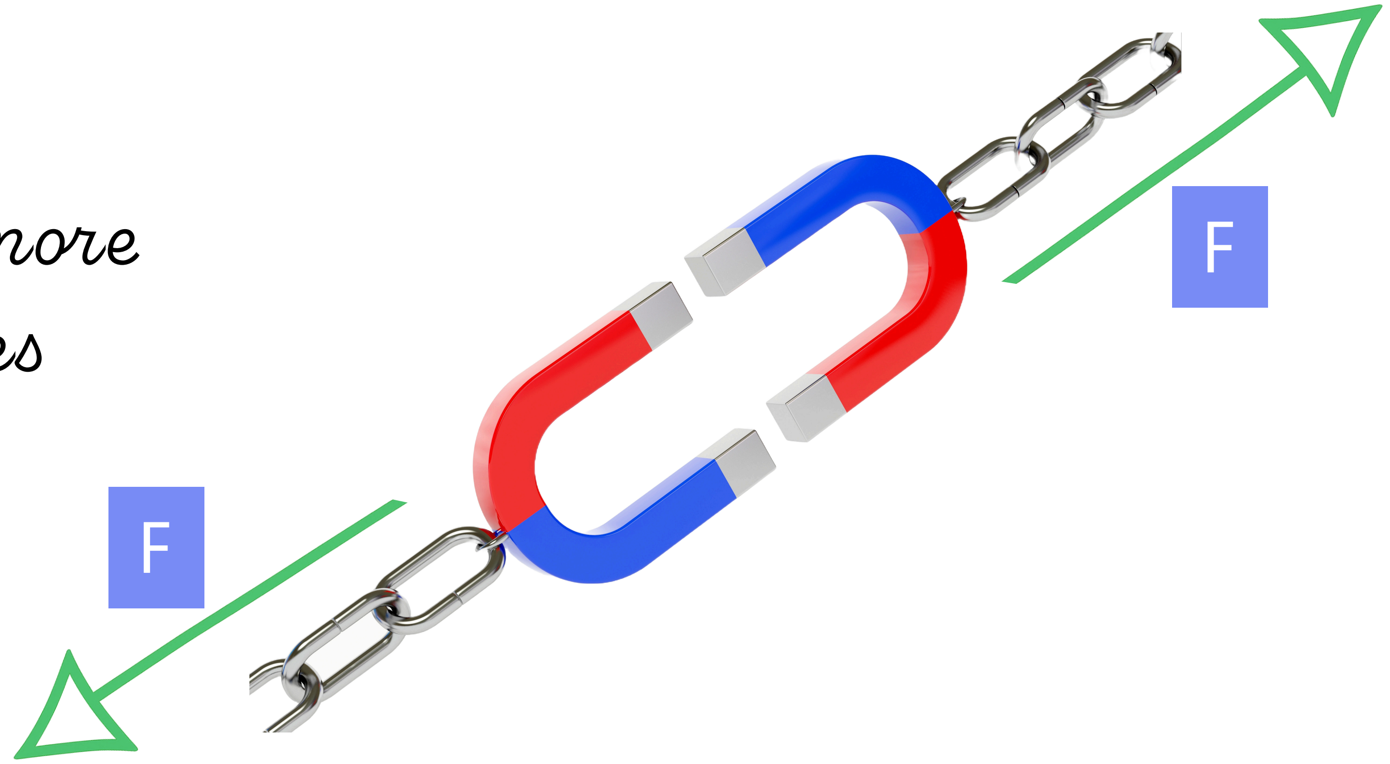


PHYSICAL MEANING OF THE FORMULA

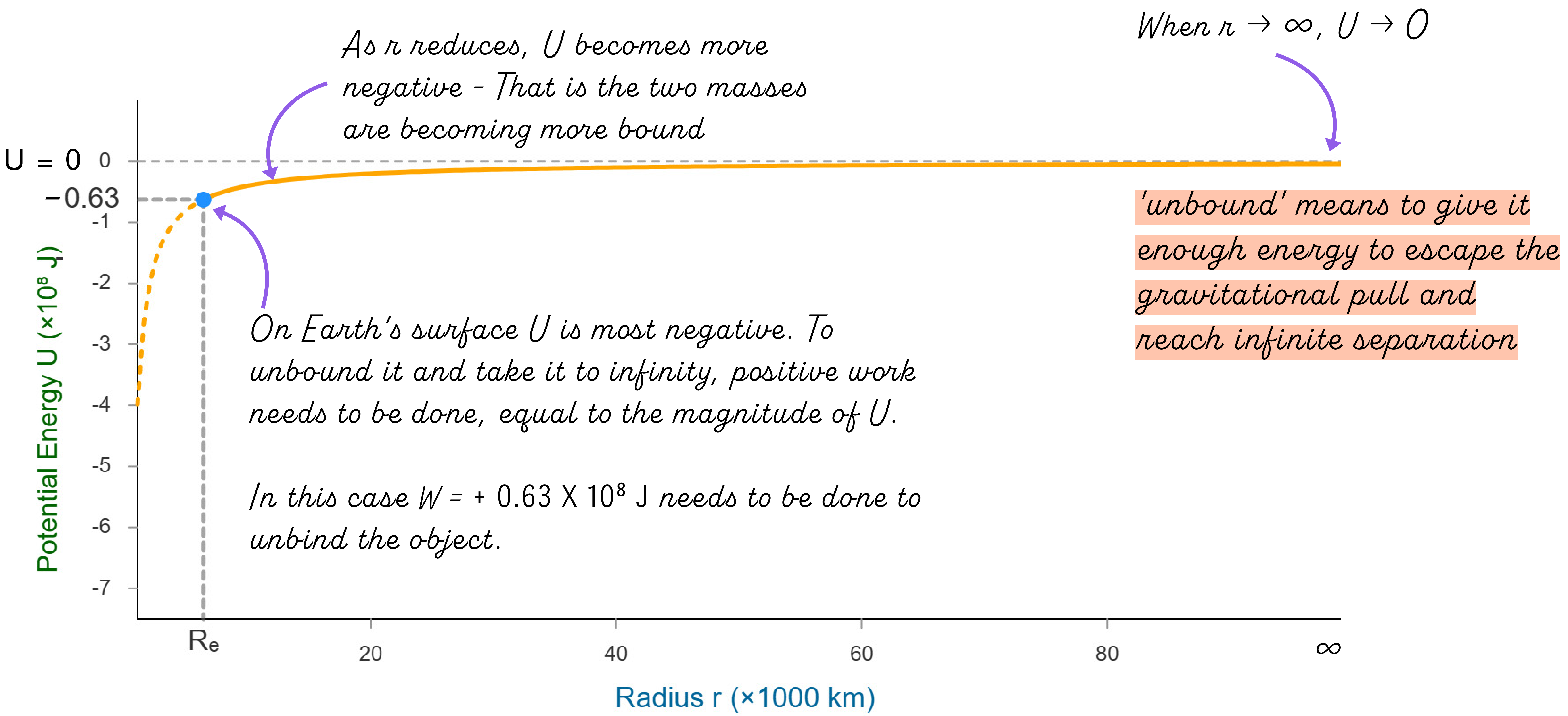
$$U = -GMm / r$$

1. The formula shows that at infinite separation ($r = \infty$), $U = 0$
2. As objects come closer – that is, as r reduces, U becomes more negative (see next slide)
3. So to increase separation, you need to add energy to the system
4. That is, work needs to be done to separate them.

Think of GPE like the effort you would require to pull two magnets away from each other – the closer they are, the more tightly they cling together. In a similar way, gravity makes masses cling more strongly as they get closer



CHANGE IN U FROM INFINITY TO EARTHS SURFACE



PHYSICAL MEANING OF THE FORMULA -

$$U = -GMm / r$$

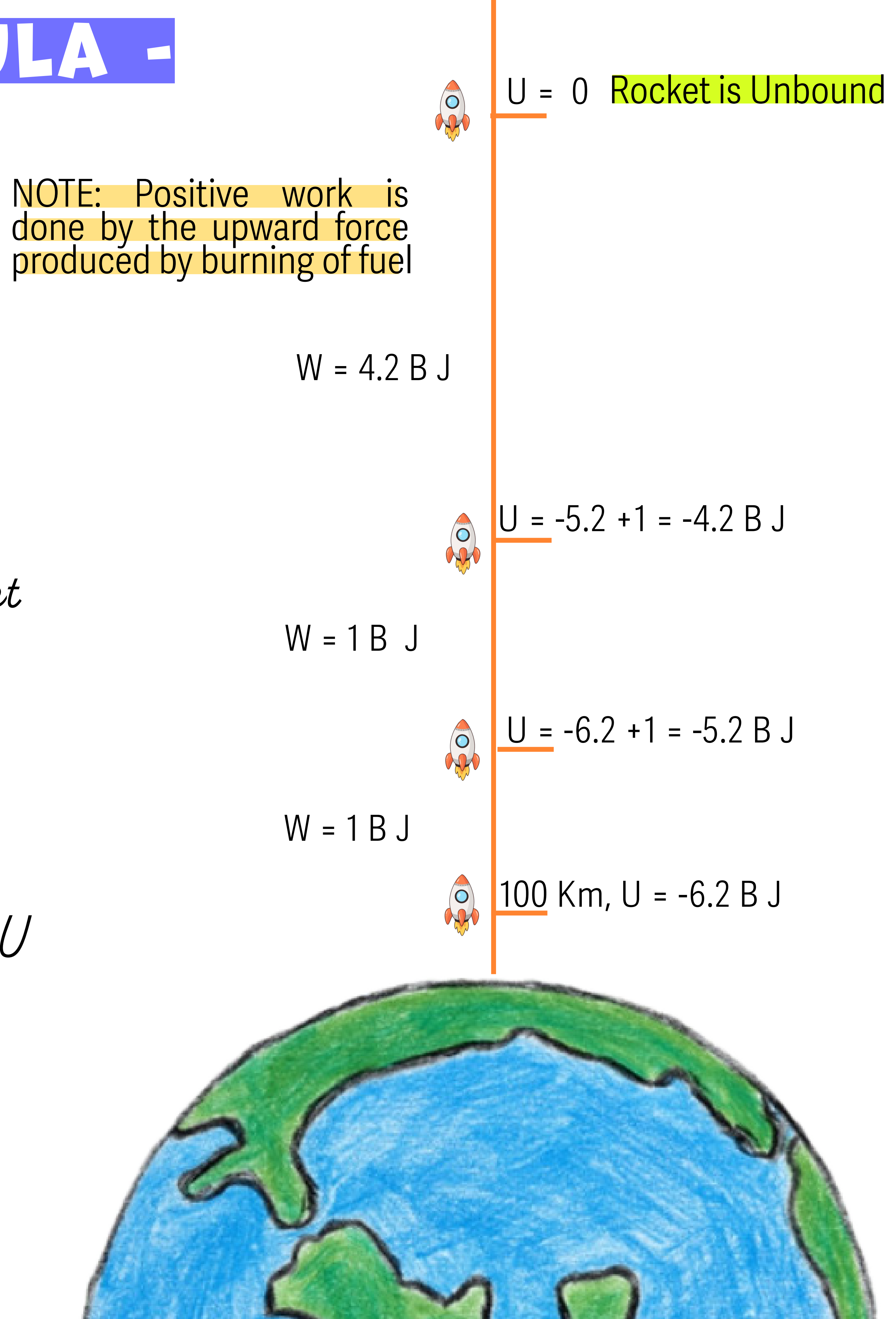
Example

(a) Take a 100 kg rocket 100 km above Earth. U is about −6.2 billion joules.

(b) If we add 1 B J of energy, U becomes −5.2 BJ. The rocket moves farther out.

(c) Add another 1 BJ, and U becomes −4.2 BJ, indicating even greater separation.

(d) Continue this way, and once a total of 6.2 BJ is added, U reaches 0 — the rocket is now infinitely far and free from Earth's gravity or unbound



COMPARING TWO REFERENCE POINTS

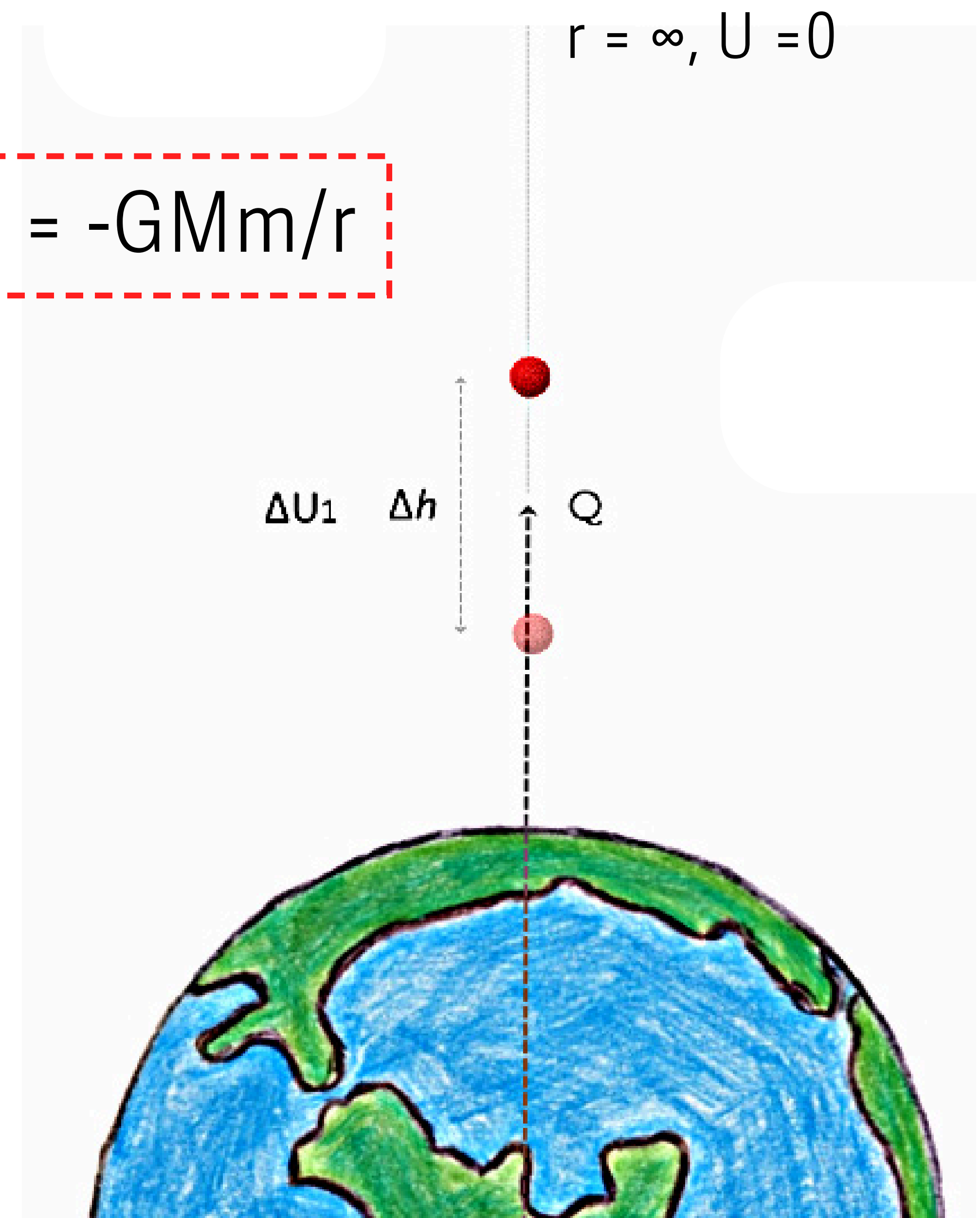
$U = 0$ at $r = \infty$

1. $U = 0$ is set at infinity – this is a common convention used in planetary physics.
2. As the object falls from higher to lower altitude, GPE becomes more negative.
3. Use this formula in space or large distance

$U = 0$ on Earth's surface

1. Here, we define $U = 0$ at Earth's surface – this is an approximation valid near Earth's surface.
2. In this model, U increases with height using $U = mgh$.
3. Use in near-surface problems

$$\boxed{U = -GMm/r}$$

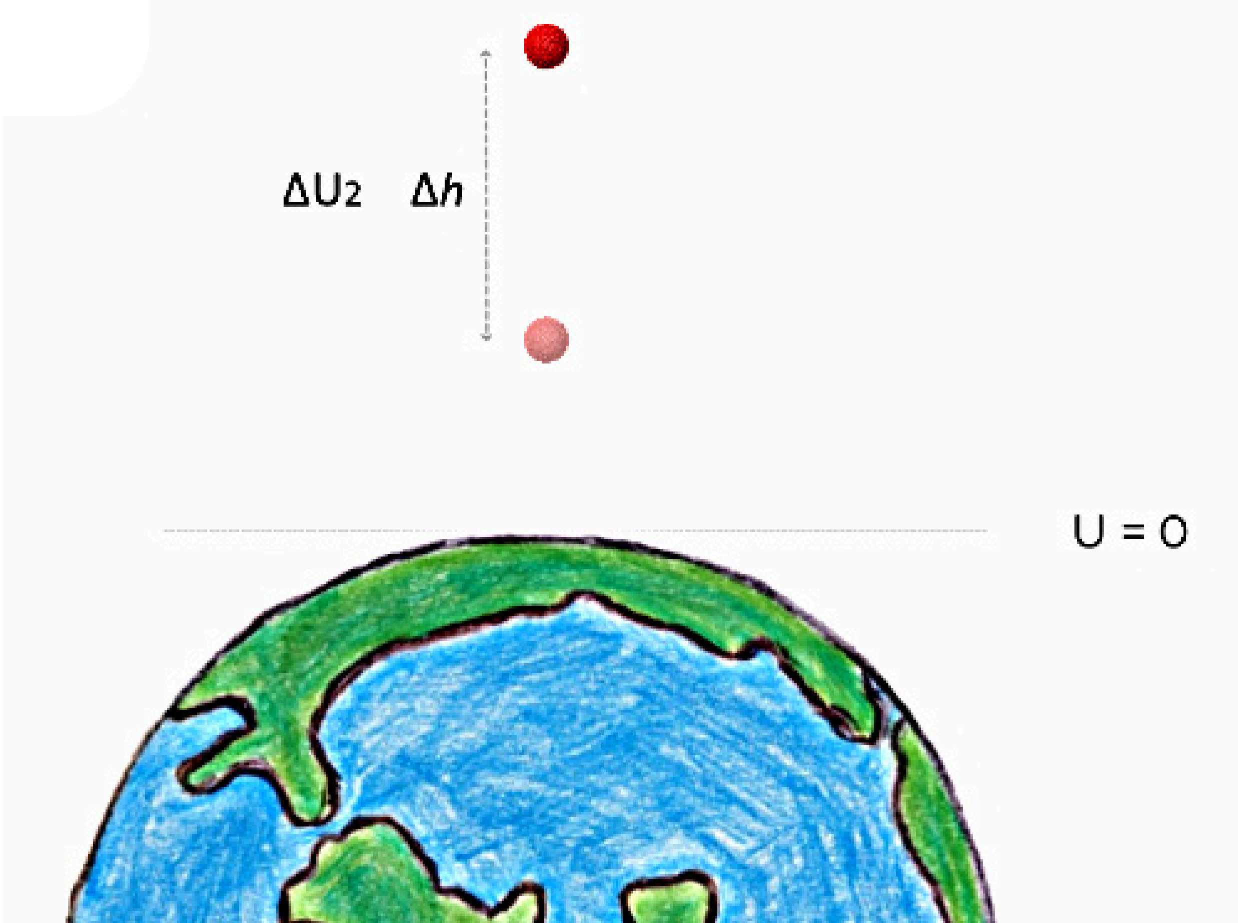


In either model the change in U is the same if an object moves from any elevation h1 to any other elevation h2

$$\Delta U_1 = \Delta U_2$$

Physics does not care about the absolute value of U. Only ΔU matters

$$\boxed{U = mgh}$$



KEY FORMULAS & EQUATIONS

THE SCIENCE CUBE

Equation	When to Use	Notes
$U = -G \cdot M \cdot m / r$	To find gravitational potential energy of two masses at a distance r.	1. Negative sign indicates the bound state of the two masses. 2. The more negative it is, the more bound they are. 3. This implies that more external positive work needs to be done to unbound them. (Reference point is at infinity where U = 0)
$\Delta U = U_2 - U_1 = -G \cdot M \cdot m\,(1/r_2 - 1/r_1)$	To calculate change in gravitational potential energy between two distances.	Useful in conservation of energy problems. In physics, this change is what is more useful to understand various phenomenon
$U = m \cdot g \cdot h$	To find gravitational potential energy near Earth's surface	Valid only for h << R (Earth's radius). Assumes g is constant.
$W = \int F \cdot dr = -\Delta U$	To relate work done by gravity to change in potential energy.	Use integral when F is variable. Essential for deriving U = −GMm/r.

COMMON MISCONCEPTIONS & CORRECT EXPLANATIONS

❌ Misconception 1: Negative GPE means "less than zero energy"

✅ Clarification: The negative sign in the universal gravitational potential energy formula ($U = -GMm/r$) does not mean "less than zero energy" in a diminishing sense.

- It signifies a "bound system" indicating that the objects are gravitationally attracted and require positive work to be done against this attraction to separate them to an infinite distance, where GPE is conventionally set to zero.
- A more negative GPE value implies a lower energy state and a stronger gravitational attraction, like being deeper in a gravitational well.

❌ Misconception 2: Gravitational energy belongs only to the object

✅ Clarification: GPE is not an intrinsic property of a single object.

- Instead, it is a property of the interacting system, such as the Earth-object system.
- GPE arises from the mutual gravitational attraction between two or more masses,
- This implies that it is stored in the gravitational field due to their relative positions, not within one object in isolation.

❌ Misconception 3: GPE and mgh mean the same thing always.

✅ Clarification: No. $U = mgh$ is a near-Earth approximation, valid only when the height h is small compared to Earth's radius—because gravity (g) is nearly constant.

- $U = -GMm/r$ is the universal formula for gravitational potential energy and must be used when dealing with space, planetary motion, or satellites, where gravity decreases with distance.
- Use mgh for ground-level problems and $-GMm/r$ for large-scale or orbital scenarios.

❌ Misconception 4: Potential energy is always meaningful on its own. That is the absolute value at various separations is what is useful

✅ Clarification: Only changes in potential energy (ΔU) have physical meaning.

- What matters is how much energy changes when an object moves.
- This change will always be the same no matter where you take the reference value of $U = 0$, as long as you are consistent. That is, the basic physics works

❌ Misconception 5: If kinetic energy is positive and potential energy is negative, the object has no energy.

✅ Clarification: No. In gravitational systems, kinetic energy (K) is always positive, and potential energy ($U = -GMm/r$) is negative. What matters is the total mechanical energy, $E = K + U$.

- For bound systems like satellites in orbit or planets around the Sun, the total energy E is negative—this means the object is gravitationally trapped. It still has kinetic energy (it's moving!), but not enough to escape.
- Only when the total energy becomes zero or positive can the object break free from gravity—that's the idea behind escape velocity. So, a negative total energy doesn't mean "no energy"—it means "not enough to escape.

A satellite in low Earth orbit has negative total energy – it's moving fast (kinetic) but still gravitationally trapped