lec 13 :- Anti Symmetric Transitive

Reflexive, Symmetric } Previous lec.

if $p = f$ then property holds.

we will check when $p = T$

Anti Symmetric :-

$\forall a, b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$A = \{1, 2, 3, 4\}$. $A \times A$.

$R = \{(1,1)\}$. Antisymmetric = ?

$(a,b) = (1,1) \in R$

$(b,a) = (1,1) \in R$

$a = b$ $1 = 1$

Anti Symmetric :-

$\forall a, b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$A = \{1, 2, 3, 4\}$.

$R = \{\} = \phi$.

$p = F$ by default the Implication = 1 property holds.

$R = \{(1,1), (2,3)\}$.

$(a,b) = (2,3) \in R$

$(b,a) = (3,2) \notin R$.

the property holds.

$A = \{1, 2, 3, 4\}$

$R = \{(1,2), (2,1), (3,2)\}$.

$(a,b) = (1,2) \in R$

$(b,a) = (2,1) \in R$

the property do not hold. $\rightarrow 2 \neq 1$

the property do not hold. $\to 2 \neq 1$

Ex7 4b2:

$A = \{1,2,3,4\}$.

$R_1 = \{(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)\}$

$(a,b) = (1,2) \in R$
$(b,a) = (2,1) \in R$ $\to a=b$
$= 1 \neq 2$

it is not antisymmetric.

$A = \{1,2,3,4\}$

$R_2 = \{(1,1),(1,2),(2,1)\}$.

it is not antisymmetric.

$(1,2) \in R \wedge (2,1) \to 1 \neq 2$

Symmetric:

$\forall a,b$ if $(a,b) \in R \to (b,a) \in R$.

AntiSymmetric:

$\forall a,b$ if $(a,b) \in R \wedge (b,a) \in R \to a = b$.

Homework Ex 7 / P 462

Do check $R_3 - R_7$ for properties.

1) Reflexive 2) Symmetric.

3) Antisymmetric.

Transitive:

$\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \to (a,c) \in R$.

$A = \{1,2,3,4\}$

$A = \{1,2,3,4\}$

$R = \phi$ Holds.

$R = \{(1,1),(1,2)\}$. $(a,b) = (1,1) \in R \wedge$
$(b,c) = (1,2) \in R$
$\rightarrow (1,2) \in R$

A relation with one element is always transitive.

Transitive.

Ex7 462:

$R_1 = \{(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)\}$

This is not transitive.

$(3,4) \in R \wedge (4,1) \in R$
$\rightarrow (3,1) \notin R$

$(3,1) \notin R$

$R_6 = \{(3,4)\}$ This is transitive.

Ex12: 463

$R = \{(a,b) \mid a \text{ divides } b\}$ $A = Z^+$

Reflexive =?
Symmetric =?
AntiSymmetric =?
Transitive =?

Reflexive: $\forall a \in A \quad (a,a) \in R$.

$\forall a \in Z^+$ a divides a

this holds $\forall a \in Z^+$.

Symmetric: $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$

$\forall a,b \in Z^+$ if a divides b $\rightarrow$ b divides a

Counter Example $\Rightarrow$ 6 divides 12 $\rightarrow$ 12 divides 6

Property do not hold.

Anti Symmetric:

$\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$

$\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$

$\forall a,b \in Z^+$ if a divides b $\wedge$ b divides a $\rightarrow a = b$.

this holds.

Transitive:- $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in Z^+$ if a divides b $\wedge$ b divides c $\rightarrow$ a divides c.

Property holds.

$A = Z^+$

$R = \phi$

R, S, AS, T ?

Reflexive. ✗ Symmetric ✓ AntiSymmetric ✓ Transitive ✓

Composite: R (a,b) $A \times B$; S (a,c) $B \times C$.

$a \in A$, $b \in B$, $c \in C$.

$S \circ R = \{(a,c) \mid (a,b) \in R \wedge (b,c) \in S\}$.

$R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$ $A \times B$.

$S = \{(1,0), (2,0), (3,1), (3,2), (4,1)\}$. $B \times C$

$A = \{1,2,3\}$

$B = \{1,2,3,4\}$.

$C = \{0,1,2\}$.

$S \circ R = \{(1,0), (1,1), (2,1), (2,2), (3,0), (3,1)\}$.

Homework #2. $R \circ S$.

$S \circ R \neq R \circ S$. (Commutative) ✗

$$S \circ R \neq R \circ S. \quad (\text{Commutative})$$

$S \circ S$

$$R \circ R = R^2$$
$$R^2 \circ R = R^3$$
$$R^3 \circ R = R^4$$
$$\vdots$$
$$R^{n-1} \circ R = R^n$$

A Relation is transitive if $R^n \subseteq R$ for $n = 1, 2, 3 \ldots$

$$\left.\begin{array}{l} R^2 \subseteq R = ?\ T \\ R^3 \subseteq R = ?\ T \\ \vdots \\ R^n \subseteq R = ?\ T \end{array}\right\} R \text{ is transitive.}$$