

lec 13 :- Anti Symmetric Transitive

Anti Symmetric :-

$$\forall a, b \in A \text{ if } (a, b) \in R \wedge (b, a) \in R \rightarrow a = b.$$

$A = \{1, 2, 3, 4\}$. AKA.

$R = \{(1, 1)\}$. Anti Symmetric ?



$(a, b) = (1, 1) \in R$

$(b, a) = (1, 1) \in R$

$a = b \quad | = |$

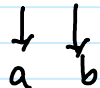
Anti Symmetric :-

$$\forall a, b \in A \text{ if } (a, b) \in R \wedge (b, a) \in R \rightarrow a = b.$$

$A = \{1, 2, 3, 4\}$.

$R = \{ \} = \emptyset$.

$R = \{(1, 1), (2, 3)\}$.



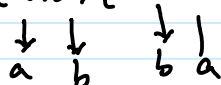
$(a, b) = (2, 3) \in R$

$(b, a) = (3, 2) \notin R$.

the property holds.

$A = \{1, 2, 3, 4\}$

$R = \{(1, 2), (2, 1), (3, 2)\}$.



$(a, b) = (1, 2) \in R$

$(b, a) = (2, 1) \in R$

the property do not hold. $\rightarrow 2 \neq 1$

Reflexive } Previous
Symmetric } Lec.

if $P \neq T$ then
Property holds.

we will check
when $P = T$

the property do not hold. $\rightarrow 2 \neq 1$

Ex 7
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$A = \{1, 2, 3, 4\}$

$R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\}$
X \downarrow \downarrow \downarrow \downarrow X
 a b b a

$(a,b) = (1,2) \in R$
 $(b,a) = (2,1) \in R \rightarrow a \neq b$

it is not antisymmetric.

$A = \{1, 2, 3, 4\}$

$R_2 = \{(1,1), (1,2), (2,1)\}$
X \downarrow \downarrow \downarrow \downarrow
 a b b a

it is not antisymmetric.

$(1,2) \in R \wedge (2,1) \rightarrow 1 \neq 2$

Symmetric:

$\forall a, b \text{ if } (a,b) \in R \rightarrow (b,a) \in R.$

Antisymmetric:

$\forall a, b \text{ if } (a,b) \in R \wedge (b,a) \in R \rightarrow a = b.$

Homework Ex 7 / P 462

Do check $R_3 - R_7$ for properties.

1) Reflexive 2) Symmetric.

3) Antisymmetric.

Transitive:

$\forall a, b, c \in A \text{ if } (a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R.$

$A = \{1, 2, 3, 4\}$

$$A = \{1, 2, 3, 4\}$$

$$R = \emptyset \quad \text{Holds.}$$

$$R = \{(1,1), (1,2)\} \quad \begin{array}{l} (a,b) = (1,1) \in R \wedge \\ (b,c) = (1,2) \in R \\ \rightarrow (1,2) \in R \end{array}$$

A relation with one element is always transitive.

Transitive.

Ex 7
462:

$$R = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\}$$

This is not transitive.

$$\begin{array}{l} (3,4) \in R \wedge (4,1) \in R \\ \rightarrow (3,1) \notin R \end{array}$$

$$R = \{(3,4)\}$$

This is transitive.

Ex 12:
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$$R = \{(a,b) \mid a \text{ divides } b\} \quad A = \mathbb{Z}^+$$

Reflexive: $\forall a \in A \quad (a,a) \in R$.

$\forall a \in \mathbb{Z}^+ \quad a \text{ divides } a$
this holds $\forall a \in \mathbb{Z}^+$.

Symmetric: $\forall a, b \in A \quad \text{if } (a,b) \in R \rightarrow (b,a) \in R$

$\forall a, b \in \mathbb{Z}^+ \quad \text{if } a \text{ divides } b \rightarrow b \text{ divides } a$

Counter Example $\Rightarrow 6 \text{ divides } 12 \rightarrow 12 \text{ divides } 6$
Property do not hold.

Anti Symmetric:

$$\forall a, b \in A \quad \text{if } (a,b) \in R \wedge (b,a) \in R \rightarrow a = b$$

Reflexive =?

Symmetric =?

Anti Symmetric =?

Transitive =?

$\forall a, b \in A$ if $(a, b) \in R \wedge (b, a) \in R \rightarrow a = b$
 $\forall a, b \in \mathbb{Z}^+$ if a divides $b \wedge b$ divides a
 $\rightarrow a = b.$

this holds.

Transitive: $\forall a, b, c \in A$ if $(a, b) \in R \wedge (b, c) \in R \rightarrow (a, c) \in R.$

$\forall a, b, c \in \mathbb{Z}^+$ if a divides $b \wedge b$ divides c
 $\rightarrow a$ divides $c.$

Property holds.

$R = \emptyset$ $A = \mathbb{Z}^+$

R, S, A.S, T ?

Reflexive. \times Symmetric \checkmark
Anti-Symmetric \checkmark
Transitive \checkmark

Composite: R S
 (a, b) (a, c) $a \in A$
 $A \times B$ $B \times C.$ $b \in B$
 $c \in C.$

$S \circ R = \{(a, c) \mid (a, b) \in R \wedge (b, c) \in S\}.$

$R = \{(1,1), (1,4), (2,3), (3,1), (3,4)\}$ $A \times B.$

$S = \{(1,0), (2,0), (3,1), (3,2), (4,1)\}$ $B \times C$

$S \circ R = \{(1,0), (1,1), (2,1), (2,2), (3,0), (3,1)\}.$

$A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4\}.$

$C = \{0, 1, 2\}.$

Homework #2. $R \circ S.$

$S \circ R \neq R \circ S.$ (Commutative) \times

So $R \neq R \circ S$. (Commutative)

S.O.S $R \circ R = R^2$

$$R^2 \circ R = R^3$$

$$R^3 \circ R = R^4$$

\vdots

$$R^{n-1} \circ R = R^n$$

A Relation is transitive

$\Rightarrow R^n \subseteq R$ for $n=1, 2, 3, \dots$

$$R^2 \subseteq R \stackrel{?}{=} T$$

$$R^3 \subseteq R \stackrel{?}{=} T$$

\vdots

$$R^n \subseteq R \stackrel{?}{=} T$$

} R is transitive.

