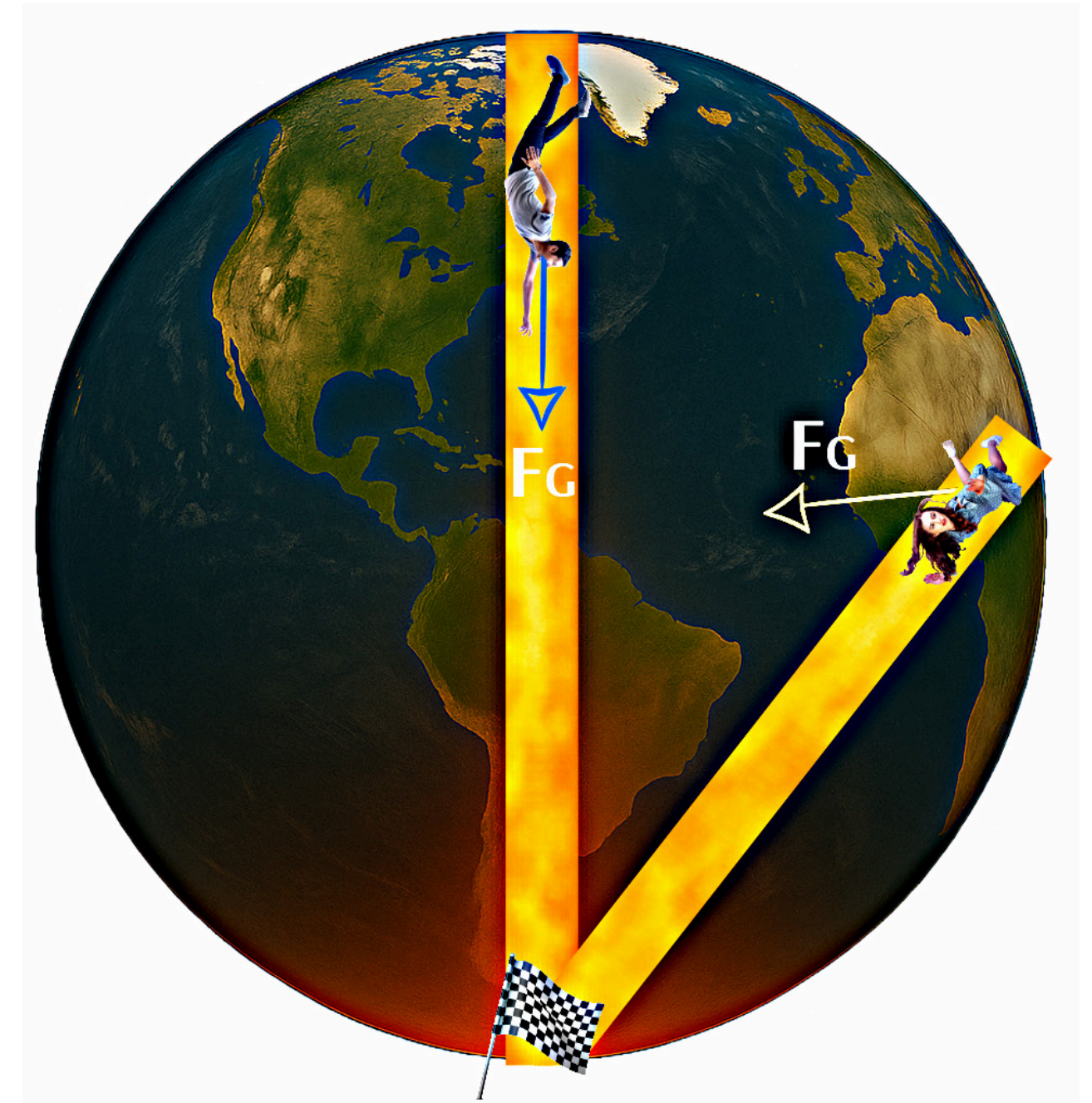


## Gravitational Force Inside Earth

1. Newton's shell theorem
2. Force inside a solid sphere & SHM in Earth's Gravity
3. Real versus ideal earth
4. Travel time between 2 points on Earth
5. Key formulas and equations
6. Misconceptions and clarifications



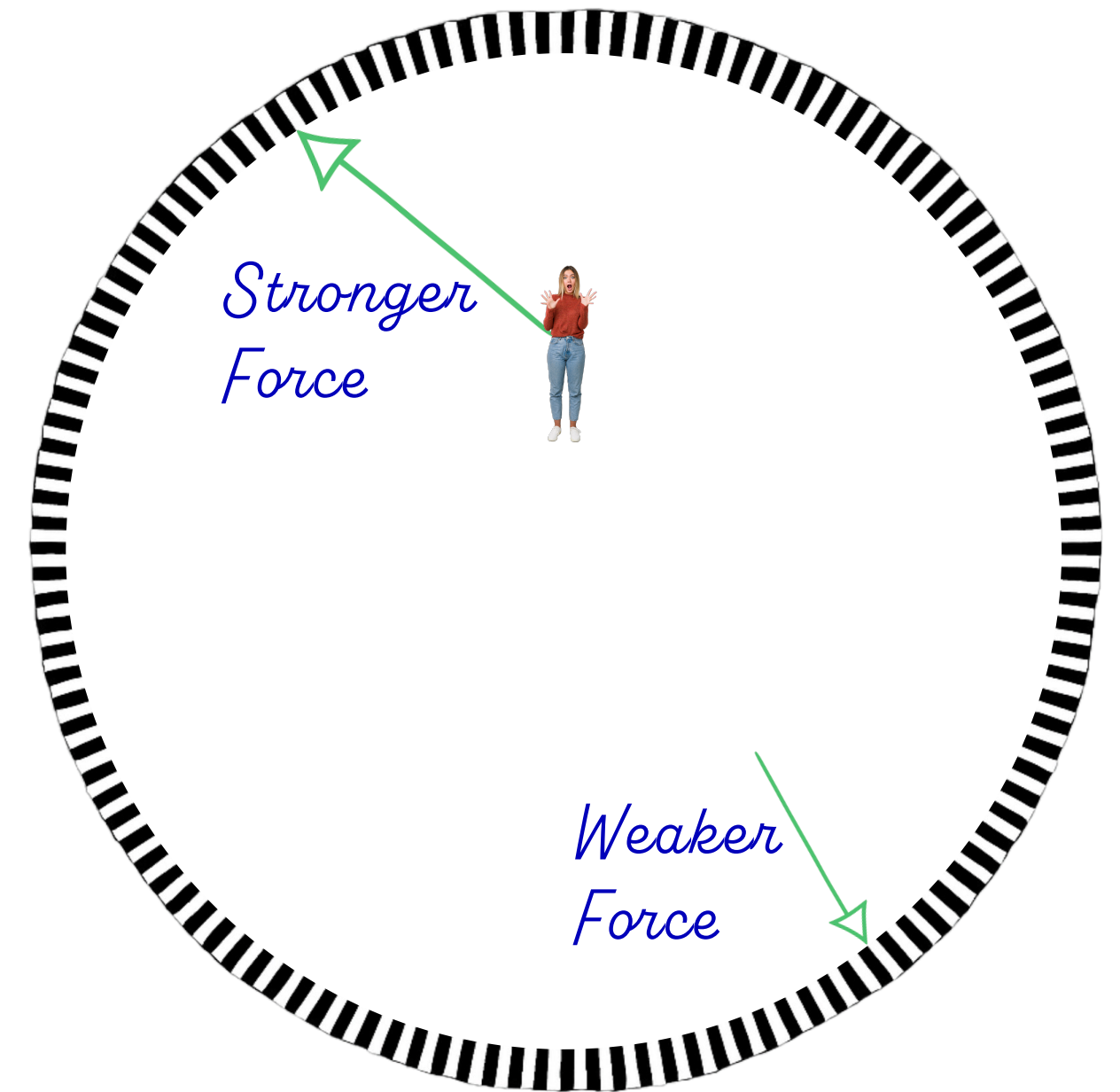
# NEWTON'S SHELL THEOREM AND HOLLOW SHELL

A spherically symmetric shell exerts **zero net gravitational force** on any particle located anywhere inside it

*Why?*

Each mass element of the shell attracts the particle, but by **spherical symmetry, forces from opposing patches cancel out exactly** due to their vector nature

**NOTE: This holds only inside a spherical shell—  
not outside or near its surface**



Though each mass element pulls on the particle, the perfect symmetry ensures a perfect cancellation — weaker forces from far-off areas are compensated by their abundance, and stronger nearby ones are few

# FORCE INSIDE A SOLID SPHERE & SHM

Consider a capsule at a distance  $r$  from Earth's center, inside a tunnel.

Only the mass  $M_i$  within radius  $r$  contributes to gravity; the outer shell has no net effect (by Shell Theorem)

$$F = G \tilde{m} \tilde{M} \tilde{r} / \tilde{R}^3$$

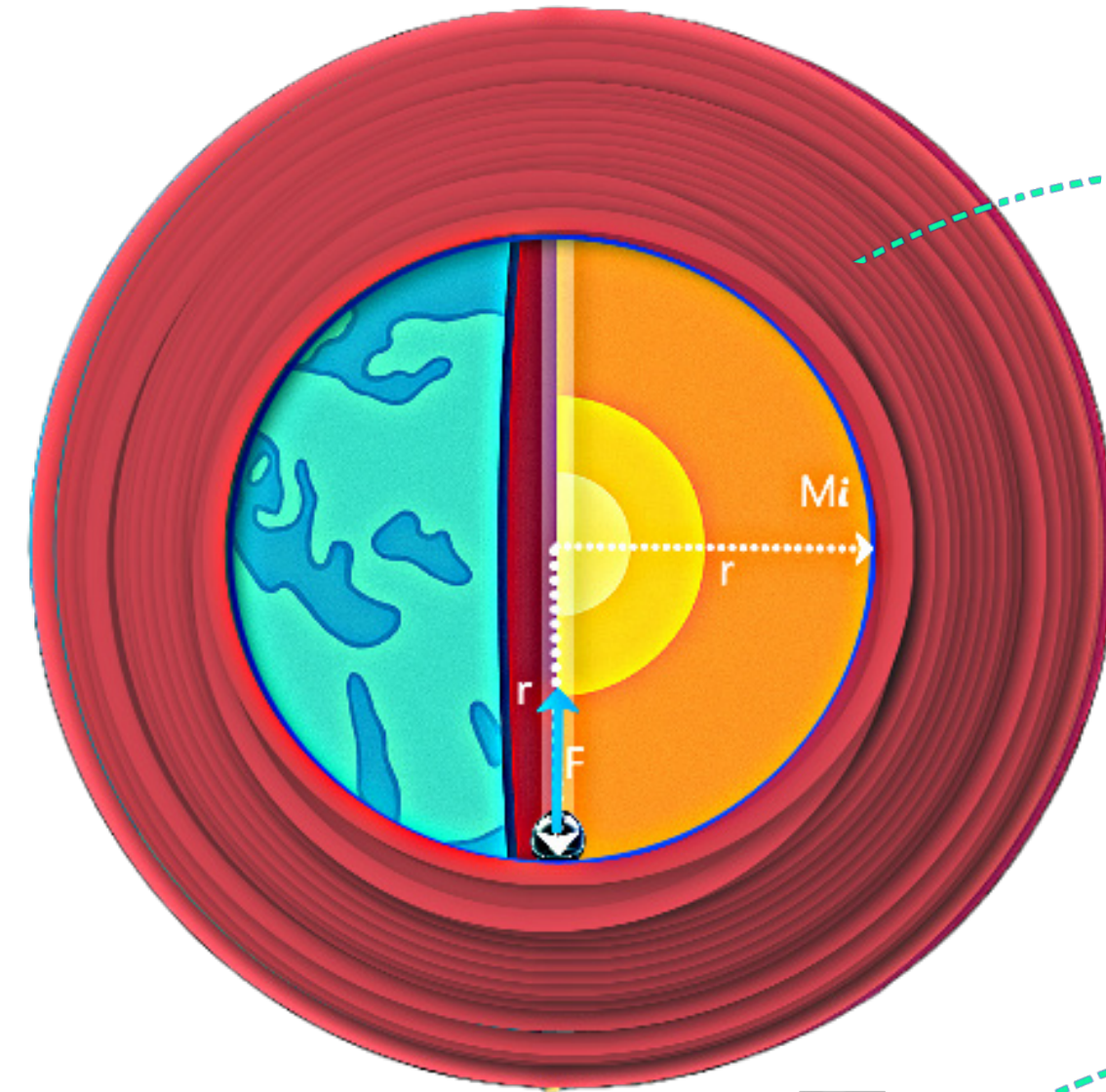
$$F = -k r$$

only true for a uniform density Earth.

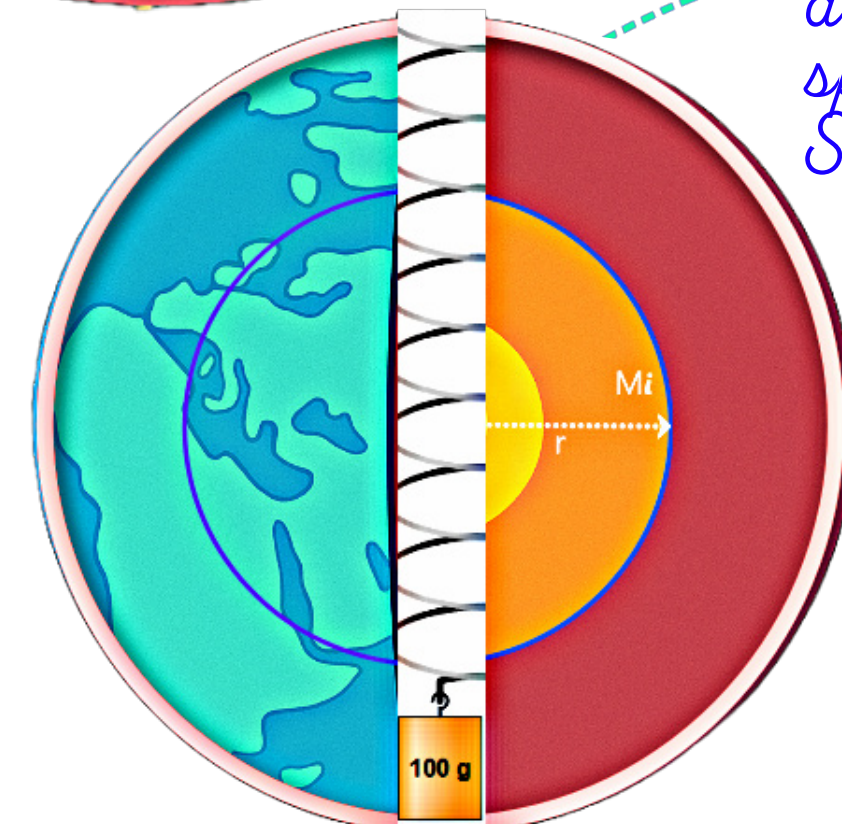
- $k = G m M / R^3$
- the minus sign shows  $F$  is directed toward the center, opposing  $r$

This is mathematically identical to Hooke's Law, so the capsule undergoes simple harmonic motion about the Earth's center.

A full oscillation (South-to-North-to-South) takes about 84 minutes, so a one-way trip is ~42 minutes



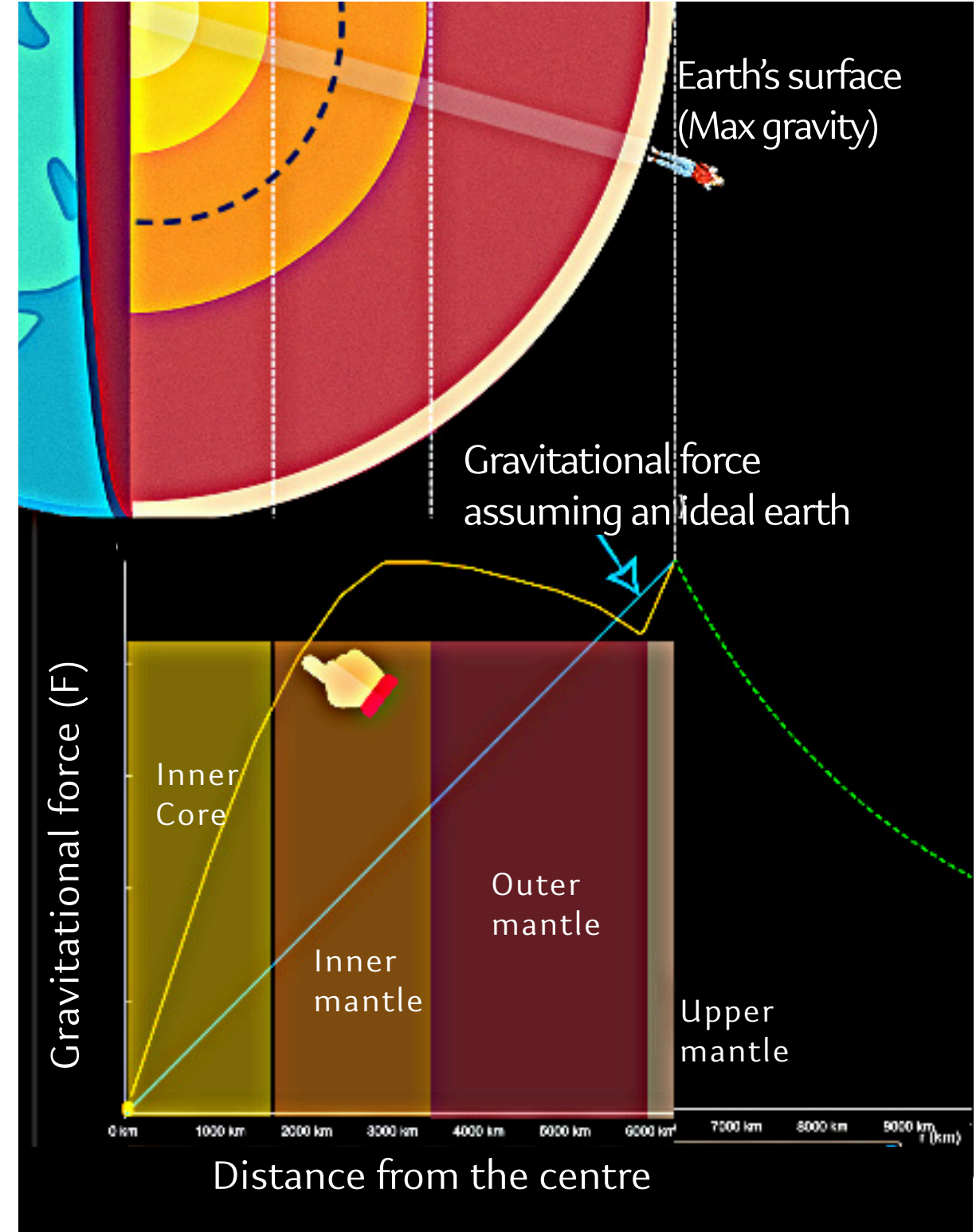
The outer shell (red) has zero contribution to gravitational force



Thus, the capsule's motion resembles that of a mass attached to a spring – a classic SHM scenario

# REAL VS. IDEAL EARTH

Earth Layer	Ideal Earth (Uniform Density)	Real Earth (PREM-based)	Explanation / Notes
Center	$F = 0$	$F = 0$	No enclosed mass → No gravitational pull at the very center.
Inner Core	$F \propto r$	$F$ increases slightly	Higher density leads to a stronger pull than ideal at similar depth.
Outer Core	$F \propto r$	$F$ reaches maximum, then starts declining	Denser outer core causes a gravitational peak before decline.
Lower Mantle	$F \propto r$	$F$ declines gradually	Density decreases slightly; still contributing to stronger gravity than upper layers.
Upper Mantle	$F \propto r$	Gravity starts to level out	Slight local dip just below the crust due to compositional transitions.
Crust (Surface)	$F = \text{maximum}$	$F = \text{maximum}$	Surface gravity is the reference value ( $\sim 9.8 \text{ m/s}^2$ ), includes Earth's rotation effects.
Above Surface	$F \propto 1/r^2$	$F \propto 1/r^2$	Newton's law applies outside any spherically symmetric mass — falls off quadratically.



In the real Earth, gravity doesn't decrease linearly — it changes due to density variations, it peaks slightly below surface due to mass distribution and eventually drops to zero at the center

# TRAVEL TIME BETWEEN 2 POINTS

1. A particle in a tunnel through Earth's diameter oscillates about the center – like a mass on a spring.
2. Travel time through the diameter of Earth (South Pole to North Pole) = 42 minutes.
3. The same 42-minute travel time applies to any straight tunnel (not just the diameter), as long as it's through a uniform Earth.
4. Reason:
  - Inside a uniformly dense Earth, gravity decreases linearly with distance from the center ( $F \propto r$ ), like a spring.
  - This creates Simple Harmonic Motion, with a fixed time period determined only by Earth's density.
  - In any straight tunnel—whether through the center or a slanted chord—the restoring force still pulls toward the center.



All straight tunnels through Earth act like S.H.M. tracks, with the same oscillation period regardless of orientation.

# KEY FORMULAS & EQUATIONS

<i>Equation</i>	<i>When to Use</i>	<i>Notes</i>
$F = G m M_i / r^2$	<i>Newton's law applied to <math>M_i</math> inside radius <math>r</math></i>	<i>Ensure <math>M_i</math> is correctly calculated for <math>r &lt; R</math>. <math>m</math> = mass of object</i>
$F = (G m M / R^3) r$ $F = -K r$	<i>Simplified, shows SHM behavior</i>	<i>Valid under uniform density. <math>M</math> is the total mass of the Earth</i>
$T \text{ (period)} = 2\pi \sqrt{R/g}$	<i>Time for full oscillation through diameter</i>	<i>Gives <math>\sim 84</math> min round-trip (<math>\sim 42</math> min one way).</i>

# COMMON MISCONCEPTIONS & CORRECT EXPLANATIONS

✘ *Misconception 1: More mass always means stronger gravity - That is, deeper inside Earth, gravity always increases due to more overlying mass.*

✔ *Clarification: The outer shells of Earth have no net gravitational effect inside them (by Shell Theorem). The only mass that matters is the mass enclosed within your current radius – and that mass actually gets smaller as you move deeper. That's why gravity decreases linearly toward the center.*

*But what about increasing density of the Earth's Core?*

- *Yes, density  $\rho$  increases into the lower mantle and core – but according to Newton's shell theorem, only the mass within radius  $r$  pulls on you.*
- *Outer layers cancel out. As you descend, you actually peel away outer mass, so the net gravitational force decreases, not increases.*

 *Misconception 2: Gravity at the center of the Earth is maximal (due to all surrounding mass)*

 *Clarification:*

- *If the Earth were of uniform density, gravity would decrease linearly as you move towards the center, becoming zero at the very center.*
- *In the more realistic model, gravity might fluctuate slightly due to varying densities of the layers, but it still approaches zero at the center.*

*The reason is that once you are inside the Earth -*

- *The gravitational pull from the mass outside your current position cancels out (Shell Theorem)*
- *...therefore you are only pulled by the mass inside your current radius.*
- *As you get closer to the center, there's less and less mass "inside" you to pull you*


✘ *Misconception 3: Within a hollow spherical shell, gravity is strong near the walls*


✔ *Clarification: Inside a hollow, uniform shell, gravity equals zero everywhere—not just at center. If you move closer to one side, that side's pull is offset by more mass on the other—which cancels gravity anywhere inside*

✘ *Misconception 4: An object dropped into a tunnel in Earth would stop at the center.*

✔ *Clarification: Due to the linear relationship between gravitational force and distance from the center ( $F = -kr$ , in an ideal uniform Earth), the object would undergo Simple Harmonic Motion (SHM).*

- *It would oscillate back and forth through the center, much like a mass on a spring, continuously moving between the two poles (or any two points on the surface if the tunnel is slanted).*
- *It would only stop if air resistance or other resistive forces were present.*

 *Misconception 5: The Shell Theorem implies that once you're inside Earth, there's no gravity at all.*

 *Clarification: The Shell Theorem states that a spherical shell of uniform density exerts no net gravitational force on a particle inside it.*

- *This doesn't mean there's no gravity at all once you enter the Earth.*
- *It means the gravitational effect of the mass above you (the "shell" of Earth's material outside your current radius) cancels out.*
- *You are still pulled by all the mass that is inside your current radius, effectively treating that inner mass as if it were concentrated at the center*

❌ *Misconception 6: The "gravity train" concept is only for tunnels going straight through the center (time taken to travel is 42 min.)*

✅ *Clarification: This 42-minute travel time through a tunnel from North to South pole is the same whether you go straight through the center of the Earth or drill a slanted tunnel between any two surface points—because the restoring force is always directed toward the center and scales linearly with distance! The travel time for a "gravity train" between any two points on the surface, ignoring resistance, is indeed the same.*

❌ *Misconception 7: Objects in a tunnel through Earth would "float" at the center.*

✅ *Clarification: At the very center, gravity is zero, but an object would not float in equilibrium unless it arrives there with zero velocity. In reality, unless stopped, it would pass through the center and continue oscillating*