Lec # 12 (Part 2).

Relation. → تعلق

Sets A & B.

A Binary Relation from A to B is a Subset of $A \times B$.

$A = \{a, b\}$

$B = \{1, 2\}$.

$A \times B = \{(a,1), (a,2), (b,1), (b,2)\}$.

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$B = \{1, 2\}$.

$A \times B = \{(a,1), (a,2), (b,1), (b,2)\}$

$2^{|A \times B|} = 2^4 = 16$.

$pow(A \times B) = \{ \emptyset, \{(a,1)\},$
$\{(a,2)\}, \{(b,1)\}, \{(b,2)\},$
$\{(a,1), (a,2)\}, \{(a,1), (b,1)\},$
$\{(a,1), (b,2)\}, \{(a,2), (b,1)\},$
$\{(a,2), (b,2)\}, \{(b,1), (b,2)\}$
$\{(a,1), (a,2), (b,1)\},$
$\{(a,1), (a,2), (b,2)\},$
$\{(a,2), (b,1), (b,2)\},$

$\}$

$R \subseteq A \times B$. $pow(A \times B)$

$= \{(a,1), (a,2),$
$(b,1), (b,2)\}$.

So $|A| \times |B|$
$= |A \times B|$

$|A| = 2$

$|B| = 2$

$2 \times 2 = |A \times B|$

$|A| = 3$.

$|B| = 4$.

How many Relations?

$R \subseteq A \times B$. $\underline{pow(A \times B)}$

$|pow(A \times B)| = 2^{|A| \times |B|}$
$= 2^{3 \times 4} = 2^{12}$ = 2 Relations.

$|A| = 4$

$|B| = 3$

How many elements in $A \times B$

$|A \times B| = |A| \times |B|$
$= 4 \times 3 = 12$.

$|A \times B| = |A| \times |B|$
$= 4 \times 3 = 12.$

1) Cardinality of $A \times B$. $= |A \times B|$

2) Relations on $A \times B$. $= 2^{|A| \times |B|}$

$(a,b) \in R \longleftrightarrow a R b.$

$(a,b) \notin R \longleftrightarrow a \not R b$

Ex 4, 461:- $A = \{1, 2, 3, 4\}$

$R = \{(a,b) \mid a \text{ divides } b\}$. $A \times A$

$|A \times A| = |A| \times |A|$
$= 4 \times 4 = 16.$
2^{16} Relations.

find R.

$A \times A = \{(1,1), (1,2), (1,3), (1,4),$
$(2,1), (2,2), (2,3), (2,4),$
$(3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$

$R = \{(1,1), (1,2), (1,3), (1,4),$
$(2,2), (2,4), (3,3), (4,4)\}.$

Ex 5, 461:- $R_1 = \{(a,b) \mid a \leq b\}$.

$A = \mathbb{Z}$

$A \times A = \mathbb{Z} \times \mathbb{Z}.$

$\{(1,1), (1,2), (2,1), (1,-1), (2,2)\}.$

$R_2 = \{(a,b) \mid a > b\}.$

$\{(1,1), (1,2), (2,1), (1,-1), (2,2)\}.$

Compute the Remaining your self.

Set builder Notation.

$\{\underbrace{\ }_{\text{type}} \mid \underbrace{\ \ldots\ }_{\text{Condition}}\}$

$\mathbb{Z} = \{-\infty \ldots +\infty\}$

Properties of Relations.

1) Reflexive.
if $\forall a \in A \ (a,a) \in R$. $A \times A$

1) Reflexive.

if $\forall a \in A \ (a,a) \in R$.

$A = \{1,2,3,4\}$.

$R = \{ (1,1),(1,2),(2,1),(2,2),$
$(3,4),(4,1),(4,4)\}$.

Is it Reflexive?

Ex7 4.62:- Do it yourself.

$A = \{1,2\}$.

$R = \Phi$ Reflexive?

$\forall a \in A \ (a,a) \in R$.

$R = \{(1,1)\}$ Not Reflexive.

$R = \{(1,1),(2,2),(2,1)\}$

Reflexive.

Symmetric:-

$\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$A = \{1,2,3,4\}$.

$R = \{(1,1)\}$. symmetric?

$R = \{ \underbrace{(1,1)}_{x}, \underbrace{(2,2)}_{x}, (\underbrace{3}_{a},\underbrace{1}_{b}) \}$.

$R = \{ (1,1),(2,2),(3,1),(1,3),$
$\underbrace{(2,1)}_{x}\}$. $(1,2) \notin R$.

Not Reflexive.

$R = \Phi$. symmetric.

Ex2 4.65 $R = \{(a,b) \mid a \text{ divides } b\}$

$A = Z$.

$A \times A$.

$\forall a \in A \Rightarrow$

$(1,1) \in R \wedge$
$(2,2) \in R \wedge$
$(3,3) \in R \wedge$
$(4,4) \in R$.

$T \wedge T \wedge F$
$= F$.

$(1,1) \in R \wedge$
$(2,2) \in R$

$R = R$.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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$R = \{(a,b) \mid a$ divides $b\}$

$A = \mathbb{Z}$.

Symmetric ?

$(2,4) \in R$. $\Rightarrow$ Counter

$(4,2) \notin R$. Example.

$(2,4) \in R \rightarrow (4,2) \notin R$.

$= F$

Not Symmetric.

$R = \{(a,b) \mid a$ divides $b\}$ $A = \mathbb{Z}$.

Reflexive = ?

$(1,1) \in R$.

$(100, 100) \in R$ Reflexive holds.

Transitive & AntiSymmetric.

Expect a Question.