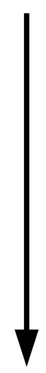
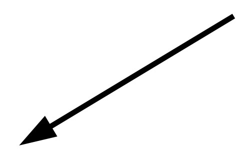
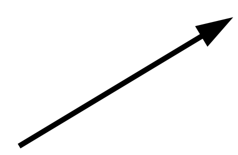
2D MOTION & VECTORS

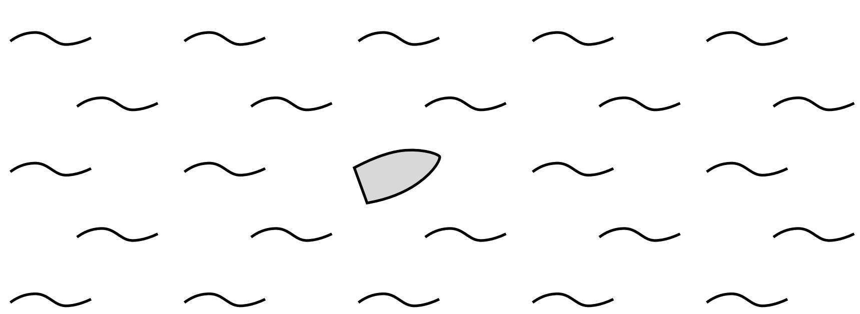
1. A boat travels a distance of d_1 in one direction and then immediately travels a distance of d_2 in a different direction, where the relative magnitudes and directions of the displacements are shown in the figure above. Which of the following best shows the boat's total displacement?

(A) 

(B) 

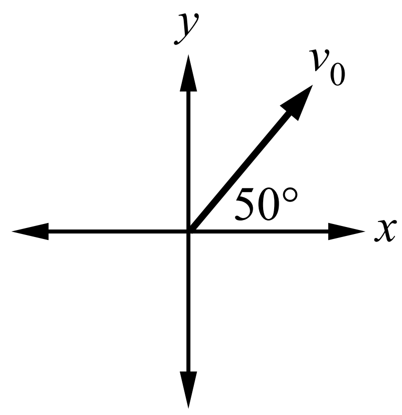
(C) 

(D) 



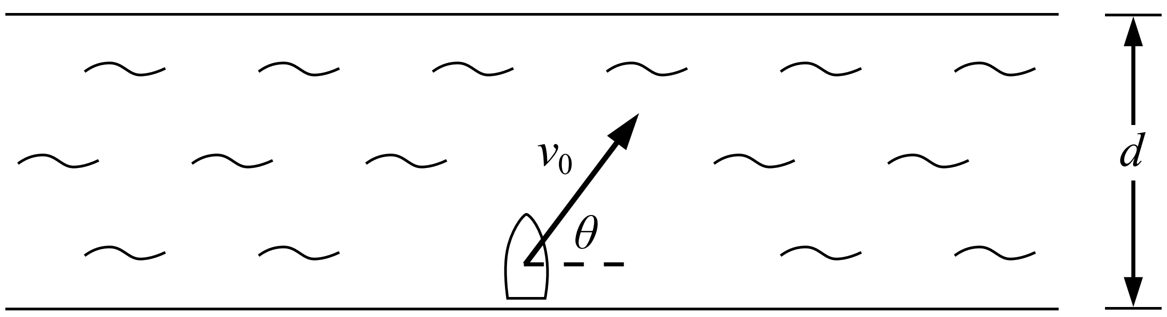
2. A boat is in the middle of a lake as shown in the figure above. The boat moves 18 m in a straight line with an unknown direction, and then it moves 25 m in a straight line with an unknown direction. Which of the following cannot be the magnitude of the boat's total displacement?

(A) 43 m
(B) 18 m
(C) 25 m
(D) 5 m



3. A biker is riding at a velocity of v_0 which has a direction shown in the figure above. If the biker maintains their speed and turns so that their velocity is at an angle of 150° from the positive x axis, the magnitude of the x component of their velocity will

(A) increase
(B) decrease
(C) be the same
(D) cannot be determined



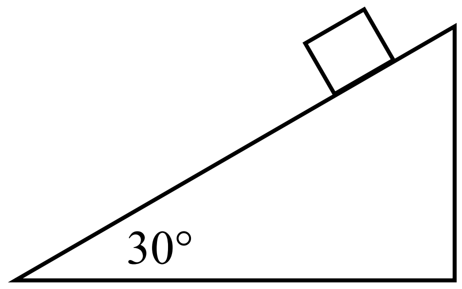
4. A boat crosses a river with a width of d. The boat points straight across the river and the river is flowing to the right. The resultant velocity of the boat is a constant v_0 as shown in the figure above. When the boat reaches the other side of the river, the distance that the boat moves downstream (to the right) is

(A) $\dfrac{d\sin(\theta)}{\cos(\theta)}$

(B) $\dfrac{d}{v_0\sin(\theta)}$

(C) $\dfrac{d\cos(\theta)}{\sin(\theta)}$

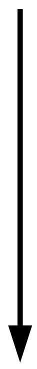
(D) $\dfrac{d}{v_0\cos(\theta)}$

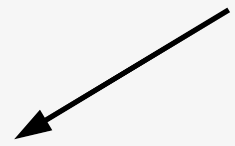


5. A block is held on an incline with negligible friction near the surface of planet X, where there is no air resistance and the gravitational acceleration is different from earth. The block is released from rest. After the block slides 2 m along the incline it is moving at a speed of 2.8 m/s. The gravitional acceleration near the surface of planet X is most nearly what percent of the gravitational acceleration near the surface of earth?

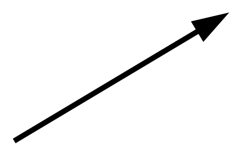
   (A) 10%
   (B) 20%
   (C) 23%
   (D) 40%

1. A boat travels a distance of d_1 in one direction and then immediately travels a distance of d_2 in a different direction, where the relative magnitudes and directions of the displacements are shown in the figure above. Which of the following best shows the boat's total displacement?

(A) 

(B) 

(C) 

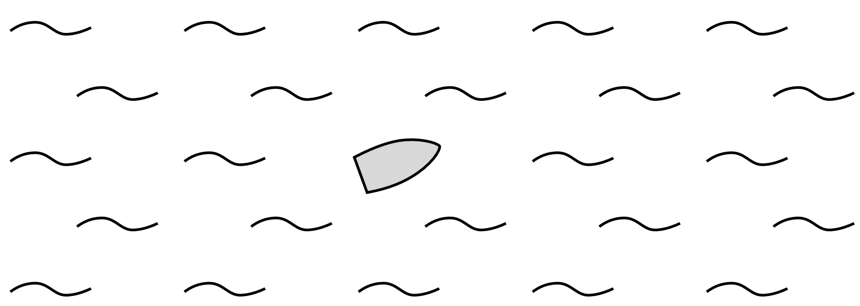
(D) 

A Incorrect
This answer shows the resultant vector of $d_2 - d_1$.

B **Correct**
The total displacement is the vector sum of $d_1 + d_2$. Both vectors have a horizontal component that points to the left so the resultant vector must also have a horizontal component that points to the left.

C Incorrect
This answer shows the resultant vector of $d_1 - d_2$.

D Incorrect
This answer shows the resultant vector of $-d_1 - d_2$.



2. A boat is in the middle of a lake as shown in the figure above. The boat moves 18 m in a straight line with an unknown direction, and then it moves 25 m in a straight line with an unknown direction. Which of the following cannot be the magnitude of the boat's total displacement?

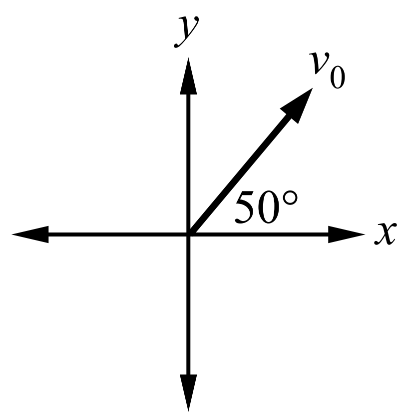
(A) 43 m
(B) 18 m
(C) 25 m
(D) 5 m

A Incorrect
This answer is within the range of possible magnitudes of the resultant displacement vector, and is the maximum possible magnitude when the two displacements are in the same direction.

B Incorrect
This answer is within the range of possible magnitudes of the resultant displacement vector.

C Incorrect
This answer is within the range of possible magnitudes of the resultant displacement vector.

D Correct
When two vectors are added, the maximum magnitude of the resultant vector is the sum of the individual vector magnitudes, which is when the vectors point in the same direction. The minimum magnitude of the resultant vector is the difference between the individual vector magnitudes, which is when the vectors point in opposite directions. For the boat's resultant displacement vector:
The minimum magnitude is: $(25\text{ m}) - (18\text{ m}) = 7\text{ m}$
The maximum magnitude is: $(25\text{ m}) + (18\text{ m}) = 43\text{ m}$
A magnitude of 5 m is not within this range.



3. A biker is riding at a velocity of v_0 which has a direction shown in the figure above. If the biker maintains their speed and turns so that their velocity is at an angle of 150° from the positive x axis, the magnitude of the x component of their velocity will

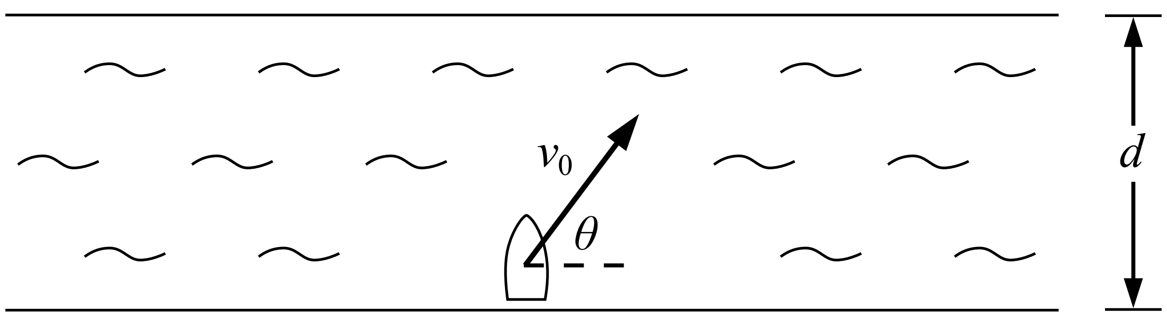
(A) increase
(B) decrease
(C) be the same
(D) cannot be determined

A Correct
The x component of the velocity vector is $v_0 \cos(\theta)$.
$v_0 \cos(50°) = 0.64\, v_0$
$v_0 \cos(150°) = -0.87\, v_0$
The second x component points in the negative x direction, but the question is asking about the magnitude which is the absolute value, so the magnitude increases.

B Incorrect

C Incorrect

D Incorrect



4. A boat crosses a river with a width of d. The boat points straight across the river and the river is flowing to the right. The resultant velocity of the boat is a constant v_0 as shown in the figure above. When the boat reaches the other side of the river, the distance that the boat moves downstream (to the right) is

(A) $\dfrac{d\sin(\theta)}{\cos(\theta)}$

(B) $\dfrac{d}{v_0\sin(\theta)}$

(C) $\dfrac{d\cos(\theta)}{\sin(\theta)}$

(D) $\dfrac{d}{v_0\cos(\theta)}$

A Incorrect
This answer switches the vertical and horizontal components of the velocity.

B Incorrect
This answer is the amount of time it takes the boat to cross the river.

C **Correct**
Using kinematics, the amount of time the boat takes to cross the river depends on the vertical component of the velocity and the width of the river. The distance the boat travels to the right depends on the time and the horizontal component of the velocity:

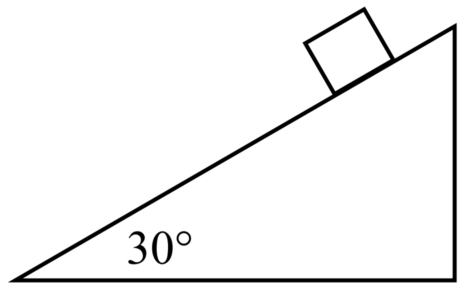
$$v_y = \frac{\Delta y}{\Delta t} = \frac{d}{\Delta t} \qquad \Delta t = \frac{d}{v_y} = \frac{d}{v_0\sin(\theta)}$$

$$v_x = \frac{\Delta x}{\Delta t} \qquad \Delta x = v_x\Delta t = v_0\cos(\theta)\frac{d}{v_0\sin(\theta)} = \frac{d\cos(\theta)}{\sin(\theta)}$$

Using only trigonometry, the boat follows a straight path across the river in the direction of the velocity. The horizontal component of the displacement vector is the distance the boat travels to the right, and the vertical component is the width of the river:

$$\tan(\theta) = \frac{\Delta y}{\Delta x} = \frac{d}{\Delta x} \qquad \Delta x = d\,\frac{1}{\tan(\theta)} = d\,\frac{\cos(\theta)}{\sin(\theta)}$$

D Incorrect
This answer would be the amount of time it takes the boat to cross the river but incorrectly uses the horizontal component of the velocity.



5. A block is held on an incline with negligible friction near the surface of planet X, where there is no air resistance and the gravitational acceleration is different from earth. The block is released from rest. After the block slides 2 m along the incline it is moving at a speed of 2.8 m/s. The gravitional acceleration near the surface of planet X is most nearly what percent of the gravitational acceleration near the surface of earth?

(A) 10%
(B) 20%
(C) 23%
(D) 40%

A **Incorrect**
This answer incorrectly switches a and g_{planet} in the vector relationship $a = g_{\text{planet}} \sin(30°)$.

B **Incorrect**
This answer incorrectly uses the acceleration of the block as the magnitude of the gravitational acceleration on the planet.

C **Incorrect**
This answer incorrectly uses $\cos(30°)$ instead of $\sin(30°)$ in the vector relationship $a = g_{\text{planet}} \sin(30°)$.

D **Correct**
This can be solved using kinematics or conservation of energy. Using kinematics, the acceleration of the block (which is the component of the gravitational acceleration that is parallel to the incline) can be found using the kinematic equation below. Then the magnitude of the gravitational acceleration can be found.

$v^2 = v_0^2 + 2a\Delta x \qquad (2.8\text{ m/s})^2 = (0\text{ m/s})^2 + 2a(2\text{ m}) \qquad a = 1.96\text{ m/s}^2$

$(1.96\text{ m/s}^2) = g_{\text{planet}} \sin(30°) \qquad g_{\text{planet}} = 3.92\text{ m/s}^2$

$$\frac{g_{\text{planet}}}{g_{\text{earth}}} = \frac{3.92\text{ m/s}^2}{10\text{ m/s}^2} = 0.4$$