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Advanced RC Structures

Slender Column 2

- Effective Length Factor
- Alignment Chart
- Moment Magnified Method
- Example

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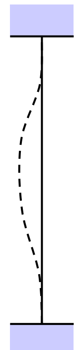
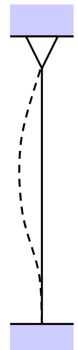
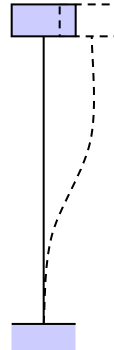
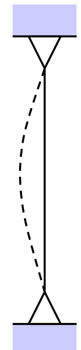
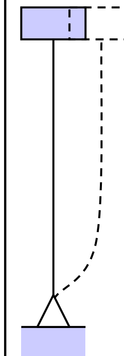
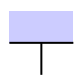
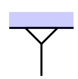
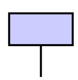
Effective Length Factor (*K*)

[illegible]						
[illegible] **K** [illegible]	0.5	0.7	1.0	1.0	2.0	2.0
[illegible] **K** [illegible]	0.65	0.8	1.2	1.0	2.1	2.0

[illegible]		
		[illegible]
		[illegible]
		[illegible]
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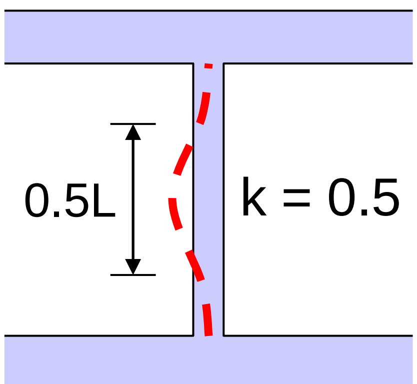
ตัวคูณความยาวประสิทธิผลของเสาในโครงข้อแข็ง

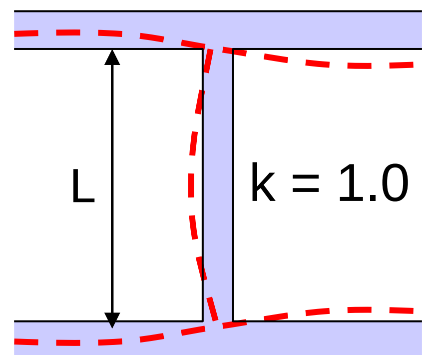
ความยาวประสิทธิผลจะขึ้นกับความต้านทานการหมุนของจุดต่อคานและเสา

ถ้าคานมีความแข็งเชิงดัดมากจะยึดปลายเสาไม่ให้หมุน จะเป็นจุดต่อแบบยึดแน่น (fix)

ถ้าคานมีความแข็งเชิงดัดน้อยจนหมุนไปตามปลายคาน จะเป็นจุดต่อแบบหมุน (pin)



Fixed Support

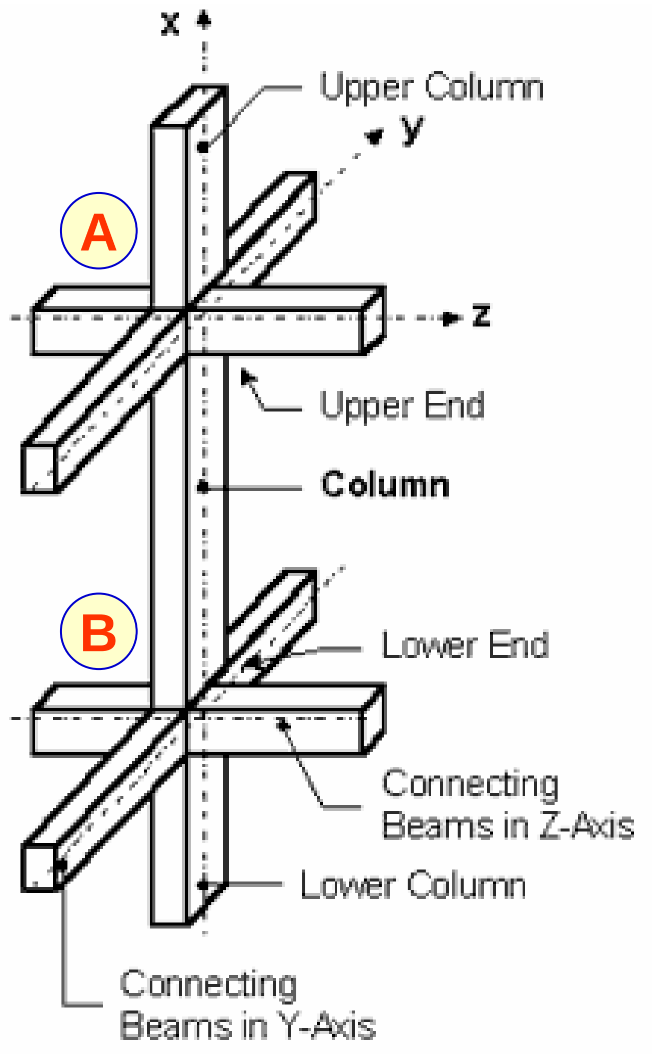


Pin Support

พฤติกรรมจริงจะอยู่ระหว่าง fix และ pin ขึ้นกับความแข็งเชิงดัดของเสาและคาน

Alignment Chart

The primary design aid to estimate the effective length factor **k**

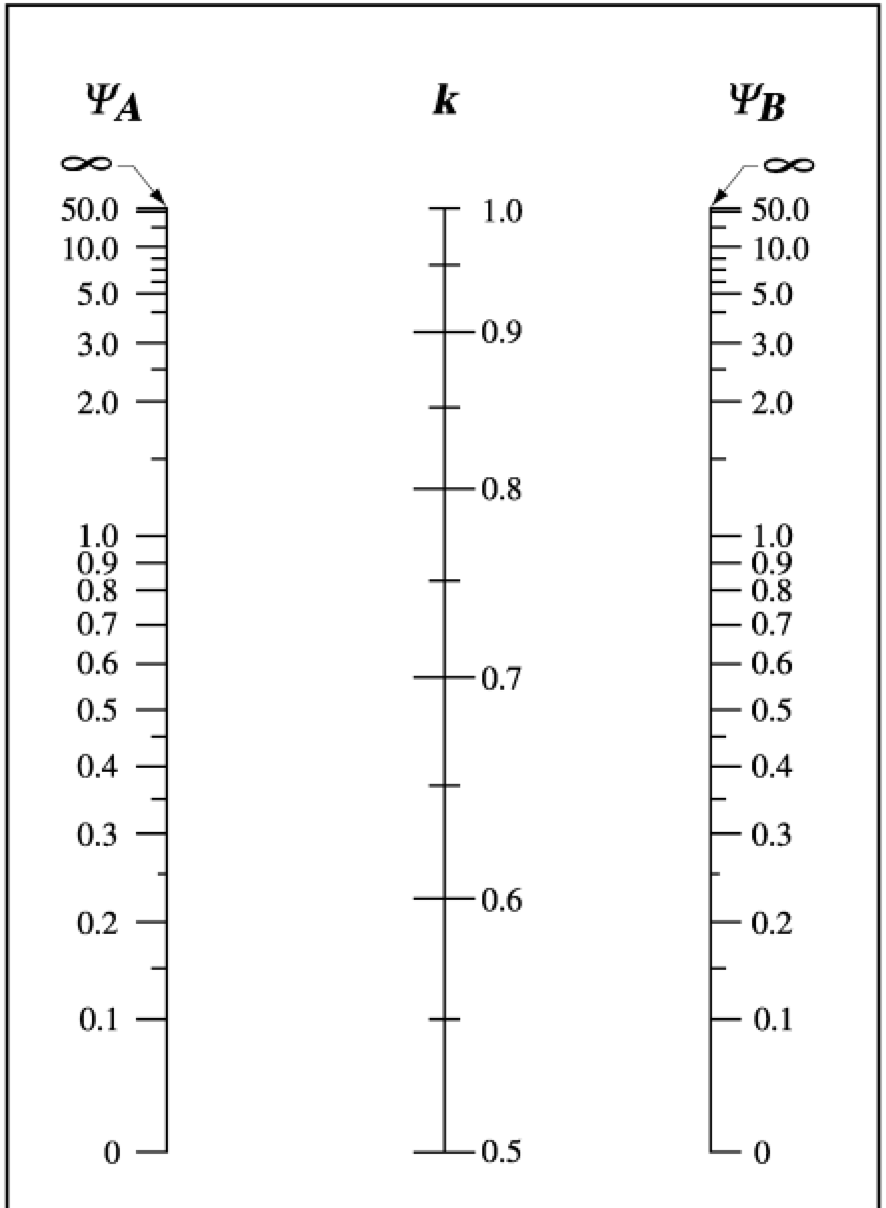


$$\psi = \frac{\Sigma EI / l_u \text{ of columns}}{\Sigma EI / l_u \text{ of beams}}$$

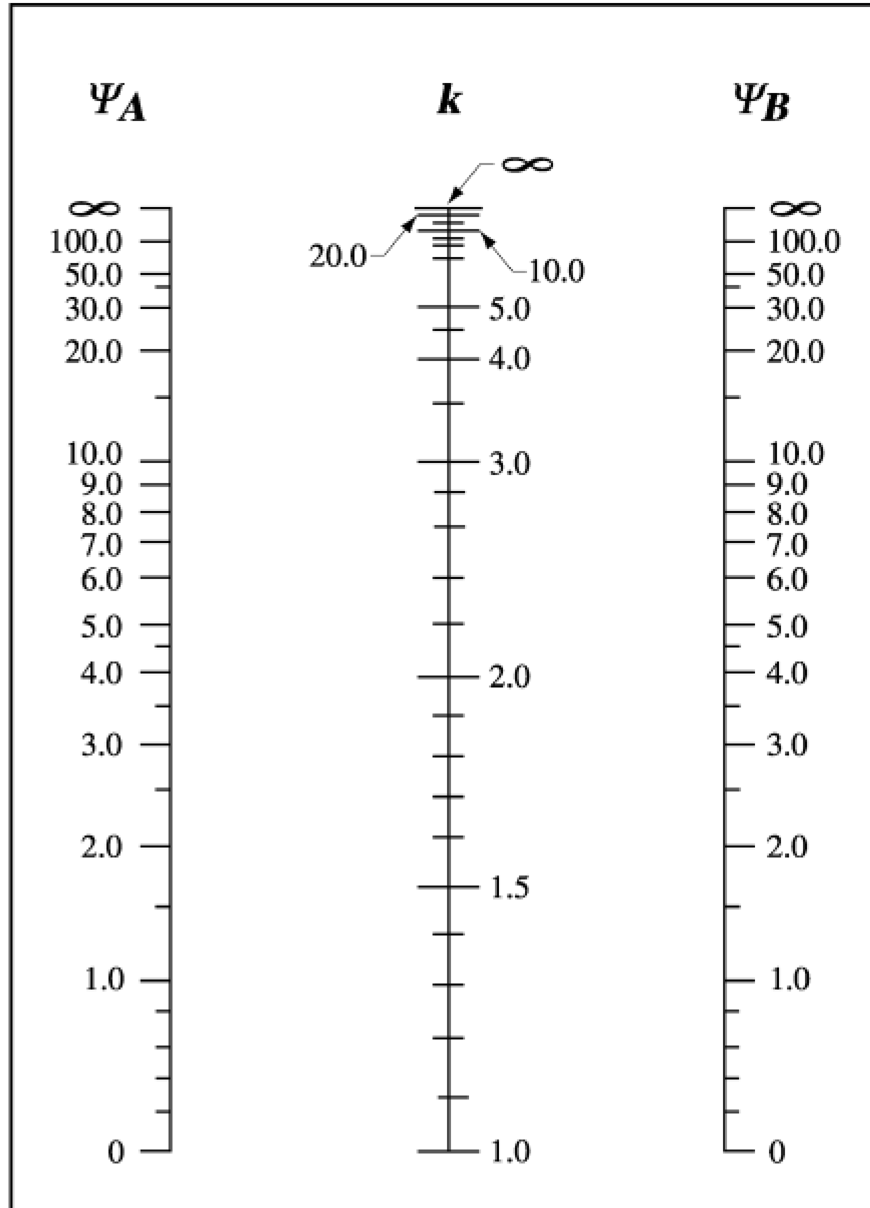
ψ_A and ψ_B are the top and bottom factors of the column.

For a hinged end ψ is infinite or 10 and for a fixed end ψ is zero or 1

Nonsway Frames



Sway Frames



Alternative Method

For computing the effective length factors for compression members in nonsway and sway frames

For columns in a nonsway frame, the effective length factors may be taken as the smaller of

$$k = 0.7 + 0.05(\psi_A + \psi_B) \leq 1.0$$

$$k = 0.85 + 0.05\,\psi_{min} \leq 1.0$$

For columns in a sway frame restrained at both ends with $\psi_m < 2$, the effective length factors may be taken as

$$k = \frac{20 - \psi_m}{20}\sqrt{1 + \psi_m}$$

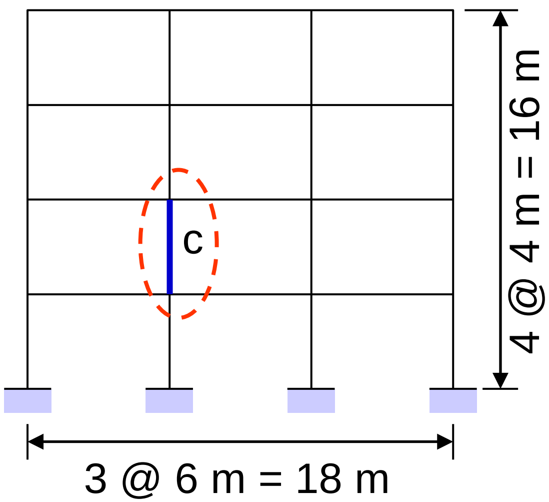
where $\psi_m = (\psi_A + \psi_B)/2$

$$k = 0.9\sqrt{1 + \psi_m}$$

For columns in a sway frame hinged at one end : $k = 2.0 + 0.3\psi$

EXAMPLE 1: Compute an effective length factor *k*

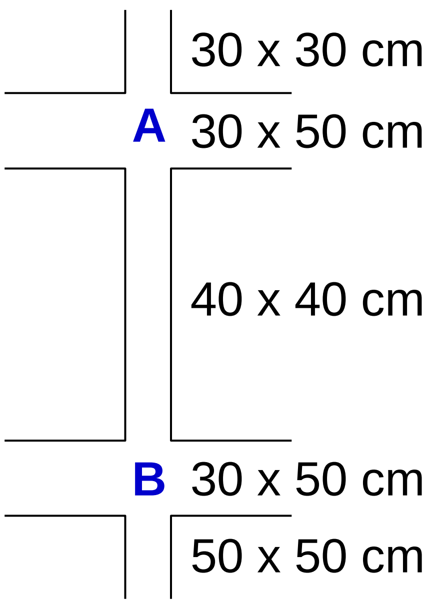
Sway Frame

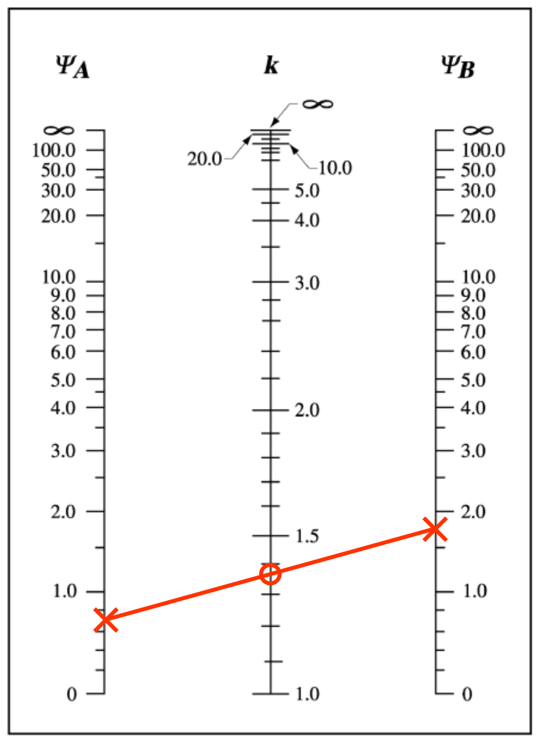


$$\psi_A = \frac{\Sigma I_c / L}{\Sigma I_b / L} = \frac{(30\times 30^3 + 40\times 40^3)/(12\times 4)}{2(30\times 50^3)/(12\times 6)}$$

$$= 0.674$$

$$\psi_B = \frac{(40\times 40^3 + 50\times 50^3)/(12\times 4)}{2(30\times 50^3)/(12\times 6)} = 1.762$$



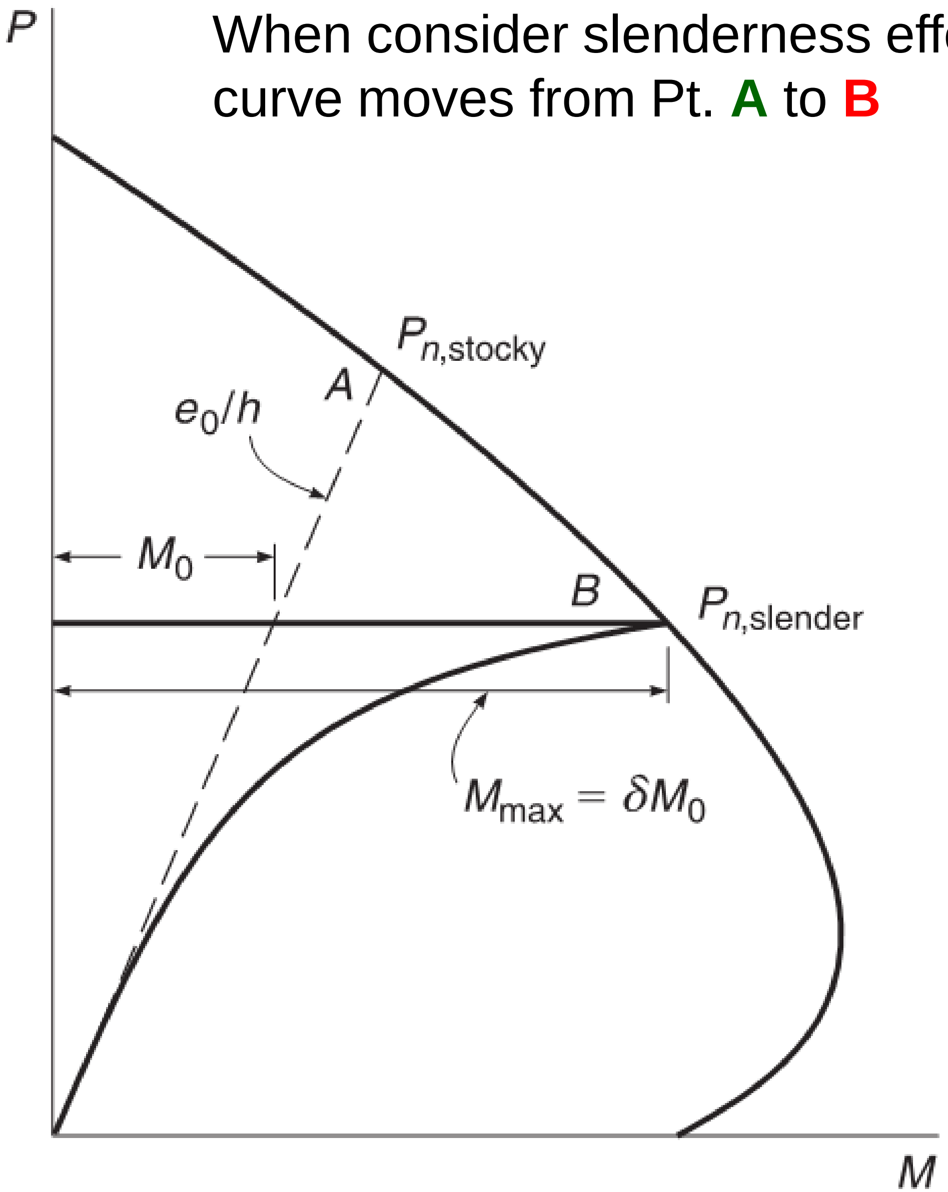


From alignment chart,

k = 1.37

Ans.

Effect of Slenderness on Carrying Capacity



When consider slenderness effect, the capacity point on interaction curve moves from Pt. **A** to **B**

M_0 increase to $M_{max} = \delta M_0$

$P_{n,stooky}$ decrease to $P_{n,slender}$

Moment Magnification Method

ACI318-14

Nonsway Frames

The factored moment used for design of columns and walls M_c shall be the first-order foctored moment M_2 amplified for the effects of member curvature.

$$M_c = \delta M_2$$

Magnification factor δ shall be calculated by:

$$\delta = \frac{C_m}{1 - \frac{P_u}{0.75 P_c}} \geq 1.0$$

M_2 shall be at least $M_{2,min} = P_u (1.5 + 0.03h)$

If $M_{2min} > M_2$ → $C_m = 1.0$ or $C_m = 0.6 - 0.4 \frac{M_1}{M_2} \geq 0.4$

Critical Buckling Load (P_c)

$$P_c = \frac{\pi^2 (EI)_{eff}}{(k \ell_u)^2}$$

Where k = effective length factor $\begin{cases} \text{for nonsway frame } k = 1.0 \\ \text{for sway frame } k \geq 1.0 \end{cases}$

$(EI)_{eff}$ shall be calculated in accordance with (a) or (b)

$$(EI)_{eff} = \frac{0.4 E_c I_g}{1+\beta_{dns}} \quad \text{———— (a)}$$

$$(EI)_{eff} = \frac{(0.2 E_c I_g + E_s I_{se})}{1+\beta_{dns}} \quad \text{———— (b)}$$

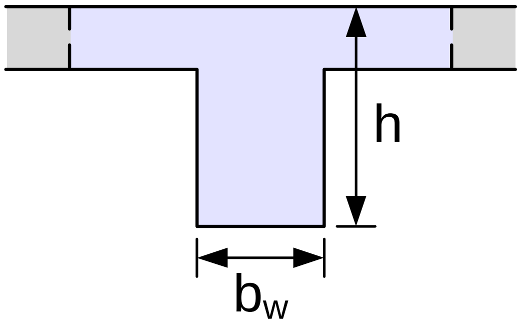
$$\beta_{dns} = \frac{\text{max. factored sustained axial load}}{\text{max. factored axial load}}$$

For simplicity, assume $\beta_{dns} = 0.6$ → $(EI)_{eff} = 0.25 E_c I_g$

Moment of Inertia and Cros-sectional Area

Unless a more rigorous analysis is used, moment of inertia and cross-sectional area of member shall be calculated in accordance with Table.

Member and condition		Moment of Inertia	Cross-sectional area
Columns		$0.70I_g$	$1.0A_g$
Walls	Uncracked	$0.70I_g$	
	Cracked	$0.35I_g$	
Beams		$0.35I_g$	
Flat plates and flat slabs		$0.25I_g$	



For T-beam, it is generally sufficiently accurate to take:

$$I_{g,T-beam} = 2\,I_{g,web} = 2\,(b_w h^3 / 12)$$

Example : Design of a slender column in a nonsway frame

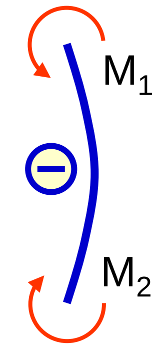
อาคารคอนกรีตหลายชั้นถูกค้ำยันอย่างเพียงพอโดยผนังเฉือนที่ช่องบันไดและปล่องลิฟท์รับน้ำหนักบรรทุก DL และ LL จากการจัดน้ำหนัก LL ดังในรูปทำให้เกิดโมเมนต์มากที่สุด โดยเสาถูกดัดเป็นรูปโค้งเดี่ยว จงออกแบบเสา C1 โดยวิธีขยายโมเมนต์ดัด

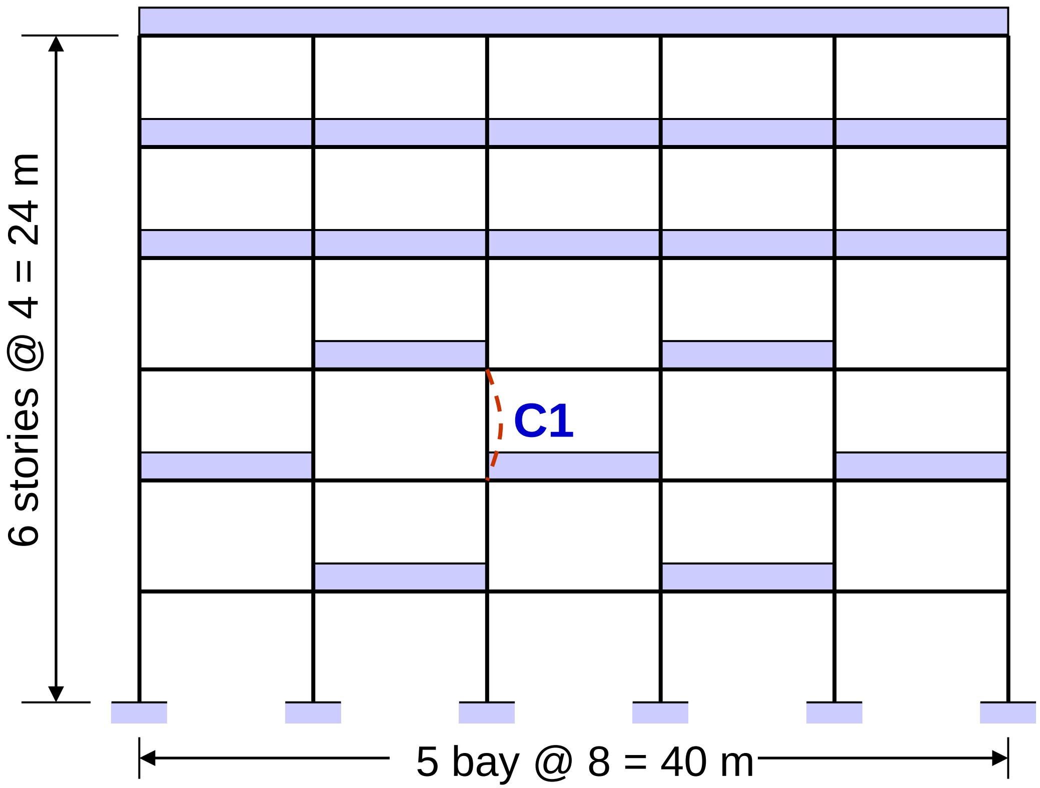
C1 Dead load
P = 100 ton
M_2 = 0.25 t-m
M_1 = -0.25 t-m

C1 Live load
P = 80 ton
M_2 = 12 t-m
M_1 = 10 t-m

All beam: 0.3 x 0.7 m
Int. column: 0.5 x 0.5 m
Ext. column: 0.4 x 0.4 m
Column height: 4.5 m

f'c = 280 ksc
fy = 4000 ksc





Solution:

Factored load

$P_u = 1.4(100)+1.7(80) = 276$ t

$M_1 = 1.4(-.25)+1.7(10) = 16.7$ t-m

$M_2 = 1.4(.25)+1.7(12) = 20.8$ t-m

$kL/r = (1.0)(450)/(0.3x50) = 30.0$

$34+12(M_1/M_2) = 34 + 12(-16.7/20.8)$

$= 24.4 < kL/r$

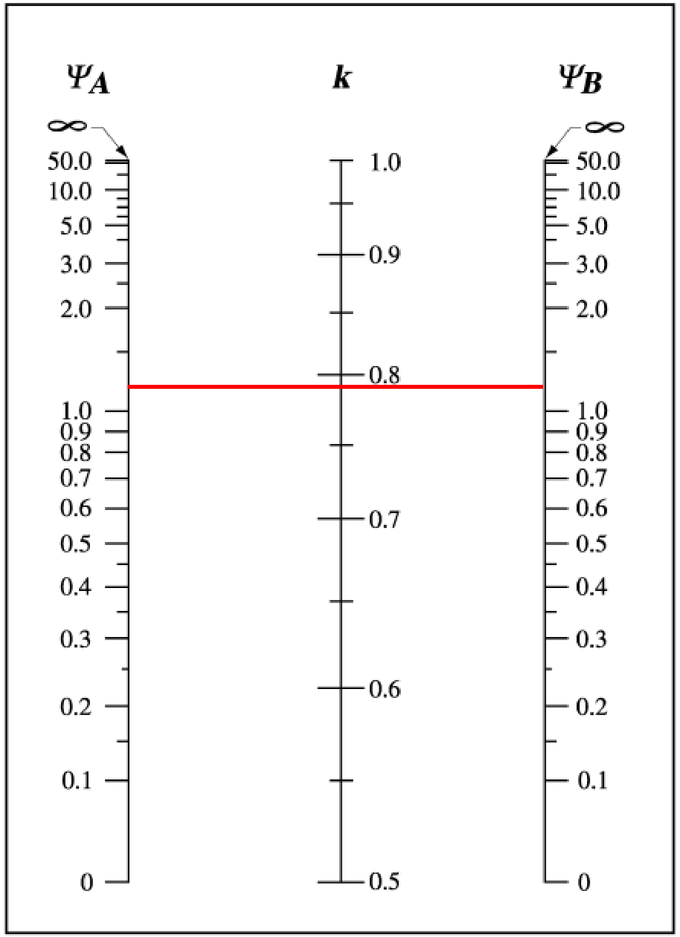
Slenderness must be considered

Concrete: $E_c = 15{,}100\sqrt{280} = 2.53\times10^5\ \text{kg/cm}^2$

Column: $0.7I_g = 0.7\times50\times50^3/12 = 3.646\times10^5\ \text{cm}^4 \rightarrow \dfrac{I}{\ell_c} = \dfrac{3.646\times10^5}{420} = 868.1\ \text{cm}^3$

Beam: $0.35I_g = 0.35\times2\times30\times70^3/12 = 6.00\times10^5\ \text{cm}^4 \rightarrow \dfrac{I}{\ell} = \dfrac{6.00\times10^5}{800} = 750.3\ \text{cm}^3$

Rotational restraint factors at the top and bottom of column are the same



$$\psi_a = \psi_b = \frac{868.1+868.1}{750.3+750.3} = 1.16$$

From nonsway alignment chart, $k = 0.78$

Or using alternative method,

$$k = 0.7+0.05(1.16+1.16) = 0.816 \quad \textbf{control}$$

$$k = 0.85+0.05\times1.16 = 0.908$$

Slenderness ratio

$$\frac{k\ell_u}{r} = \frac{0.816\times450}{0.3\times50} = 24.48 > 34+12(M_1/M_2)$$

Slenderness must be considered

Minimum moment: $M_{2,min} = P_u(1.5 + 0.03h)$

$$= 276(1.5 + 0.03 \times 50)/100 = 8.28 \text{ t-m} < M_2$$ **OK**

$$C_m = 0.6 - 0.4\frac{M_1}{M_2} = 0.6 - 0.4\left(-\frac{16.7}{20.8}\right) = 0.92$$

Factor β_{dns} is the ratio of factored dead load to factored load

$$\beta_{dns} = \frac{1.4 \times 100}{1.4 \times 100 + 1.7 \times 80} = 0.51$$

$$(EI)_{eff} = \frac{0.4 E_c I_g}{1 + \beta_{dns}} = \frac{0.4 \times 2.35 \times 10^5 \times 50 \times 50^3 / 12}{1 + 0.51}$$

$$= 3.24 \times 10^{10} \text{ kg/cm}^2$$

Buckling load: $P_c = \frac{\pi^2 (EI)_{eff}}{(k\ell_u)^2} = \frac{\pi^2 \times 3.24 \times 10^{10}}{(0.816 \times 450)^2} = 2.37 \times 10^6 \text{ kg} = 2{,}370 \text{ tons}$

Magnification factor: $\delta = \frac{C_m}{1 - P_u / 0.75P} = \frac{0.92}{1 - 276/(0.75 \times 2{,}370)} = 1.09$

Magnified moment: $M_c = \delta_{ns} M_2 = 1.09 \times 20.8 = 22.7 \text{ t-m}$

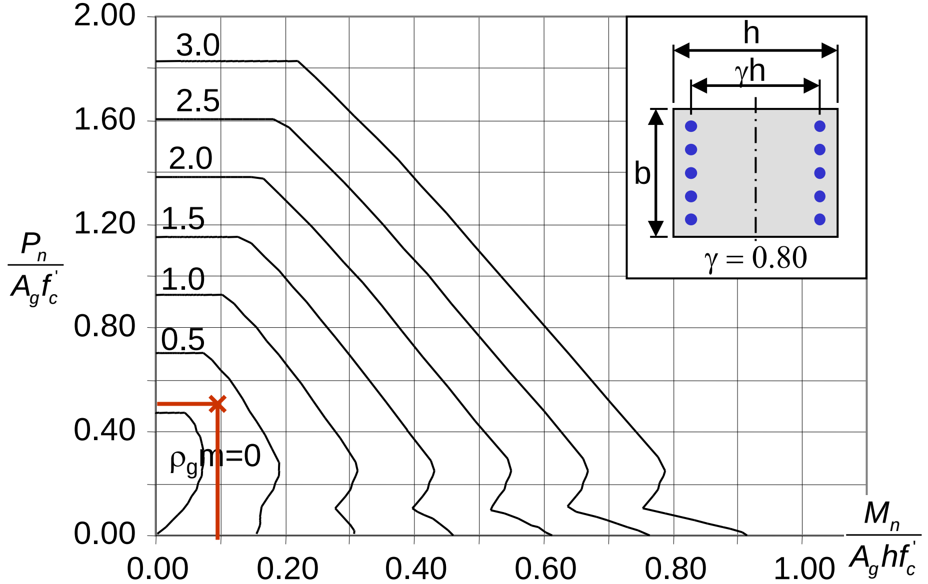
Using an interaction diagram

$$\frac{P_u}{\phi f'_c A_g} = \frac{276}{0.7 \times 0.28 \times 2500} = 0.56$$

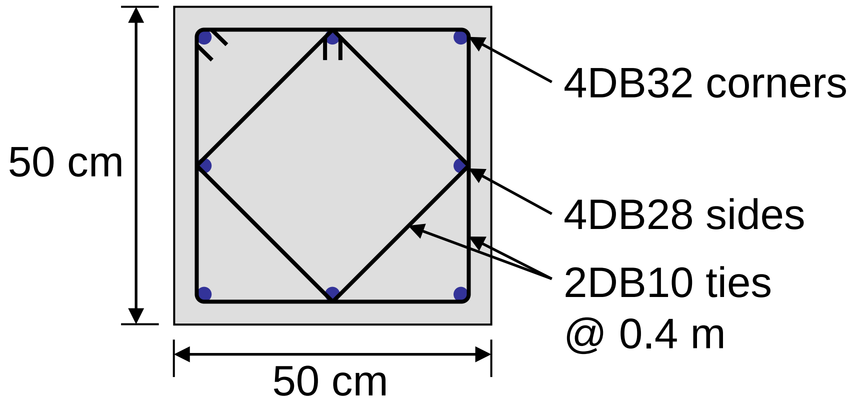
$$\frac{M_u}{\phi f'_c A_g h} = \frac{22.7 \times 100}{0.7 \times 0.28 \times 2500 \times 50} = 0.093$$

$$\rho_g = 0.36 \times 0.85 \times 280 / 4000 = 0.021$$

$$A_{st} = 0.021 \times 2500 = 53.6 \text{ cm}^2$$



Use 4DB32+4DB28 (A_{st} = 56.8 cm²)



End of Lecture