Ascend Education 嘉善書室

GCSE Maths Essentials 數學常規班

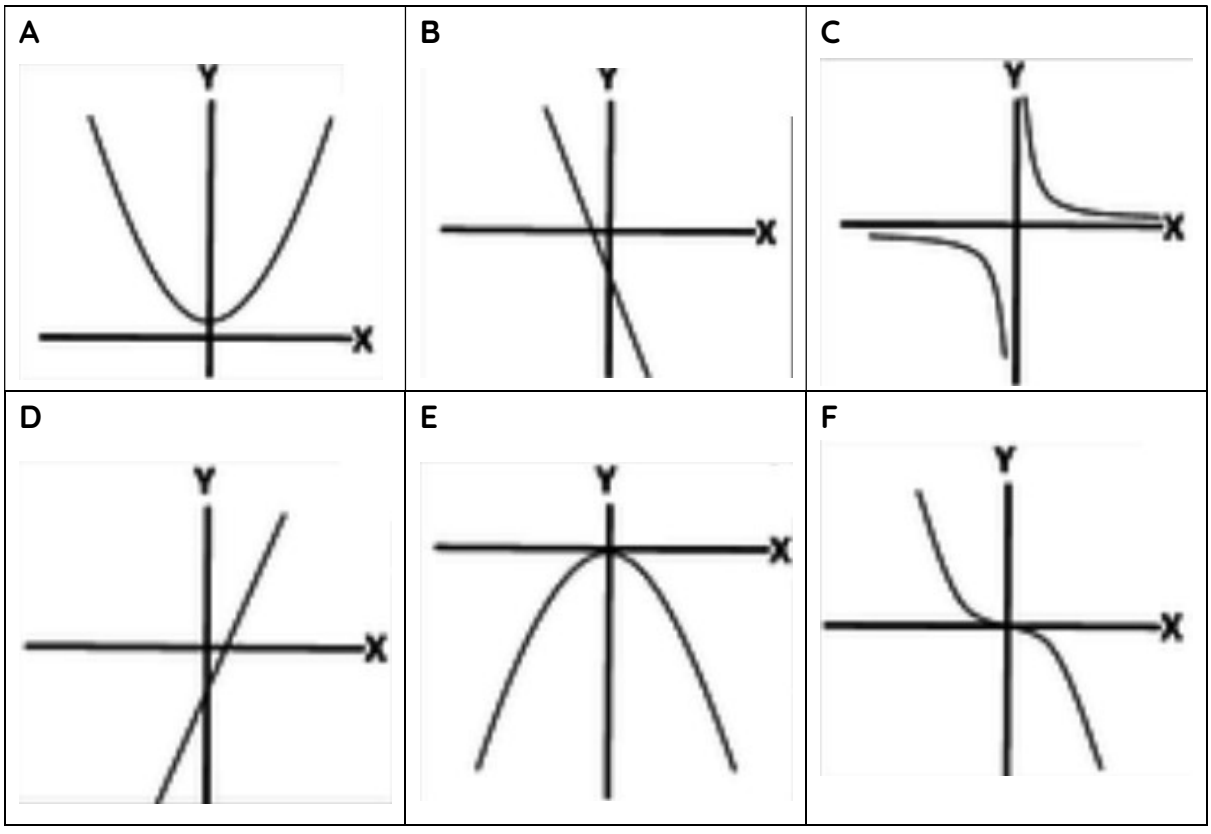
Introduction to graphs

Lesson Objectives:

- Type of graphs
- Solving simultaneous equations graphically
- Solving harder equations graphically

Warm up (3 minutes)

Match the following graphs.

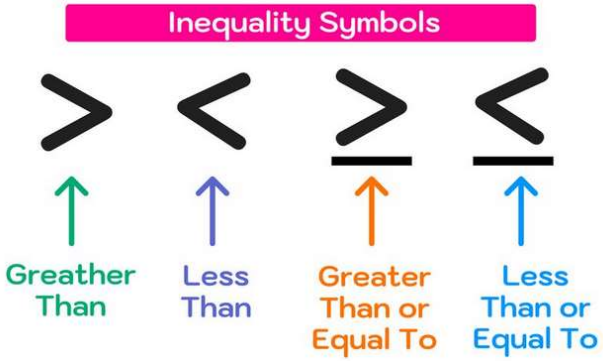


$y = -x^2$		$y = 2x - 1$	
$y = -x^3$		$y = x^2 + 1$	
$y = -2x - 1$		$y = \frac{1}{x}$	

Solving inequalities algebraically

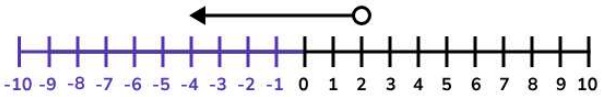
- We did linear graphs and quadratic graphs last lesson.
- What about inequalities? Can we solve them on graphs?
- Yes, but we need a revision on inequalities first…

- Inequalities compare numbers or expressions in order of size.

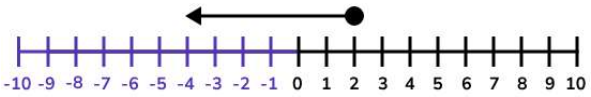


- You can express inequalities on **a number line**
  - An open circle 空心圓點 = **doesn't include** the value (ONLY $>$ AND $<$)
  - A closed circle 實心圓點 = **does include** the value (ONLY $\geq$ AND $\leq$)
- Let's look at some examples:

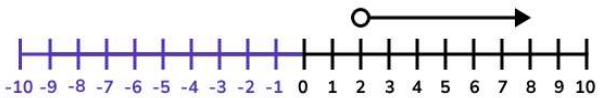
"x is less than 2" = $x < 2$



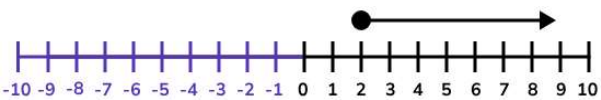
"x is less than or equal to 2" = $x \leq 2$



"x is greater than 2" = $x > 2$



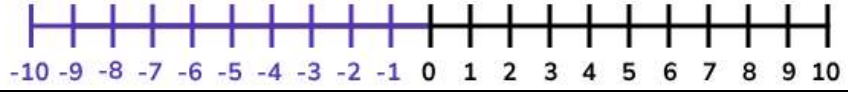
"x is greater than or equal to 2" = $x \geq 2$



- Realize that you can combine two or more inequalities together

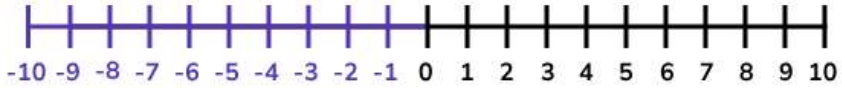
Write down all the possible integer values when $x < 4$ and $x > 2$.

Write down all the possible integer values when $x \geq -6$ and $x \leq 1$.



Well, sometimes the inequalities don't give any result. Be aware of that!

Write down all the possible integer values when $x \leq -7$ and $x > 4$.

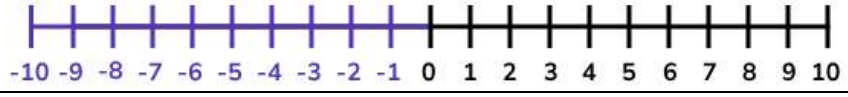


Most difficult questions in GCSE ask you to combine more complex inequalities.

Write down all the possible integer values when

$$-8 \leq x \leq 4$$

and $2 < x < 9$



Solving linear inequalities

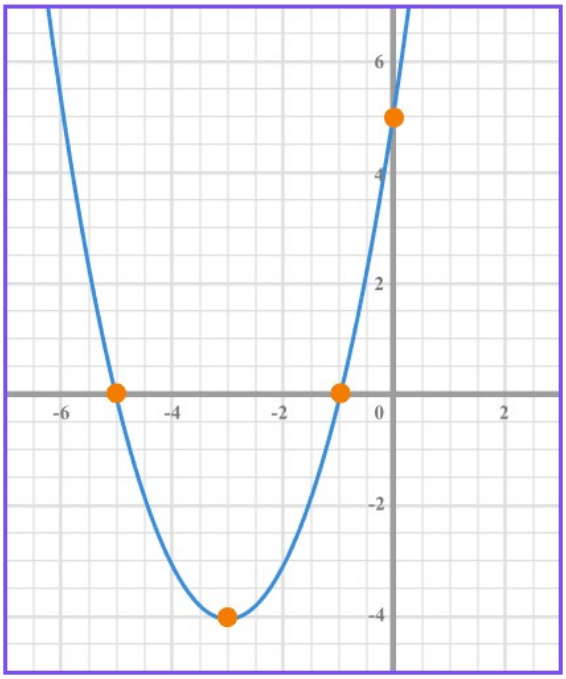
1. Solve $3(x+7) < 31 - x$	2. Solve $4x - 3 \geq 2x + 25$
3. Solve $2(x-5) \leq 8$	4. Solve $6x - 5 > 4x + 1$
5. Solve $\frac{x-4}{-2} > 6$	6. Solve $-4 < 3x + 2 \leq 5$

Quadratic inequalities

- If you happen to encounter a quadratic inequality in GCSE exam (which is definitely among the more difficult exam questions), you need to do the followings:
  - Sketch a graph (to help you think)
  - Factorization the expression
  - Expect your answer to be either "$a > x > b$" or "$x < a \text{ or } x > b$ ". (where ***a*** and ***b*** are the roots to the quadratic equations)

Worked Example

Solve $x^2 + 6x + 5 \leq 0$.



What happen if we switch the inequality sign – Solve $x^2 + 6x + 5 \geq 0$.

Exercise

1. Solve $x^2 + 7x + 10 < 0$

2. Solve $x^2 + 3x + 10 \leq 0$

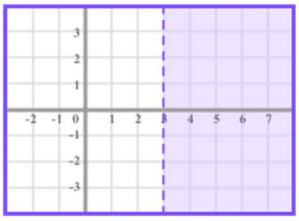
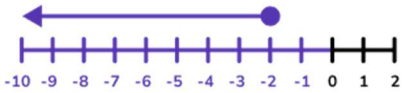
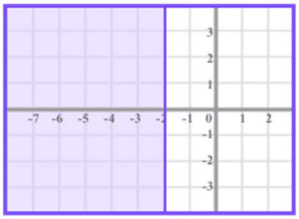
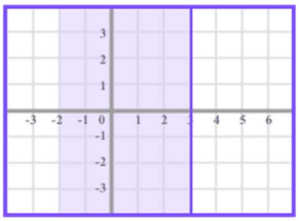
3. List the integer that satisfy $2x^2 + 9x + 4 \leq 0$

4. Solve $x^2 - 9 > 0$

Solving inequalities graphically

- Inequalities on a graph allow us to visualise the regions that satisfy one or more inequalities.
- In GCSE exam, these inequalities are often linear
  - which means they can be expressed using straight line graphs.
- You can use two types of lines to express region of inequalities
  - **Solid line 實線**: the value is included (ONLY ≥ AND ≤)
  - **Dashed line 虛線**: the value is NOT included (ONLY > AND <)

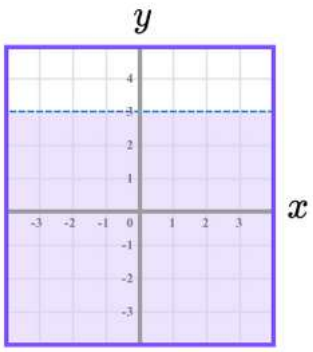
- For vertical line $x = k$,

On a number line	On a graph
$x > 3$ 	$x > 3$ We use a dashed line for $x = 3$ and can shade the region required to the right of the line. 
$x \leq -2$ 	$x \leq -2$ We use a solid line for $x = -2$ and can shade the region required to the left of the line. 
$-2 < x \leq 3$ 	$-2 < x \leq 3$ We can use a dashed line for $x = -2$ and a solid line for $x = 3$. We can shade the region required in between the lines. 

- For horizontal line $y = k$,

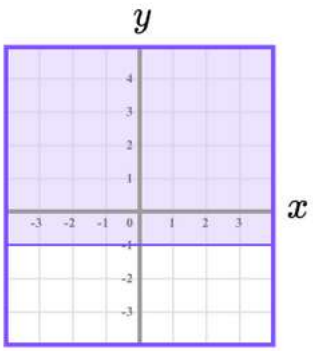
$y < 3$

We need y to be small to make it less than 3. This means the region required is under the line.



$y \geq -1$

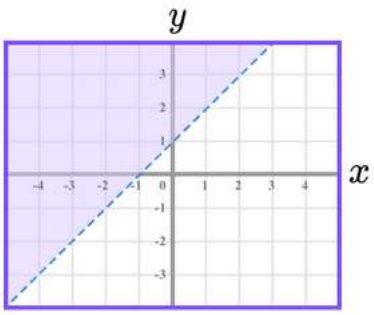
We need y to be large to make it greater than -1. This means the region required is above the line.



- For linear graphs $y = mx + c$ (or any other form $ax \pm by = c$)

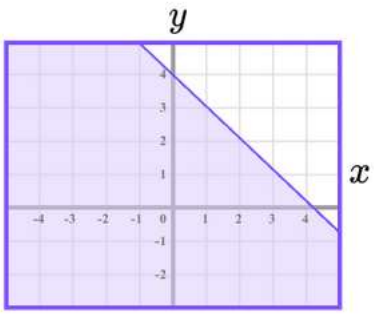
$y > x + 1$

We need y to be greater than $x + 1$ so y should be large.
The region required is above the line.



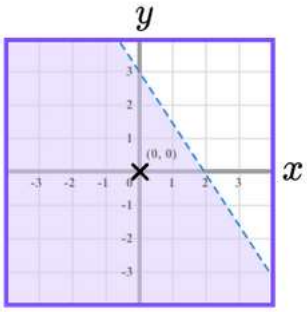
$y \leq 4 - x$

We need y to be less than $4 - x$ so y needs to be small.
The region required is under the line.



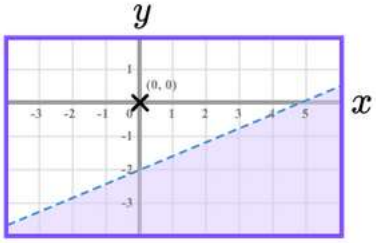
$3x + 2y < 6$

We can use the origin (0, 0) to check the region.
3(0) + 2(0) < 6 so (0, 0) is in the required region.

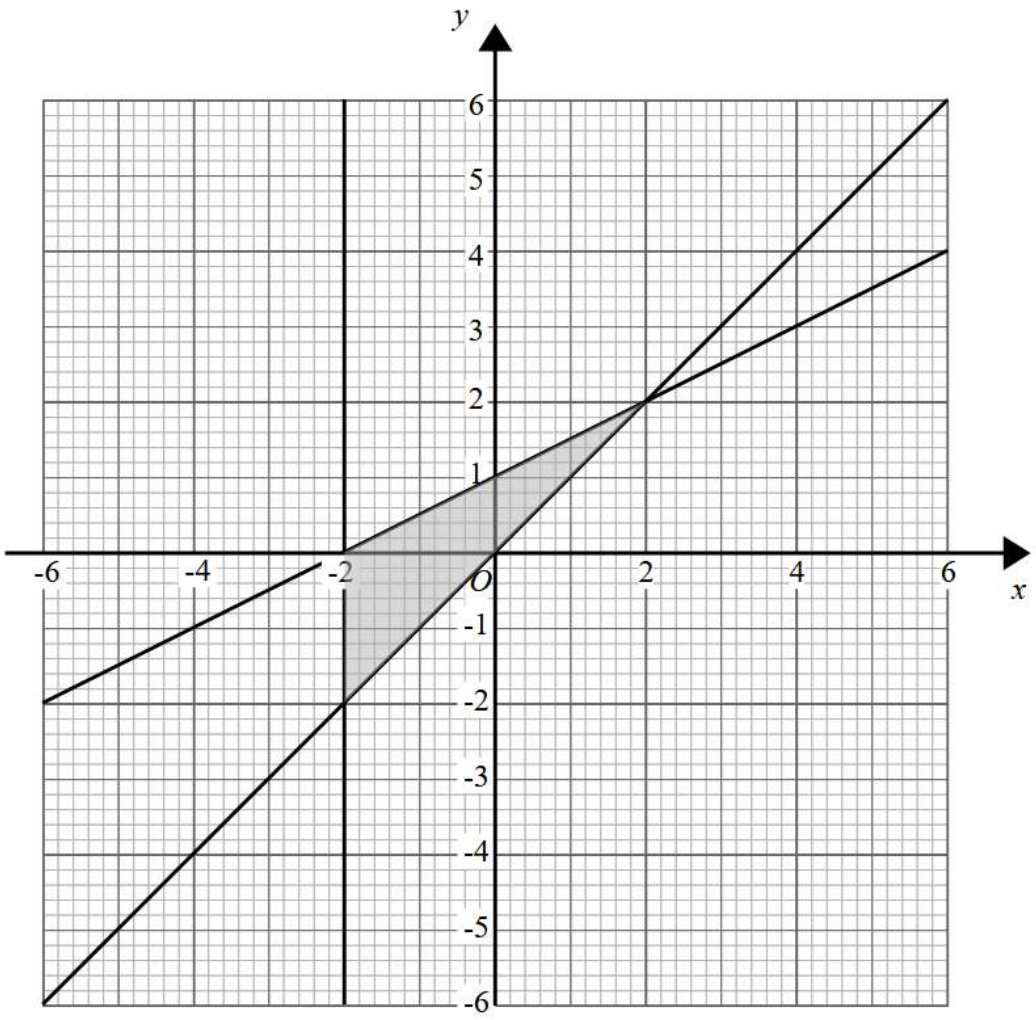


$2x - 5y > 10$

We can use the origin (0, 0) to check the region.
2(0) - 5(0) < 10 **not** > 10 so (0, 0) is **not** in the required region.



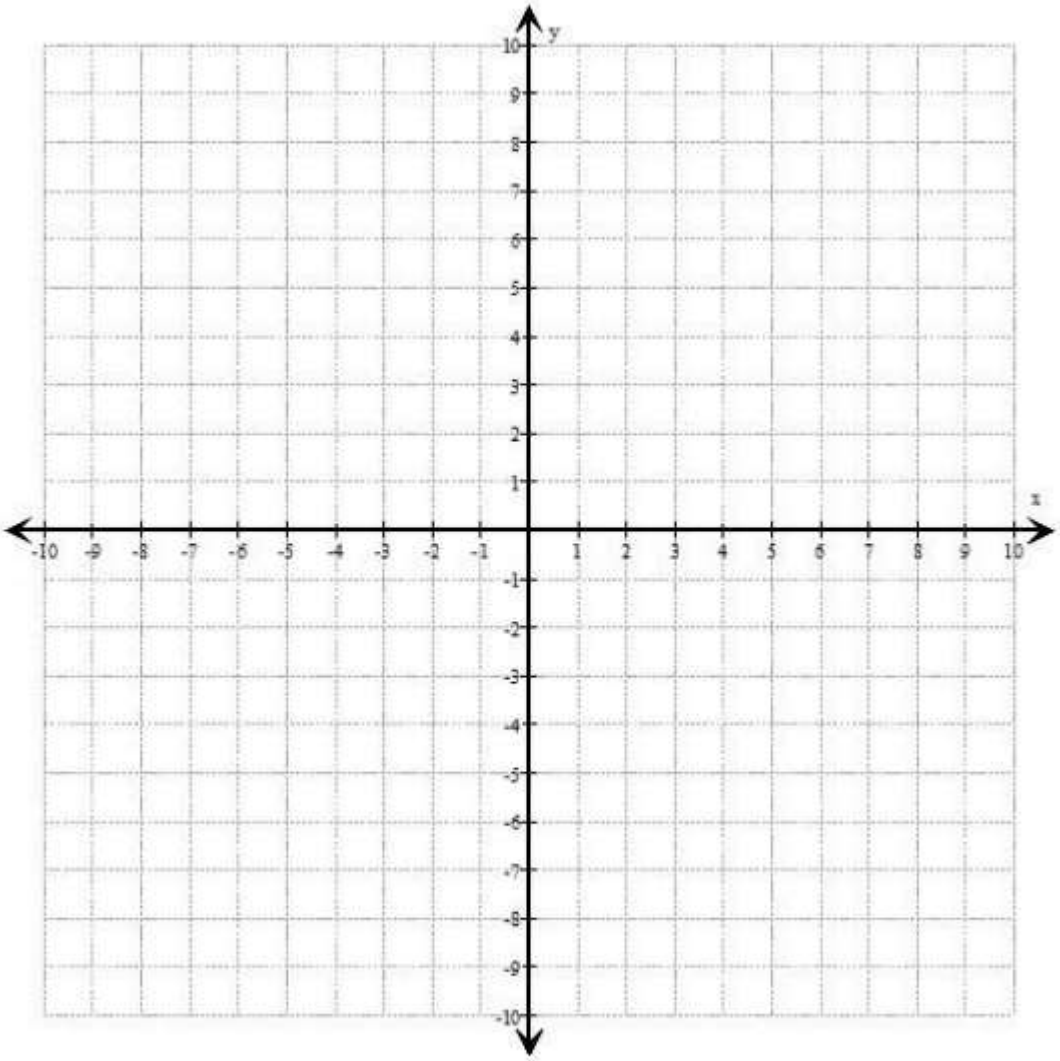
Worked Example



Write down the three inequalities that define the shaded region

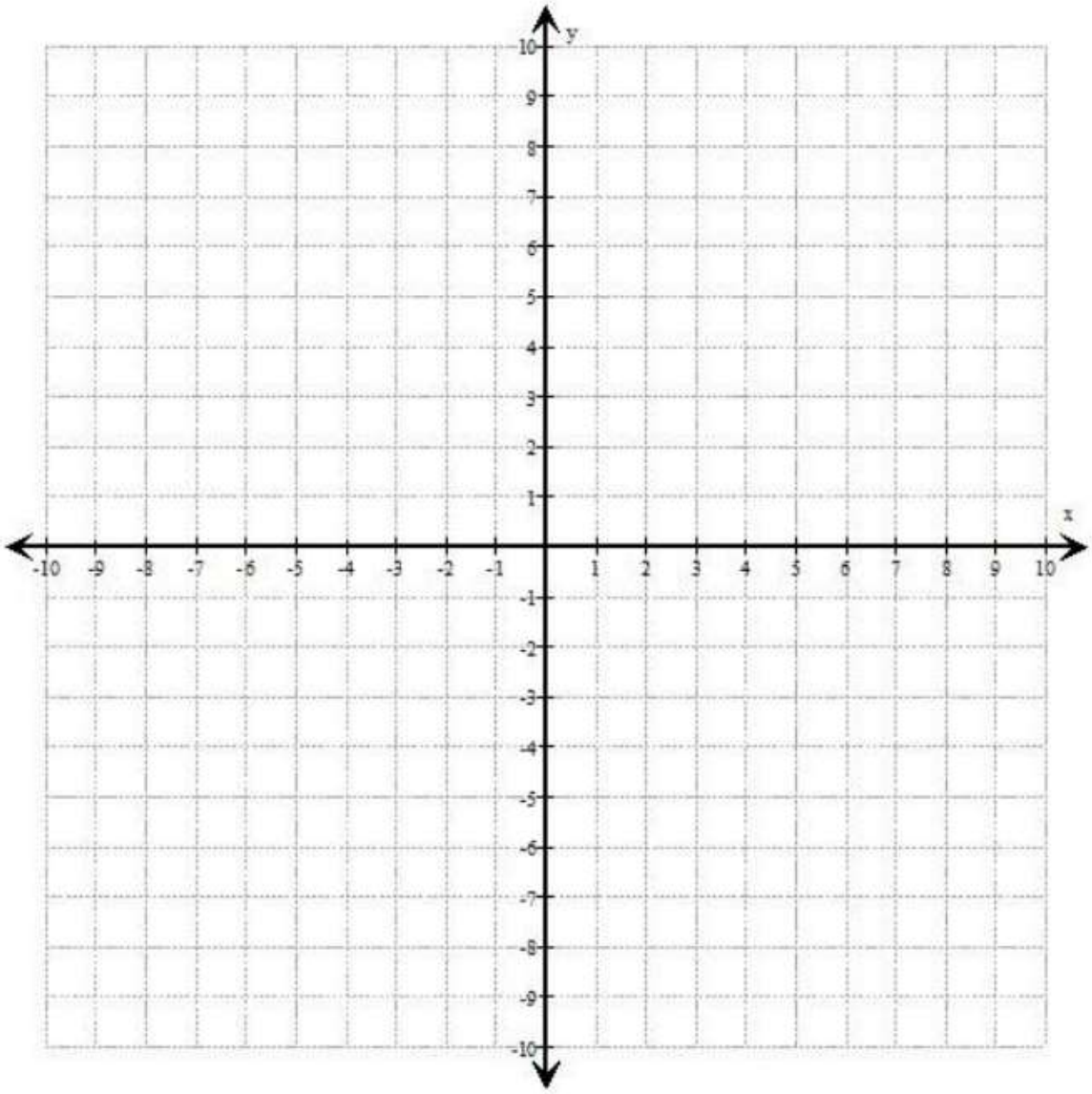
1. On the axes below, shade the region which satisfies all the inequalities

a. $y \leq 5$, $y \geq x$, $y > -x + 1$



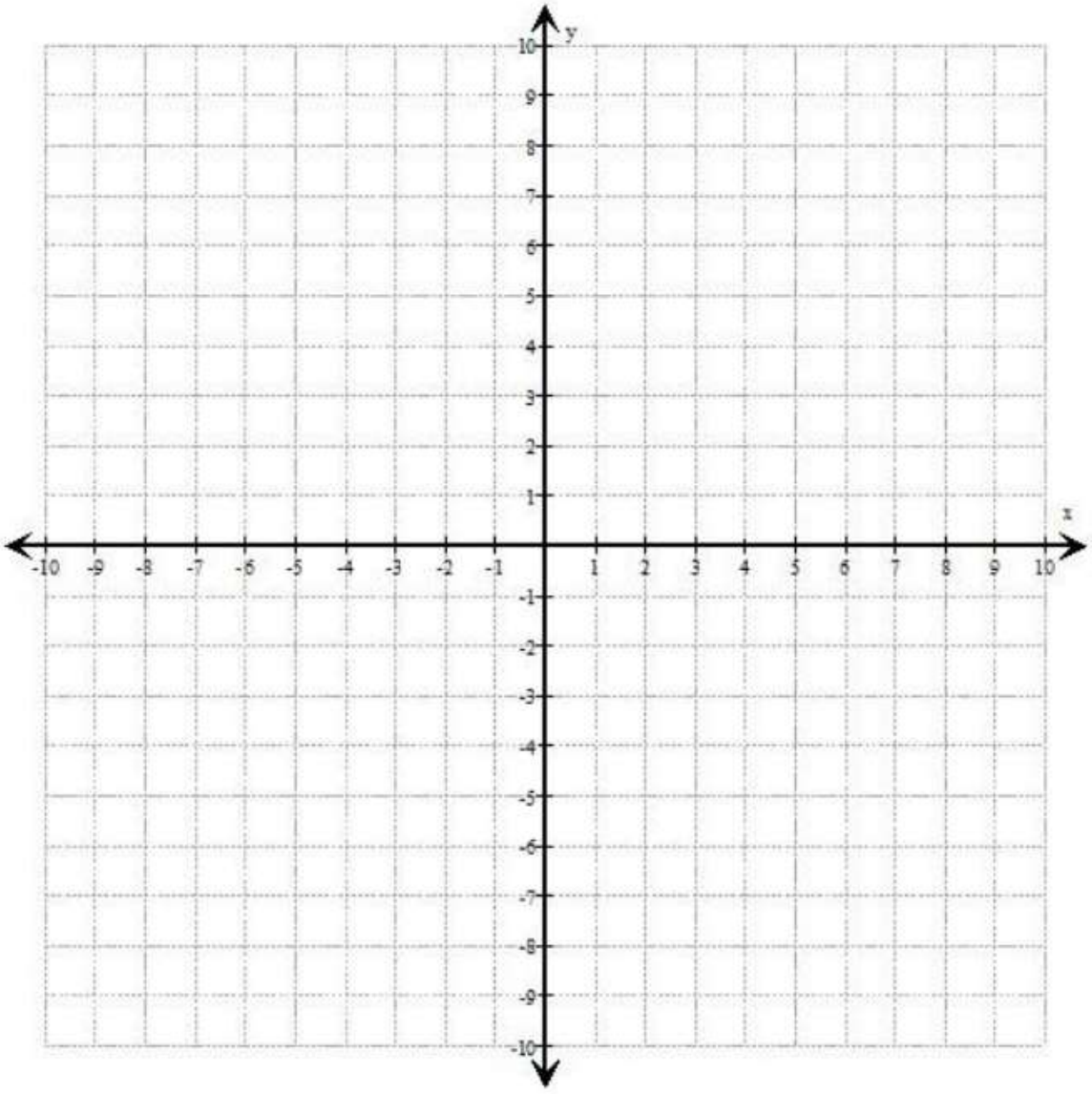
(4 Marks)

b. $y \leq 4, \quad y \geq -2, \quad y > x - 1, \quad y < x + 5$



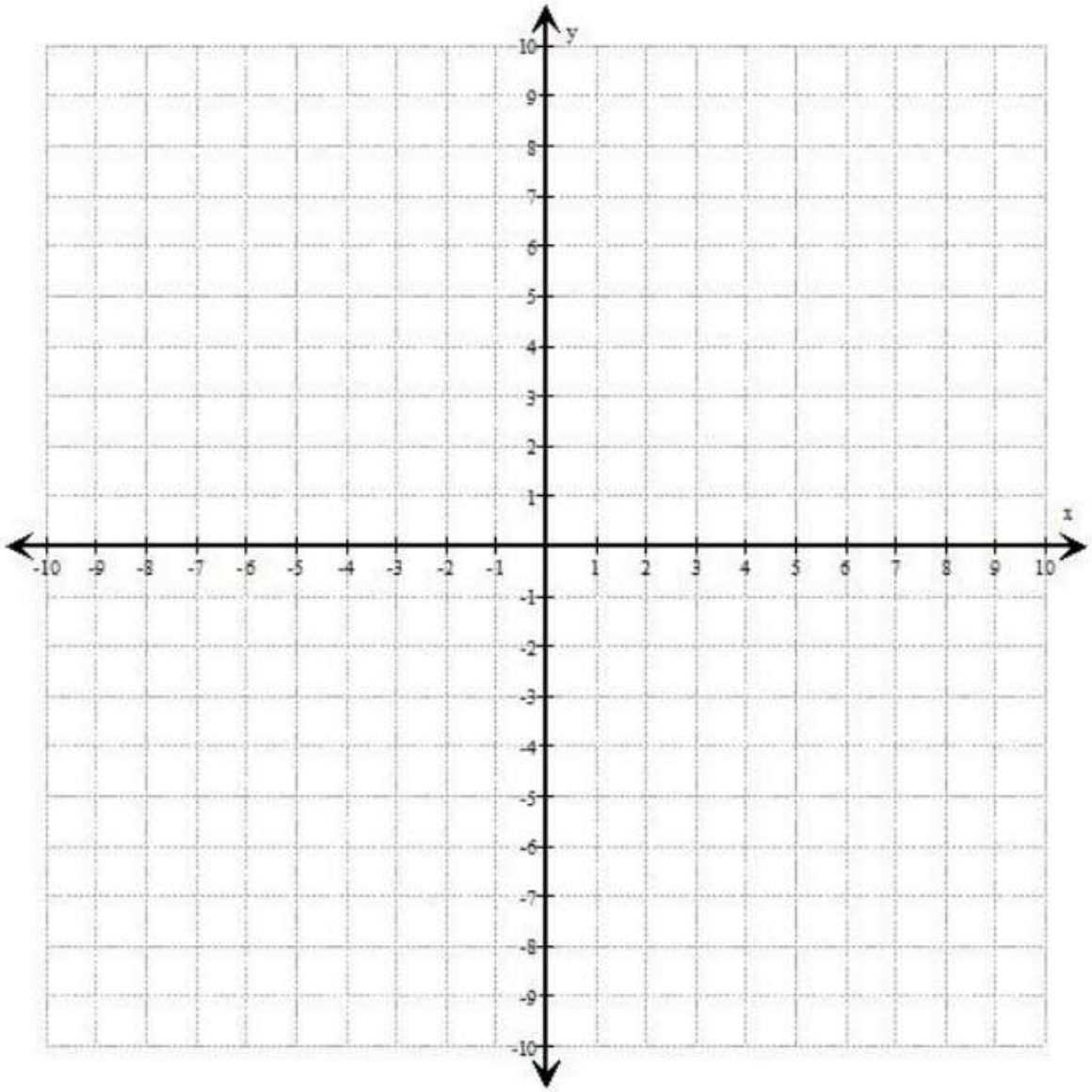
(4 Marks)

c. $y \leq 5, \qquad y \geq 7 - x, \qquad y \geq x - 4$



(4 marks)

d. $y < 9, \quad y > -9, \quad y \leq 9 - 2x, \quad y \geq -2x - 4$



(4 Marks)

Answer

Linear Inequalities

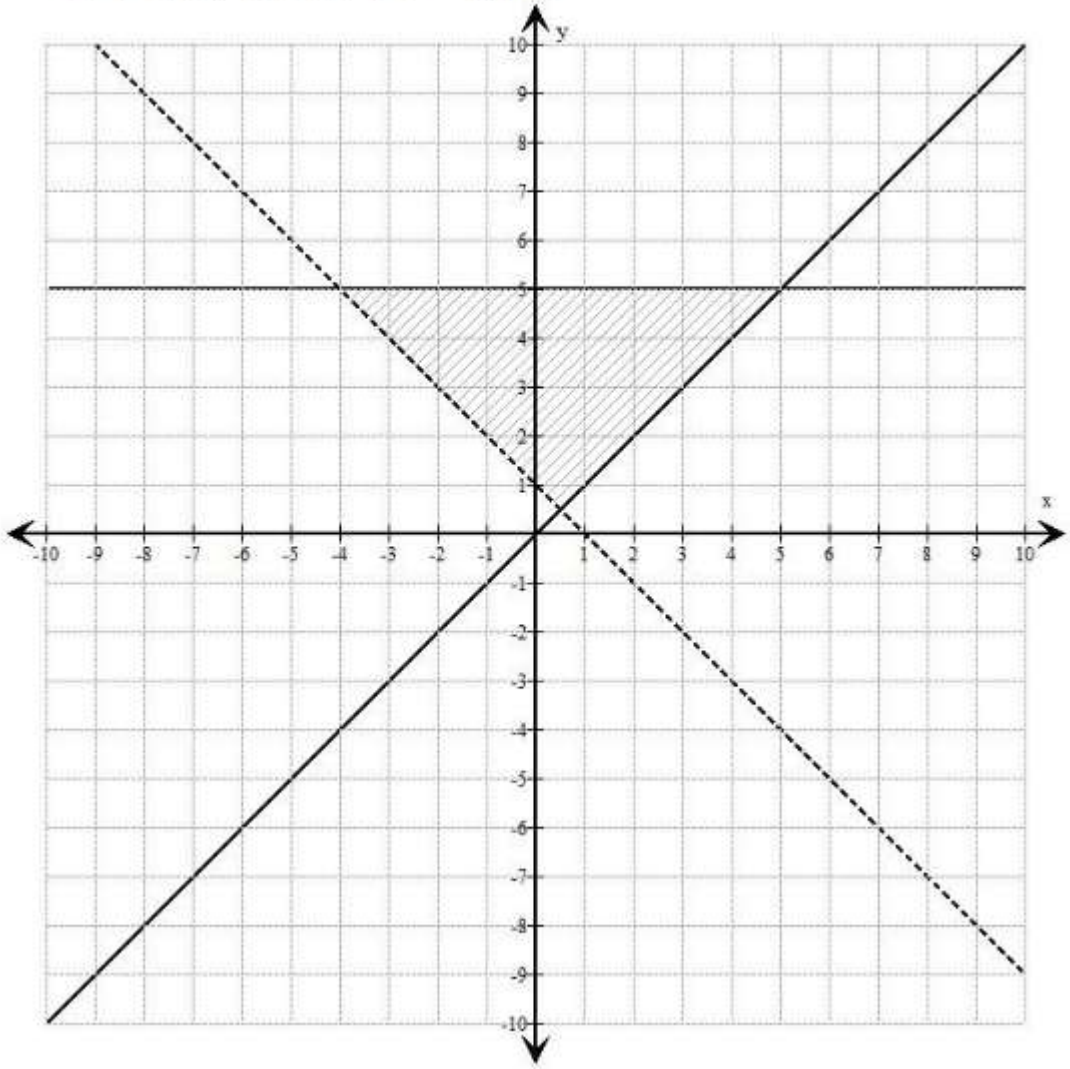
1. $x < 2.5$
2. $x \geq 14$
3. $x \leq 9$
4. $x > 3$
5. $x < -8$
6. $-2 < x \leq 1$

Quadratic inequalities

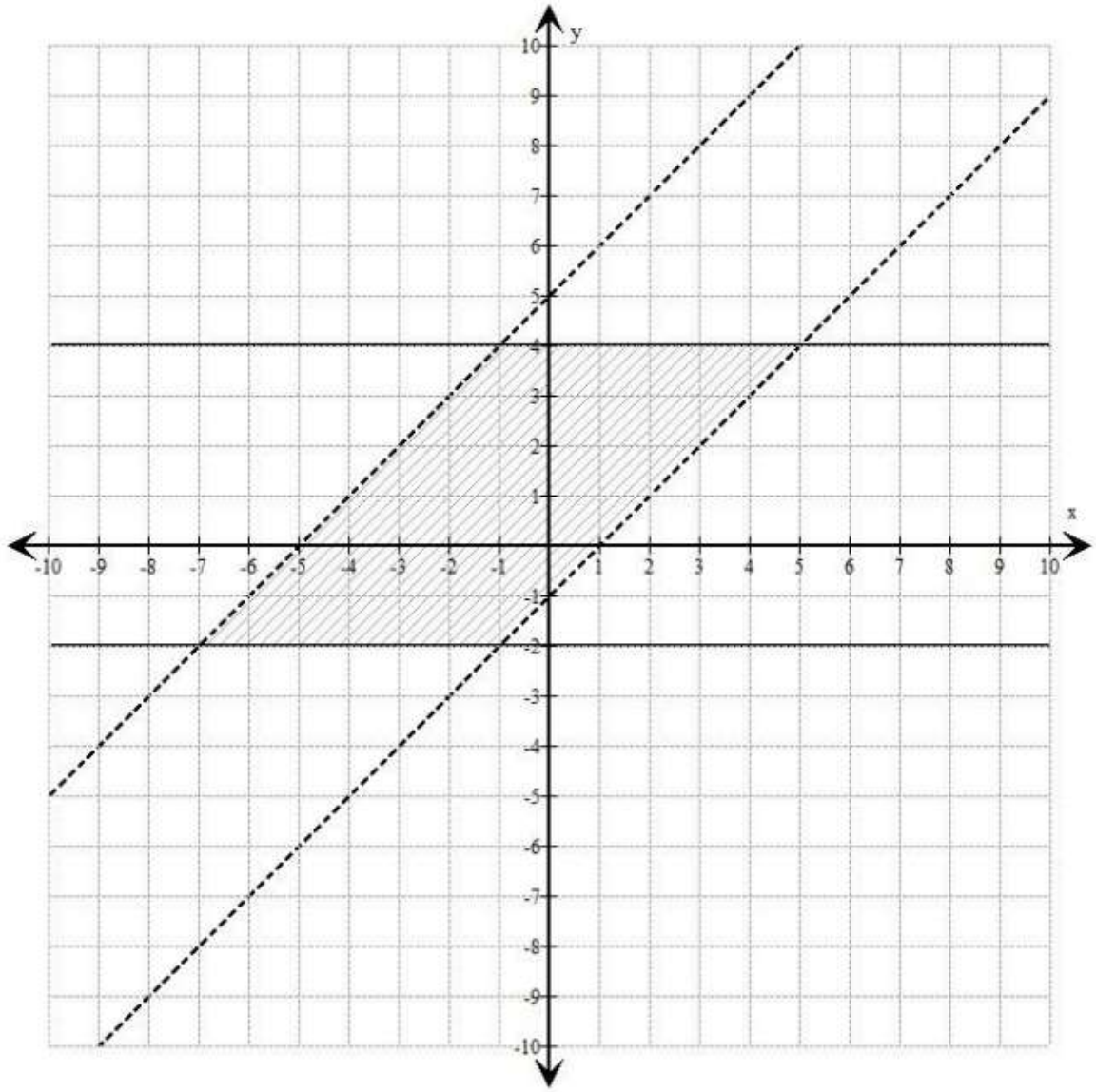
1. $-5 < x < -2$
2. $-5 \leq x \leq 2$
3. -4,-3,-2,-1
4. x>3 and x<-3

1. On the axes below, shade the region which satisfies all the inequalities

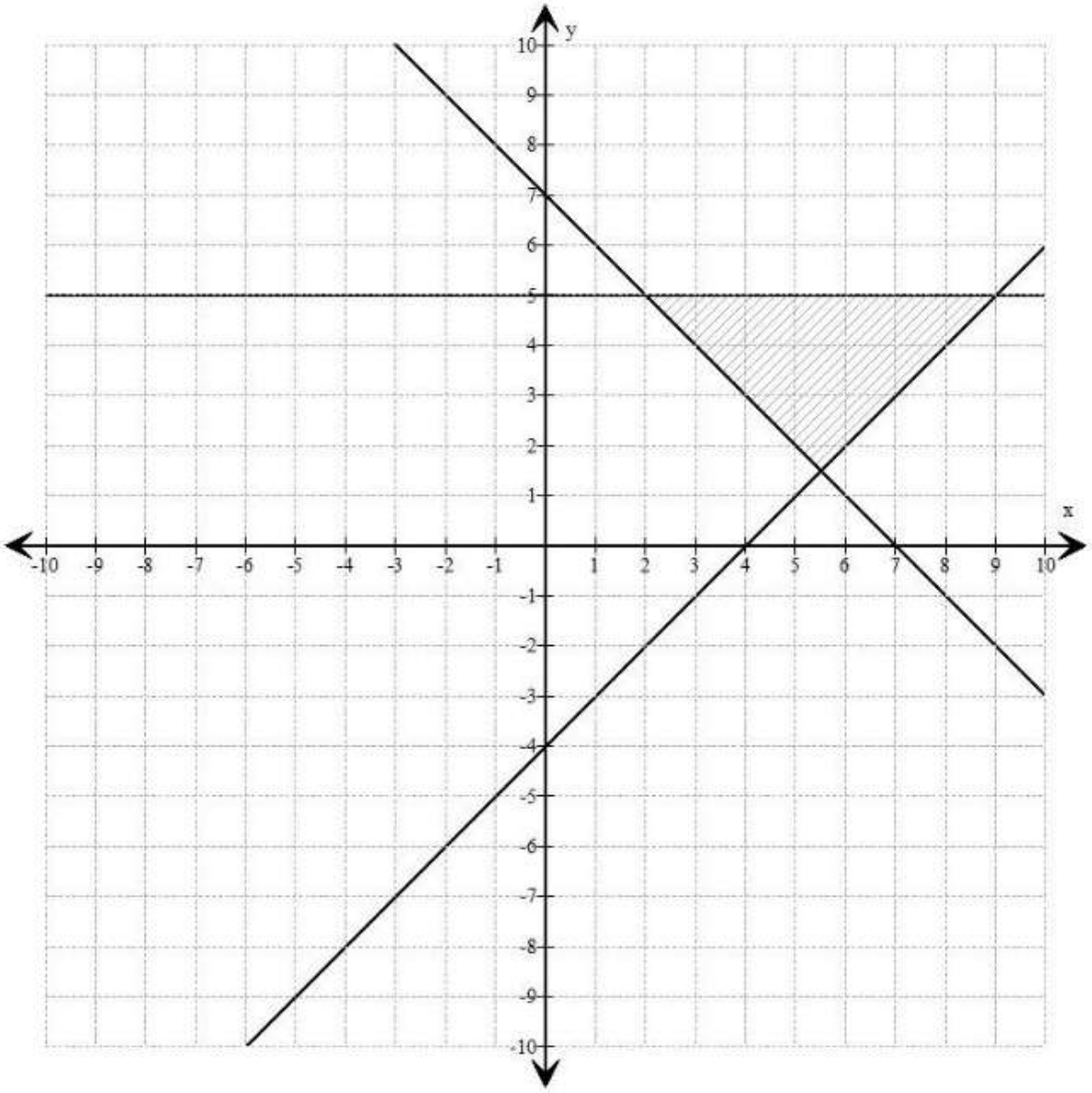
   a. $y \leq 5$, $y \geq x$, $y > -x + 1$



b. $y \leq 4, \quad y \geq -2, \quad y > x - 1, \quad y < x + 5$



c. $y \leq 5, \qquad y \geq 7 - x, \qquad y \geq x - 4$



d. $y < 9, \quad y > -9, \quad y \leq 9 - 2x, \quad y \geq -2x - 4$