lec 16:- $\underline{S \circ R}$

$M_R = [r_{ij}]$ $\qquad$ $M_S = [s_{ij}]$

$M_{S \circ R} = [t_{ij}]$.

$$t_{ij} = \begin{cases} 1 & \exists k \quad r_{ik} = s_{kj} = 1 \\ 0 & \text{otherwise.} \end{cases}$$

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_{\underline{S \circ R}} = M_R \odot M_S$$

$$M_{S \circ R} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$t_{ij} = \begin{cases} 1 & \exists k \quad r_{ik} = s_{kj} = 1 \\ 0 & \text{otherwise.} \end{cases} \qquad t_{11} = \begin{cases} 1 & \exists k \quad r_{1k} = s_{k1} = 1 \\ 0 & \end{cases}$$

$k = 1, 2, 3$

t_{13} $\quad \exists k \quad r_{1k} = s_{k3} = 1$

$i = 1 \quad k = 1 \quad \times \; r_{11} = s_{13} = 1 \checkmark$

$j = 3 \quad k = 2 \quad \times \; r_{12} = s_{23} = 1 \checkmark$

$k = 3 \quad \checkmark \; r_{13} = s_{33} = 1$

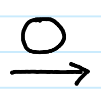
$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

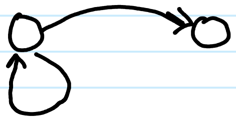
Homework: Compute the Remaining (6) Values.

Representing Relations Using Graphs.

Set of Vertices.

Set of Edges.





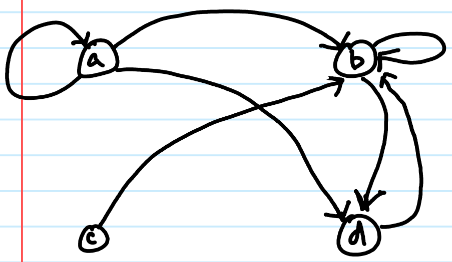
→ Set A (which is the Set based on which the Relation is defined).

→ Corresponds to tuples in Relation.

→ Corresponds to tuples in Relation.

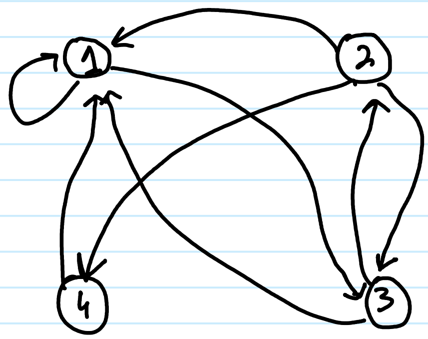
Set of Vertices = $V = \{a, b, c, d\}$.

Set of Edges. $E = \{(a,b), (a,d), (b,b),$
$(b,d), (c,a), (c,b),$
$(d,b)\}$.



p480 Ex8 $E = R = \{(1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2),$
$(4,1)\}$

$V = A = \{1,2,3,4\}$.   Draw the Graph.



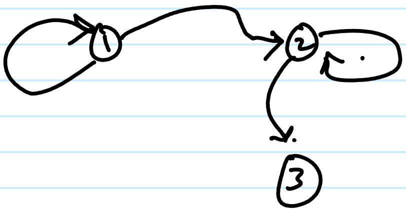
Graph → Relations.
↓
Matrix.

Key observation: 3 Interchangeable forms

Given one form you can obtain the other forms.
{ Relation in Set
Matrix
Graph.

Properties in Graph.

1) Reflexive. $\forall a \in A \quad (a,a) \in R$.

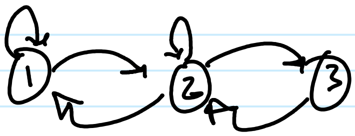


reflexive ?
No.

Examples.

1   2   reflexive?
No.

reflexive?
No.



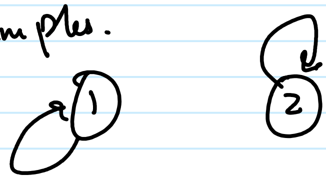
reflexive
No.

$R = \phi$.
reflexive = ?
Yes.

Symmetric $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

(1,2)

Examples.

① 

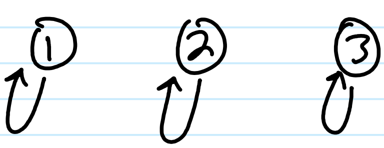
Symmetric ?
Yes.

② 

Not.
(2,1) is absent

③ 

Yes.

④ 

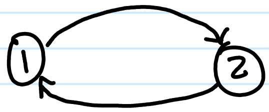
Yes.

⑤ empty Graph $R = \phi$.
Yes.

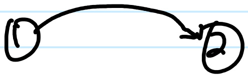
A. 1: Symmetric

Anti Symmetric.

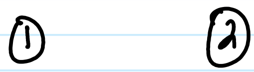
$\forall a,b \in A$ if $(a,b)\in R \wedge (b,a)\in R$
$\rightarrow a=b$.

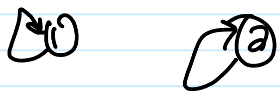
Not $\Leftarrow$ 

$(1,2)\in R \wedge (2,1)\in R$
$\rightarrow 1\neq 2$.

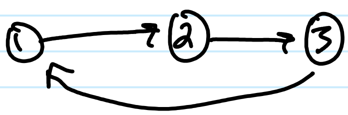


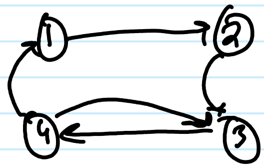
Examples.

1  Yes.

2  Yes.

3  Yes.

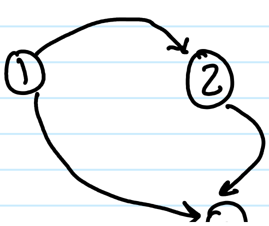
4  Yes.

5  No.

6 $R=\phi$ empty Yes.

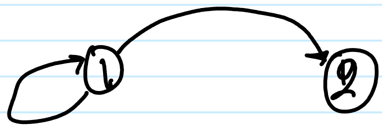
Transitive.

$\forall a,b,c \in A$ if $(a,b)\in R \wedge (b,c)\in R$
$\rightarrow (a,c)\in R$.



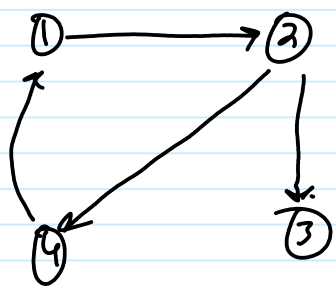
Examples ① empty Graph $R = \phi$.
Yes.

②



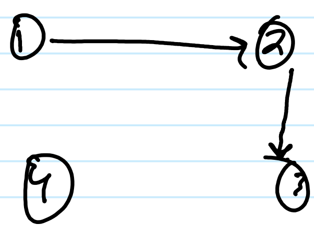
$$\begin{array}{ccc} (1,1) & (1,2) & \to (1,2). \\ \downarrow\ \downarrow & \downarrow\ \downarrow & \\ a\ \ b & b\ \ c & \end{array}$$

③



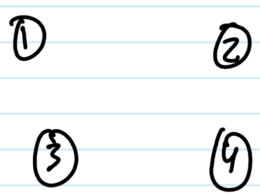
(1,2) (2,4)
(1,4)
No.

④



No
(1,3)
absent

⑤



Yes.

$\bar{R}$ $\quad R^{-1}$ for the Relations R
in matrix form.

$\bar{R} = \{(a,b) \mid (a,b) \notin R\}$.

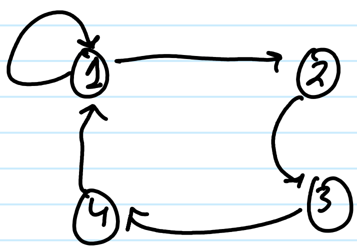
$M = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$ $M = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$

$\bar{R} = \{(a,b) \mid (a,b) \notin R\}$.

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad M_{\bar{R}} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

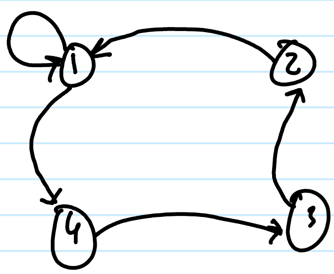
$R^{-1} = \{(b,a) \mid (a,b) \in R\}$.

$$M_{R^{-1}} = [M_R]^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

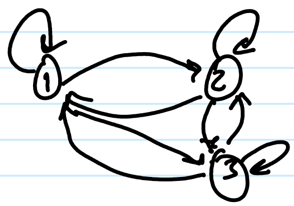


$\Rightarrow R$

$R^{-1} =$



Reverse the arrows.

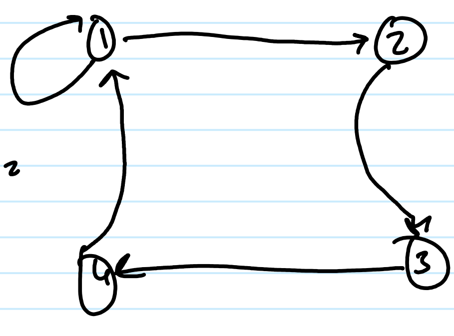


$\bar{R}$ = Homework.

think over it

& the Compute $\bar{R}$ for the following.

$R =$



$\bar{R} = ?$