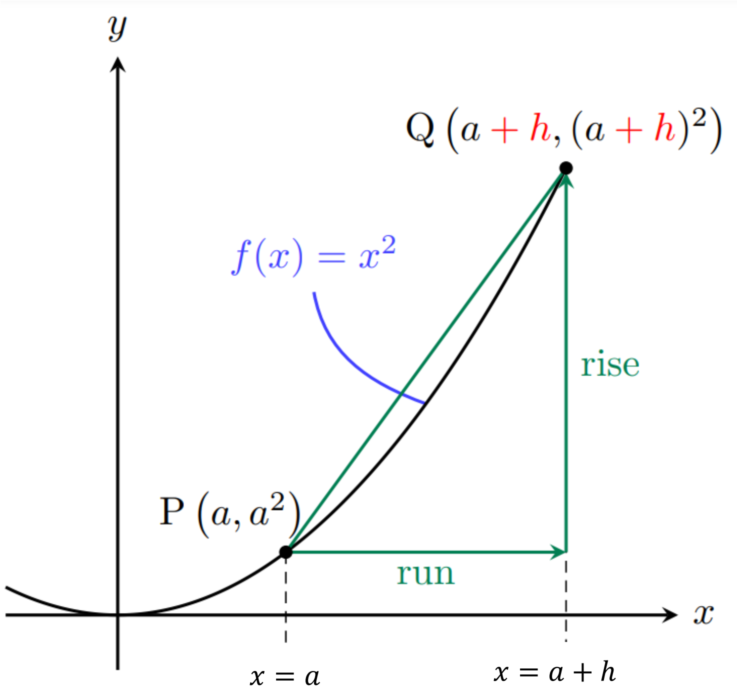
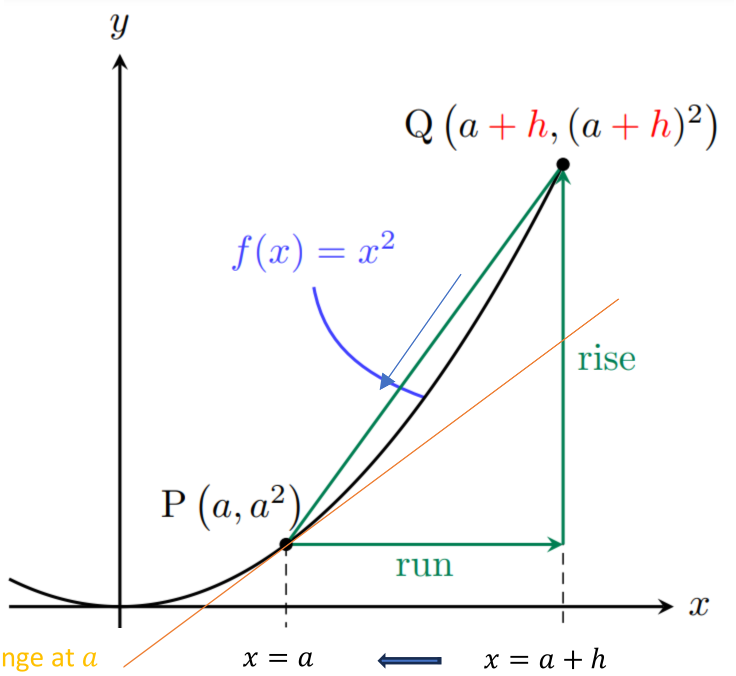
First principle and tangent



Gradient between P and Q $= \frac{rise}{run} = \frac{f(a+h)-f(a)}{(a+h)-a} = \frac{f(a+h)-f(a)}{h} = \frac{(a+h)^2-a^2}{h} = \frac{a^2+2ah+h^2-a^2}{h} = 2a + h$



instantaneous rate of change at a

$f'(a) = 2a$

As h is approaching to 0: $\lim_{h \to 0} \frac{f(a+h)-f(a)}{(a+h)-a} = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h \to 0} 2a + h = 2a$

so, we can say the value of derivative of $f(x)$ at $x = a$ can be approximated by

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = 2a$$

$$\therefore f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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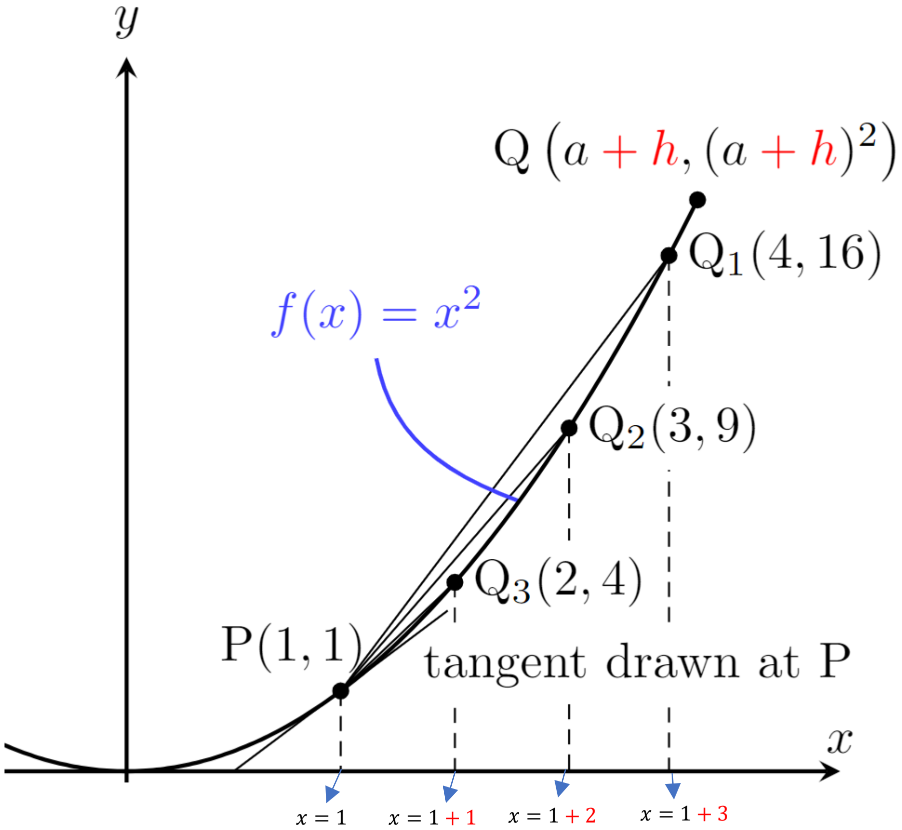
Exercise 1) Let $y = x^2$, $P(1,1)$, $Q_1(4,16)$, $Q_2(3,9)$, $Q_3(2,4)$. Find the average rate(gradient) at which y changes with respect to x over each of the following intervals:



a. PQ_1

b. PQ_2

c. PQ_3

d. Find the instantaneous rate of change of y with respect to x when $x = 1$.