

# lec 15:- Representing Relations.

Why? We use Sets.

Computers are good at processing Numbers.

We want to convert Sets to Some Numbers.

to convert the Sets into Numbers.

→ MATRICES. [ ]

Rows & Columns.

Sets. { }

→ Unique

P476-  $A = \{a_1, a_2, \dots, a_m\}$   $|A| = m$

$B = \{b_1, b_2, \dots, b_n\}$   $|B| = n$ .

Size of  $M_R$   $|A| \times |B|$

$m \times n$   
↓ ↓  
rows columns.

$R \subseteq A \times B$

$M_R = [m_{ij}]$

↓  
 $m_{ij} =$  A particular entry of  $M_R$  is located at  $i$ th row &  $j$ th col.

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$$

$A = \{a_1, a_2, \dots, a_m\}$   $|A| = m = 3$

$B = \{b_1, b_2, \dots, b_n\}$   $|B| = n = 2$

$A = \{1, 2, 3\}$   $B = \{1, 2\}$   
↓ ↓ ↓ ↓ ↓  
 $a_1 \ a_2 \ a_3$   $b_1 \ b_2$

3x2

$[m_{11} \ m_{12}]$

$$M = \begin{matrix} & a_1 & a_2 & a_3 & a_4 & a_5 \\ \begin{matrix} m_{11} \\ m_{21} \\ m_{31} \end{matrix} & \begin{matrix} m_{12} \\ m_{22} \\ m_{32} \end{matrix} \end{matrix} \quad 3 \times 2$$

$$R = \{(2,1), (3,1), (3,2)\}$$

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$$

$$(a_1, b_2) = (1, 2) \notin R$$

$$(a_2, b_1) = (2, 1) \in R$$

$$M = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R \end{cases}$$

$$a_1 = 1 \quad a_2 = 2 \quad a_3 = 3$$

$$b_1 = 1 \quad b_2 = 2$$

$$(a_1, b_1) = (1, 1) \notin R$$

P477 Ex 2:

$$A = \{a_1, a_2, a_3\}$$

$$B = \{b_1, b_2, b_3, b_4, b_5\}$$

$$R \subseteq A \times B$$

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$m_{12} = (a_1, b_2)$$

$$m_{21} = (a_2, b_1)$$

Try find R.

$$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2, b_4), (a_3, b_1), (a_3, b_3), (a_3, b_5)\}$$

M → tuples of Relations also.

How to investigate the properties.

1) Reflexive:  $\forall a (a, a) \in R$

$$R \subseteq A \times A$$

$$a_i \in A$$

$$m_{ij} \rightarrow (a_i, b_j)$$

\*  $\forall_i (m_{ii} = 1)$

$$\begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & 1 & \\ & & & & & 1 \end{bmatrix}$$

$$\begin{matrix} m_{11} \\ m_{22} \\ m_{33} \\ m_{44} \\ m_{55} \end{matrix}$$

→ Reflexive.

$[1]$  Reflexive.

$[0]$  Reflexive?

~~$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$~~

No

No

$m_{11} = 1$   $\forall i m_{ij} = 1$ .  
 $m_{22} \neq 1$ .

2) Symmetric.

$\forall a, b$  if  $(a, b) \in R \rightarrow (b, a) \in R$ .

$\forall i, j$  if  $m_{ij} = 1 \rightarrow m_{ji} = 1$ .

$M_R = \begin{bmatrix} & 1 & \\ 1 & & \\ & & 1 \end{bmatrix}$

$m_{12} = 1$   
 $m_{21} = 1$ .

$m_{23} = 1$   
 $m_{32} = 1$ .

$m_{ij} = m_{ji}$

$(M_R)^T = M_R$

Symmetric.

$M^T = M$   
Symmetric.

Examples:-

$[0]$

Symmetric.  
Yes.

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Yes.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Yes.

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

No

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

No

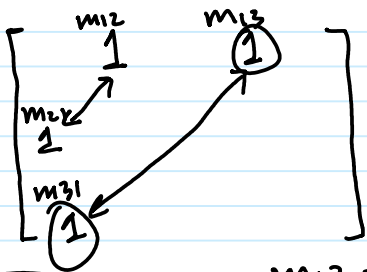
Anti Symmetric:-

$\forall a, b \in A$  if  $(a, b) \in R \wedge (b, a) \in R \rightarrow a = b$ .

$\forall i, i$  if  $m_{ii} = 1 \wedge m_{ii} = 1 \rightarrow i, i$

Tab 2A if  $(a,b) \in R$  and  $(b,c) \in R \rightarrow a \in R$ .

$\forall i, j$  if  $m_{ij} \geq 1 \wedge m_{ji} \geq 1 \rightarrow i \sim j$

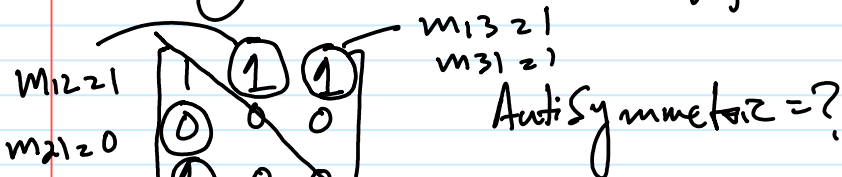


$1 \neq 2$ .

$i \neq j$   
 $m_{ij} \geq 0$  or  
 $m_{ji} \geq 0$

P 2 f

$R \rightarrow R^{-1}$



$\Downarrow$  ignore  $1 \neq 3$   
 Not AntiSymmetric

$\begin{bmatrix} 0 \\ \end{bmatrix}$  Yes.

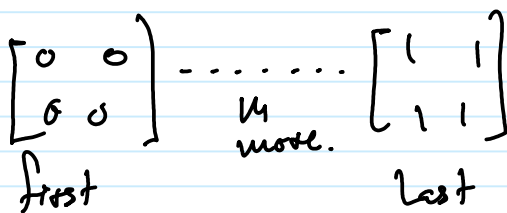
$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$  Yes.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  Yes.

Home work :- Give a 2x2 matrix

(a) Find all those matrices which Represent AntiSymmetric Relations.

(b) Find all those matrices which Represents reflexive & Symmetric.



out of 16

find all which are reflexive.

first

last

$$M_{R1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$M_{R2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_{R1} M_{R2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{R1} M_{R2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

matrix are  
reflexive.

Symmetric

→ Antisymmetric.

RECAP :-

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Do first

do questions.

- 1) How to Represent Relations Using Matrices.
- 2) Properties in Matrices.
- 3) Union & Intersection

