lec 15:- Representing Relations.

why? We use Sets.

Computers are good at processing Numbers.

We want to Convert Sets to Some Numbers.

to Convert the Sets into Numbers.

→ MATRICES. [ ]

Rows & Columns.

Sets. → { }

→ Unique

P476:- $A = \{a_1, a_2, \dots, a_m\}$ $|A| = m$

$B = \{b_1, b_2, \dots, b_n\}$ $|B| = n.$

Size of M_R $|A| \times |B|$

$m \times n$

↓ rows ↓ colums.

$R \subseteq A \times B$

$M_R = [m_{ij}]$

↓ m_{ij} = A particular entry of M_R is located at ith row & jth col.

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$$

$A = \{a_1, a_2, \dots, a_m\}$ $|A| = m = 3$

$B = \{b_1, b_2, \dots, b_n\}$ $|B| = n. = 2$

$A = \{1, 2, 3\}$ $B = \{1, 2\}.$

↓ a_1 ↓ a_2 ↓ a_3 ↓ b_1 ↓ b_2

3×2

M $[m_{11} \quad m_{12}]$

3x2

$M_R = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{32} \end{bmatrix}$

$R = \{(2,1), (3,1), (3,2)\}$

$m_{11} = 0$

$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$

$m_{11} = \begin{cases} 1 & \text{if } (a_1, b_1) \in R \\ 0 & \text{if } (a_1, b_1) \notin R \end{cases}$

$a_1 = 1 \quad a_2 = 2 \quad a_3 = 3$

$b_1 = 1 \quad b_2 = 2$

$(a_1, b_1) = (1,1) \notin R$

$(a_1, b_2) = (1,2) \notin R$

$(a_2, b_1) = (2,1) \in R$

$M = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$

P477 Ex2:- Let $A = \{a_1, a_2, a_3\}$

$B = \{b_1, b_2, b_3, b_4, b_5\}$

$R \subseteq A \times B.$

$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$

$m_{12} = (a_1, b_2)$

$m_{21} = (a_2, b_1)$

Try find R.

$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3),$
$(a_2, b_4), (a_3, b_1), (a_3, b_3),$
$(a_3, b_5)\}.$

$M \rightarrow$ tuples of Relations also.

How to investigate the properties.

1) Reflexive:- $\forall a \; (a, a) \in R$

$R \subseteq A \times A.$

$a_i \in A.$

$\underbrace{(a_i, a_i)}$ $\in R.$

$m_{ij} \rightarrow (a_i, b_j).$

★ $\boxed{\forall_i \; (m_{ii} = 1.)}$

$\begin{bmatrix} 1 & & & & & & \\ & 1 & & & & & \\ & & 1 & & & & \\ & & & 1 & & & \\ & & & & 1 & & \\ & & & & & 1 & \\ & & & & & & 1 \end{bmatrix} \rightarrow$ Reflexive.

m_{11}
m_{22}
m_{33}
m_{44}
m_{55}

$[\quad\quad 1] \rightarrow$ Reflexive.

$[0]$ Reflexive?

No

$\cancel{\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}}$ No

$m_{11} = 1$ $\quad \forall i \; m_{ii} = 1.$

$m_{22} \neq 1.$

2) Symmetric.

$\forall a,b$ if $(a,b) \in R \rightarrow (b,a) \in R.$ $\quad R \subseteq A \times B.$

$\forall i,j$ if $m_{ij} = 1 \rightarrow m_{ji} = 1.$

$m_{ij} = m_{ji}$

$$M_R = \begin{bmatrix} & 1 & \\ 1 & & 1 \\ & 1 & \end{bmatrix}$$

$m_{12} = 1$

$m_{21} = 1.$

K.

$m_{23} = 1$

$m_{32} = 1.$

$(M_R)^T = M_R$

Symmetric.

$\underbrace{M^T = M}_{\text{Symmetric.}}$

Examples:- $[0]$ Symmetric.

Yes.

$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ Yes.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Yes.

$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ No

$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ No

Anti Symmetric :-

$\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b.$

$\forall i,j$ if $m_{ij} = 1 \wedge m_{ji} = 1 \Rightarrow i = j$

$\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall i,j$ if $\underline{m_{ij} = 1 \wedge m_{ji} = 1} \Rightarrow i = j$

$$\begin{bmatrix} & \overset{m_{12}}{1} & \overset{m_{13}}{1} \\ \overset{m_{21}}{1} & & \\ \overset{m_{31}}{1} & & \end{bmatrix}$$

$1 \neq 2.$

$i \neq j$

$m_{ij} = 0$ or $m_{ji} = 0$

$m_{12} = 1$ $m_{13} = 1$

$m_{21} = 0$ $m_{31} = 1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Antisymmetric = ?

Ignore

$1 \neq 3$

$\Downarrow$

Not Antisymmetric

$\begin{bmatrix} 0 \end{bmatrix}$ Yes.

$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ Yes.

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Yes.

Home work :- Give a 2x2 matrix

(a) find all those matrices which Represent Antisymmetric Relations.

(b) find all those matrices which Represents reflexive & Symmetric.

$\underset{\text{first}}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}} \ldots\ldots\ldots \underset{\text{Last}}{\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}$

14 more.

out of $\underline{16}$

find all which are reflexive.

$p = f$

$p \rightarrow q = T.$

First Last

$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$M_{R_1 \cap R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$M_{R_1 \cup R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

reflexive.

→ Symmetric

→ Antisymmetric.

RECAP :-

1) How to Represent Relations Using Matrices.

2) Properties in Matrices.

3) Union & Intersection

p 476–478.

Do first 10 Questions.