# Inverse function

Find the inverse of the function $f(x) = 2x - 3$

step. 1 replace $f(x)$ with $y$

$$f(x) = 2x - 3$$

$$y = 2x - 3$$

step. 2 interchange $x$ and $y$

$$x = 2y - 3$$

step. 3 solve the equation for $y$

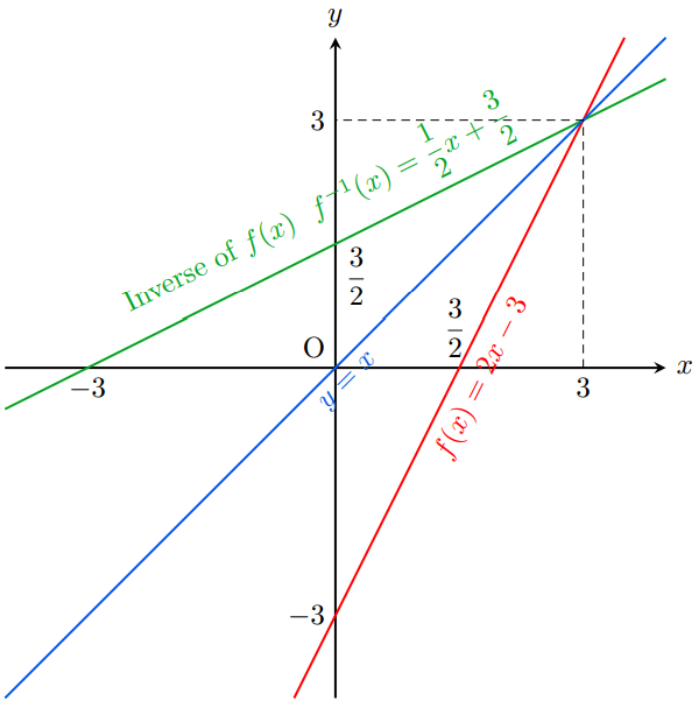
$$x + 3 = 2y$$

$$y = \frac{x+3}{2}$$

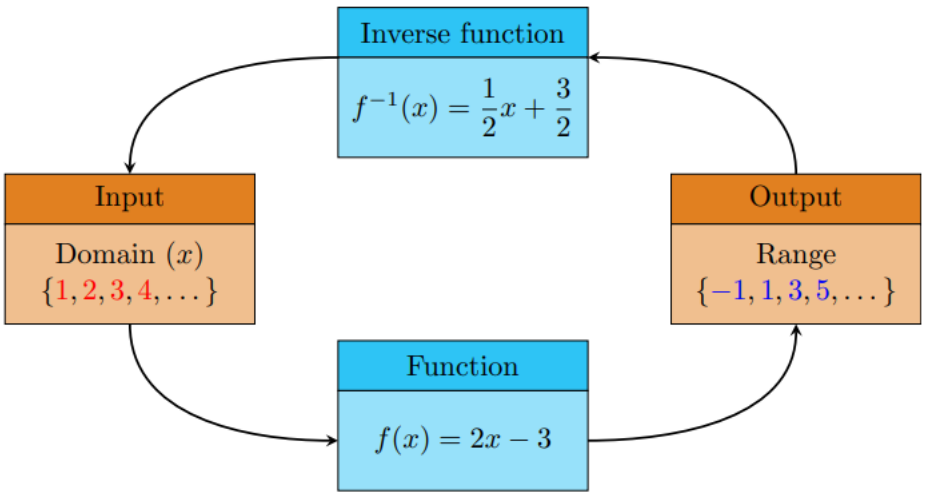
step. 4 replace $y$ with $f^{-1}(x)$

$$y = \frac{x}{2} + \frac{3}{2}$$

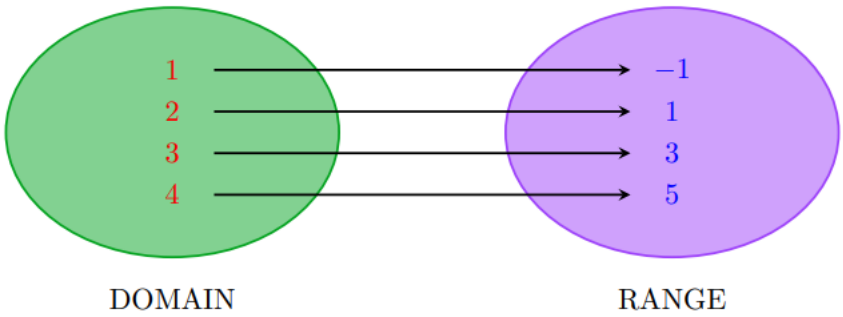
$$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$$



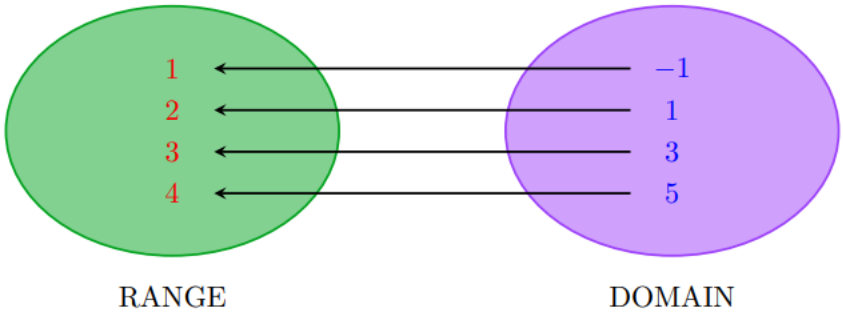

$f(x)$ and $f^{-1}(x)$ are mirror images about the line $y = x$



$f(x) = 2x - 3$




$f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$




$x$ and $y$ have been swapped

| $x$ | $y$ |
|---|---|
| 1 | $-1$ |
| 2 | 1 |
| 3 | 3 |
| 4 | 5 |

$\{(1,-1),(2,1),(3,3),(4,5)\}$

**Domain:** $\{1,2,3,4\}$ **Range:** $\{-1,1,3,5\}$

$\longleftrightarrow$

$\{(-1,1),(1,2),(3,3),(5,4)\}$

**Domain:** $\{-1,1,3,5\}$ **Range:** $\{1,2,3,4\}$

# Exercise

**1** For each of the following, find the inverse function and state its domain and range:

**a** $f : \mathbb{R} \to \mathbb{R},\ f(x) = 6 - 2x$

**b** $f : [1,5] \to \mathbb{R},\ f(x) = 3 - x$

**c** $f : \mathbb{R}^+ \to \mathbb{R},\ f(x) = x + 4$

**d** $f : (-\infty, 4] \to \mathbb{R},\ f(x) = x + 4$

**e** $f : [-1,7] \to \mathbb{R},\ f(x) = 16 - 2x$

**2** Find the inverse function of each of the following. State the domain and range of $f^{-1}$.

**a** $f : [0,\infty) \to \mathbb{R},\ f(x) = x^2$

**b** $f : [2,\infty) \to \mathbb{R},\ f(x) = (x-2)^2 + 3$

**c** $f : (-\infty, 4] \to \mathbb{R},\ f(x) = (x-4)^2 + 6$

**d** $f : [0,1] \to \mathbb{R},\ f(x) = \sqrt{1-x}$

**e** $f : [0,4] \to \mathbb{R},\ f(x) = \sqrt{16 - x^2}$

**f** $f : [0,\infty) \to \mathbb{R},\ f(x) = (x+4)^2 + 6$