CONSERVATION OF ENERGY, WORK & POWER

The Law of Conservation of Energy

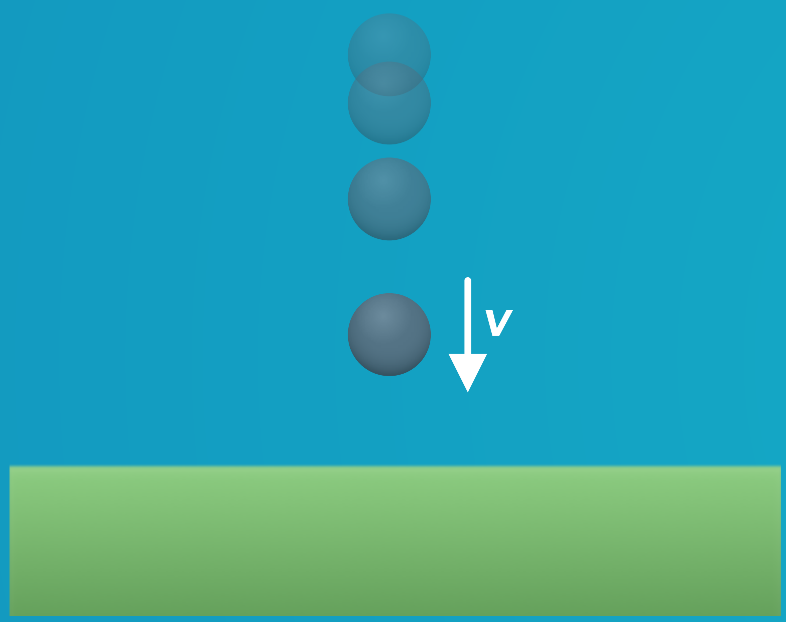
- **The law of conservation of energy**: the total amount of energy in an isolated system (and the universe) is conserved, which means it is constant and doesn't change over time.
- There are many different types of energy (kinetic energy, gravitational potential energy, etc.) and energy can be converted or transformed between those different types, but it cannot be created or destroyed.

Variables		SI Unit
E	energy	J
K	kinetic energy	J
U_g	gravitational potential energy	J
U_{sp}	spring potential energy	J

- Energy can be converted or transformed from one type of energy to another type of energy.

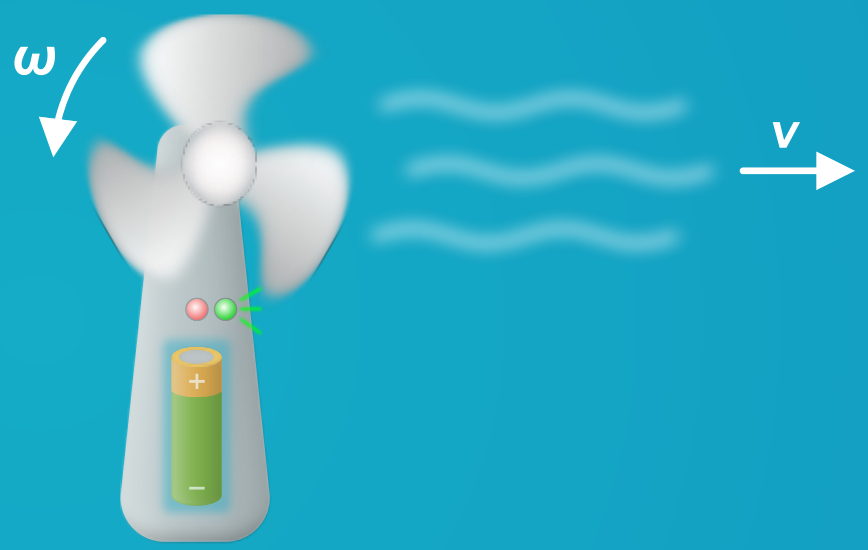
Gravitational potential energy is converted into kinetic energy as a ball falls towards the earth

$$U_g \rightarrow K$$

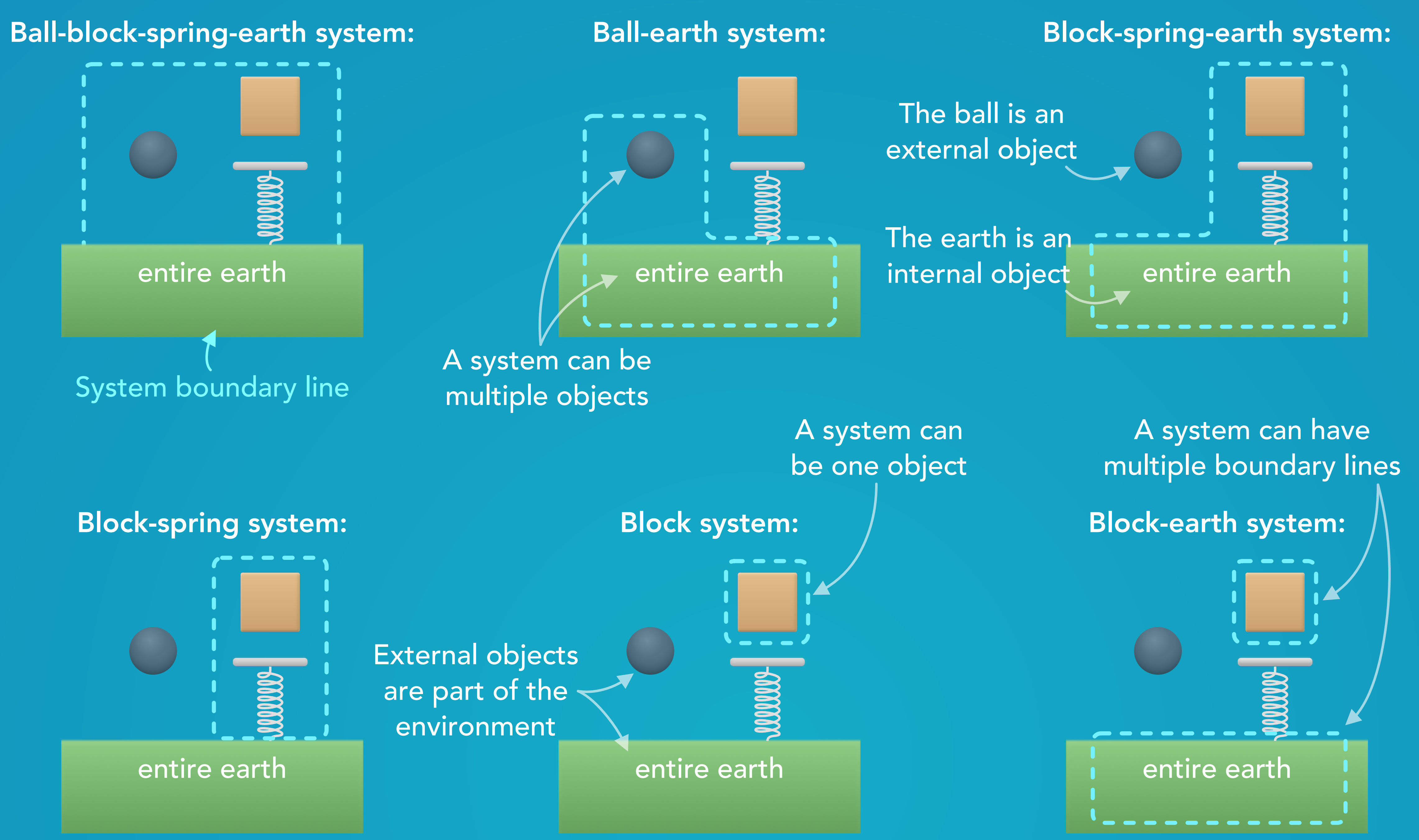


Chemical energy in a fan battery is converted into electrical energy, which is converted into rotational kinetic energy, translational kinetic energy, sound energy, light energy and thermal energy

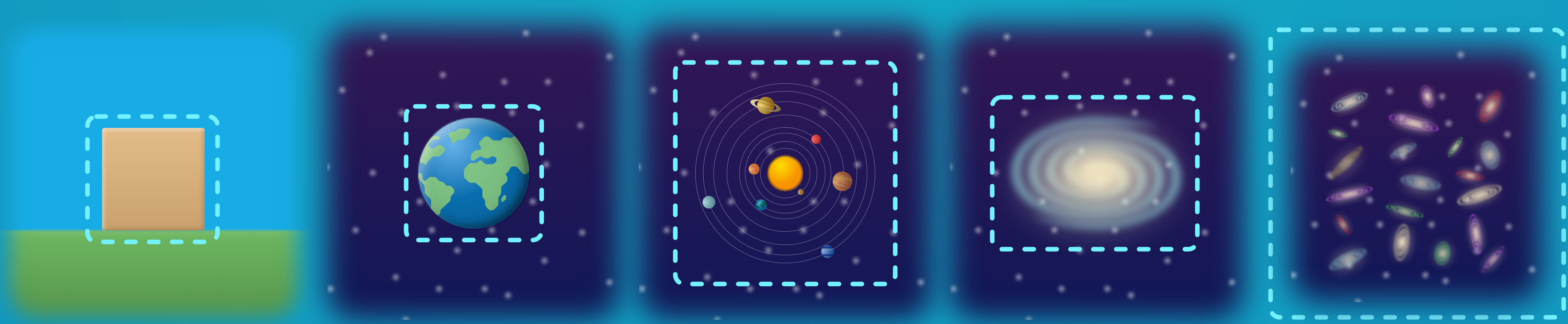
$$E_{chem} \rightarrow E_{elec} \rightarrow K_{rot} + K + E_{sound} + E_{light} + E_{therm}$$



- A **system** is a chosen group of objects which is separated from the **environment** by a chosen boundary line. There are no predefined or existing systems, we choose how to define a system for a given scenario.
- Once a system is chosen, everything in the universe (objects, forces, energy) is either inside the system (internal) or outside the system (external) at any moment in time.
- The objects in a system do not have to be in contact, and multiple separate boundary lines can be drawn to define the system so that only specific objects are included.



A system can be defined as any size, from one object to the entire universe



- An **isolated system** is defined as a system where **energy does not enter or leave the system**. If energy does enter or leave the system, it is not an isolated system.
- According to the law of conservation of energy, **the total amount of energy within an isolated system is conserved over time**. Energy within the system can be converted back and forth between different types, but the **total** amount of energy stays the same.

Conservation of energy (in an isolated system)

$$\Delta E_{total} = 0 \quad , \quad E_{total\ i} = E_{total\ f}$$

Isolated system: no energy moves into or out of the system, no work is done on the system, and no net external forces are acting on the system

The total amount of energy in the system is the sum of every type of energy

Isolated system

$$E_{total} = K + U_g + U_{sp} + E_{therm} + E_{light} + E_{sound} + E_{chem} + E_{elec} + \ldots$$

$\Delta E_{total} = 0$ (total energy does not change)

$$K \leftrightarrow U_g \leftrightarrow U_{sp} \leftrightarrow E_{therm} \leftrightarrow E_{elec} \leftrightarrow E_{chem} \leftrightarrow E_{sound} \leftrightarrow E_{light} \leftrightarrow K$$

Energy can be converted between different types within the system

The environment also has energy, but no energy enters or leaves an isolated system to or from the environment

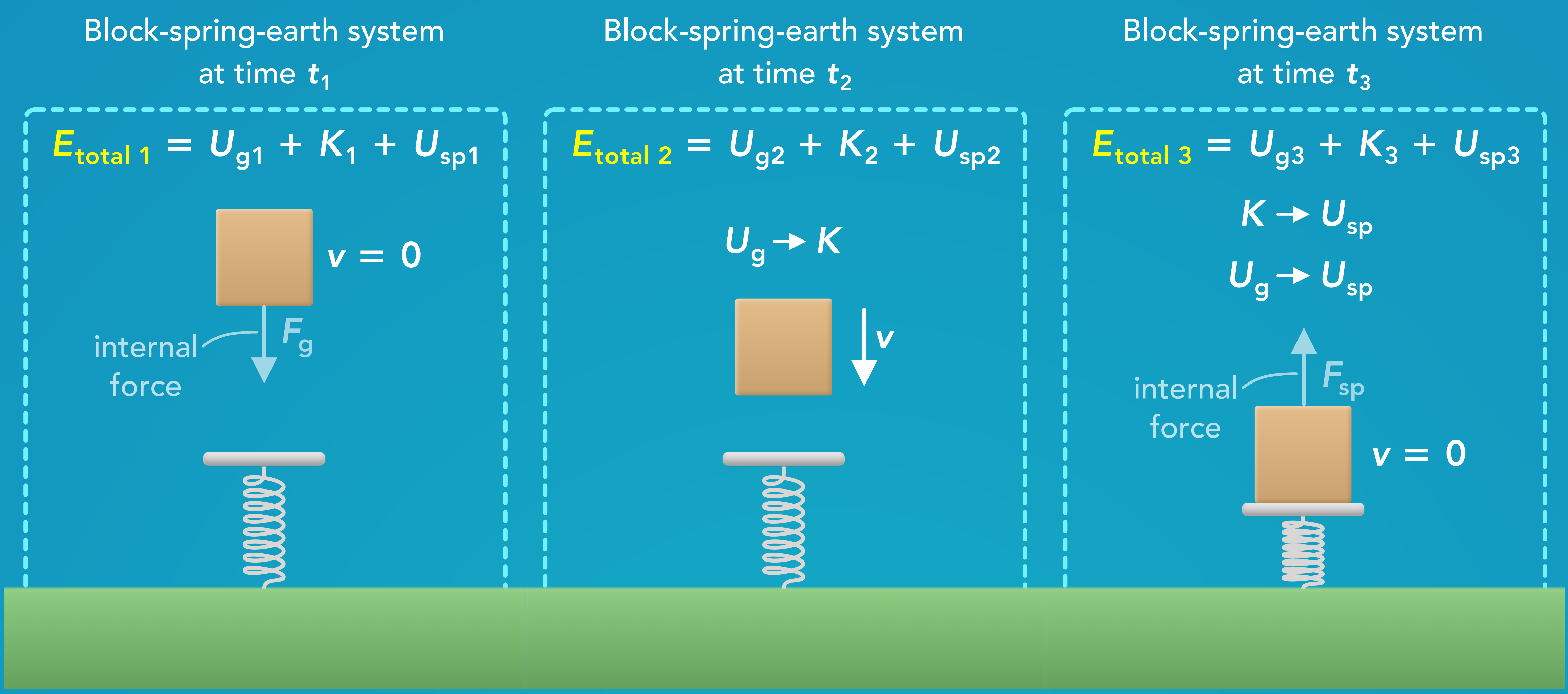
Environment

$$E_{total} = K + U_g + U_{sp} + E_{therm} + E_{light} + E_{sound} + E_{chem} + E_{elec} + \ldots$$

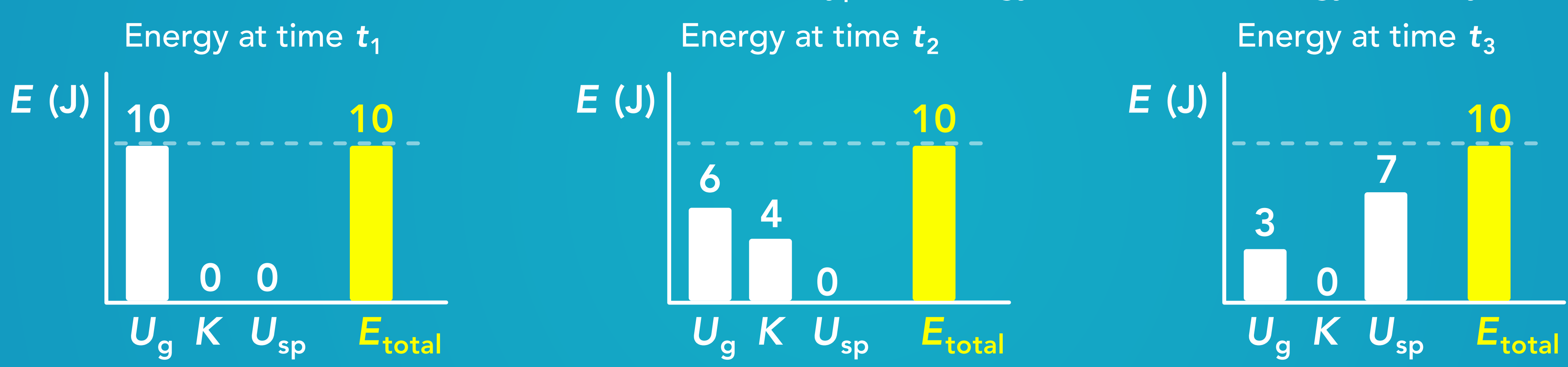
$$\Delta E_{total} = 0$$

$$K \leftrightarrow U_g \leftrightarrow U_{sp} \leftrightarrow E_{therm} \leftrightarrow E_{elec} \leftrightarrow E_{chem} \leftrightarrow E_{sound} \leftrightarrow E_{light} \leftrightarrow K$$

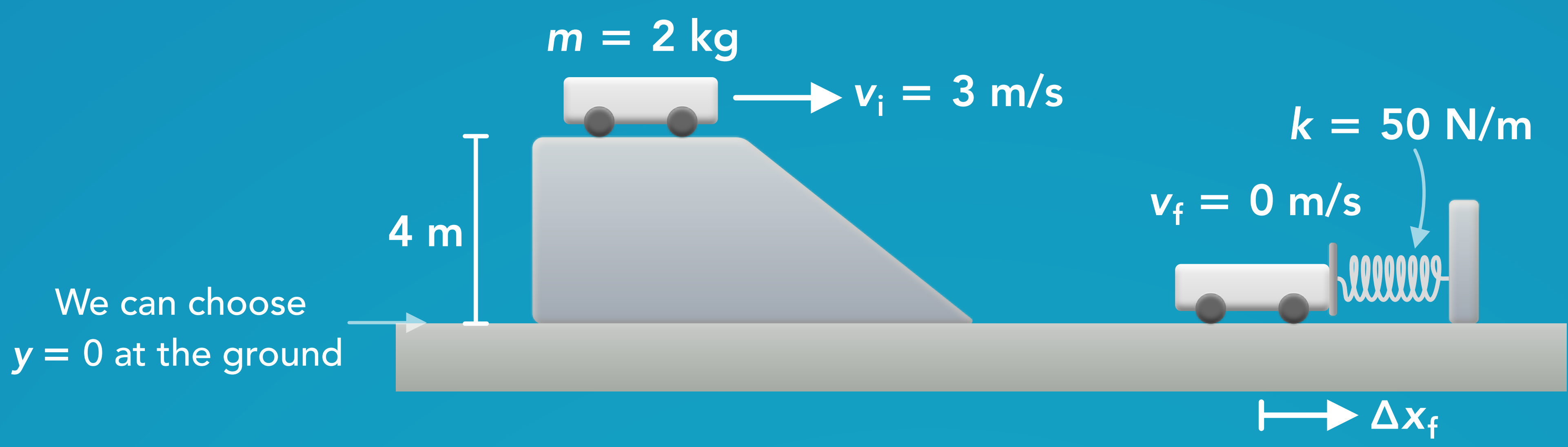
Example: In the isolated system below, the block, the spring and the earth are inside the system. The gravitational force between the block and the earth, and the spring force between the block and the spring, are internal forces which convert energy between different types within the system. The gravitational force and the spring force do not change the total amount of energy in the system.



We can use bar charts to show the amount of each type of energy and the total energy in the system



Example: A 2 kg cart is 4 m above the ground with a speed of 3 m/s. It rolls down a ramp and contacts a spring with a spring constant of 50 N/m. The cart compresses the spring and momentarily comes to a stop. At that moment, how much is the spring compressed? Assume there is negligible friction.



We're assuming that the **cart-spring-earth system** is an isolated system so the total energy in the system is conserved over time. The initial total energy is equal to the final total energy:

$$E_{\text{total i}} = E_{\text{total f}}$$

$$K_i + U_{g\,i} + U_{sp\,i} = K_f + U_{g\,f} + U_{sp\,f}$$

$$\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}k\Delta x_i^2 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}k\Delta x_f^2$$

$$\frac{1}{2}(2\text{ kg})(3\text{ m/s})^2 + (2\text{ kg})(9.8\text{ m/s}^2)(4\text{ m}) + \frac{1}{2}(50\text{ N/m})(0\text{ m})^2 = \frac{1}{2}(2\text{ kg})(0\text{ m/s})^2 + (2\text{ kg})(9.8\text{ m/s}^2)(0\text{ m}) + \frac{1}{2}(50\text{ N/m})\Delta x_f^2$$

$$9\text{ J} + 78.4\text{ J} + 0\text{ J} = 0\text{ J} + 0\text{ J} + 25\Delta x_f^2$$

$$1.87\text{ m} = \Delta x_f$$

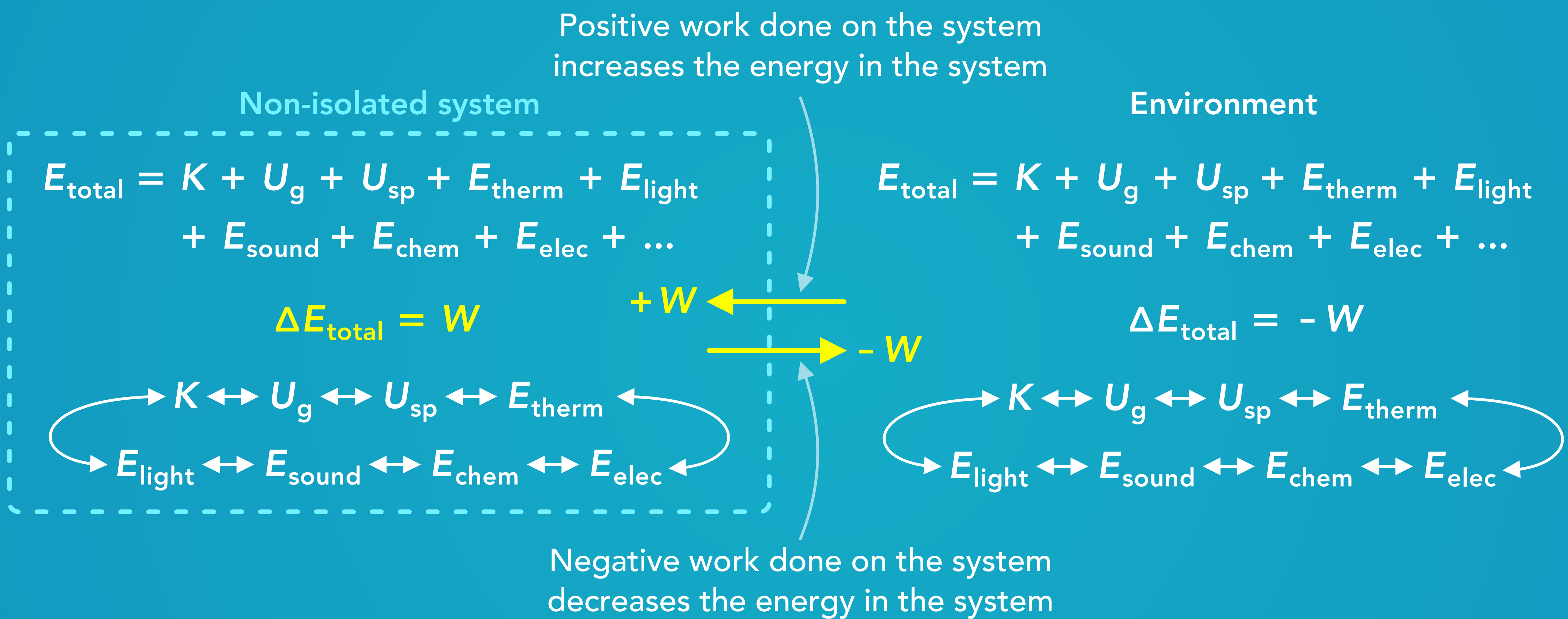
Work

Variables		SI Unit
W	work	$J = N \cdot m$
E	energy	J
F	force	N
d	displacement	m

- The law of conservation of energy says the total amount of energy within an isolated system stays the same over time. But if the system is not isolated, energy can move into or out of the system and the total energy in the system changes over time (it is not conserved).
- **Work** is the amount of energy transferred into or out of a system when an external force is applied over some displacement.
- The SI unit of work is a joule (J), the same as the unit of energy. 1 J = 1 N·m.
- If energy is transferred **into** the system, the work done on the system is **positive** and the total amount of energy in the system **increases**.
- If energy is transferred **out** of the system, the work done on the system is **negative** and the total amount of energy in the system **decreases**. We might also say that "work is done by the system on the environment".

Work done on a system

$$W = \Delta E_{system} = E_f - E_i$$

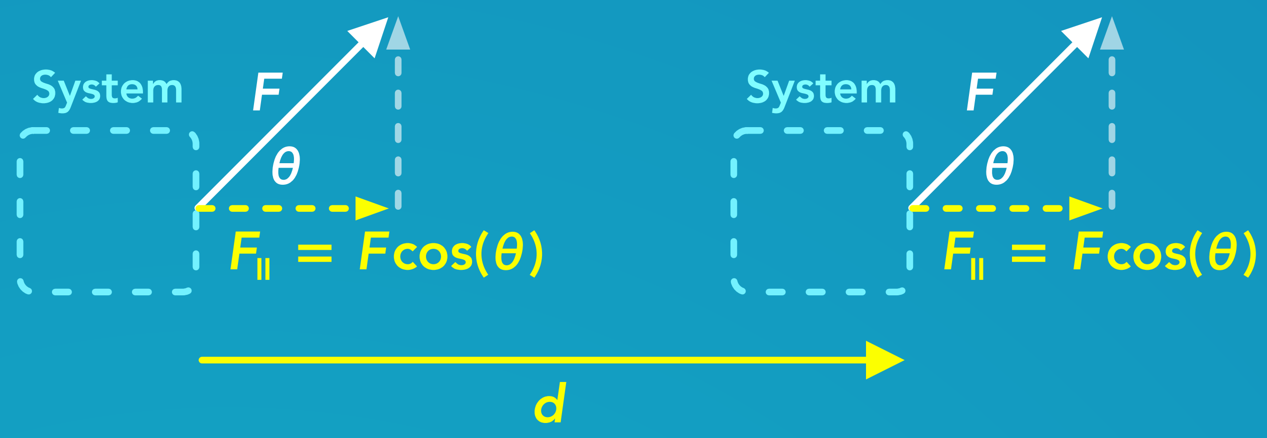


- Work is done on a system (energy is transferred into or out of a system) by an external force when the system moves some displacement. Only the component of the force that is parallel to the displacement does work.
- The work done on the system is positive if the parallel component of the force is in the same direction as the displacement, and the work is negative if they are in opposite directions.

Work

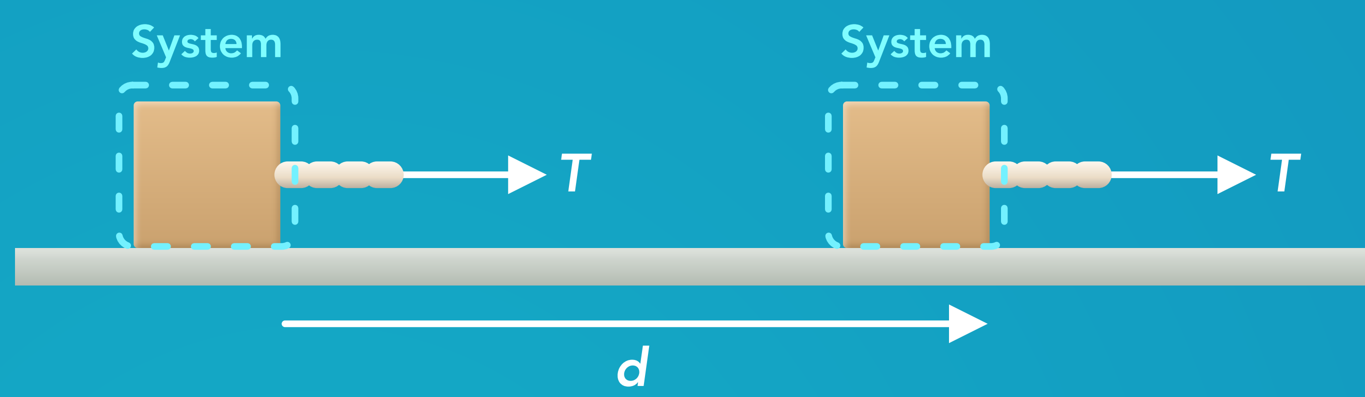
$$W = F_{\parallel} d = F\cos(\theta)\, d$$

$F_{\parallel}$: component of force parallel to d
d : displacement of the system
θ : angle between F and d
F must be an external force



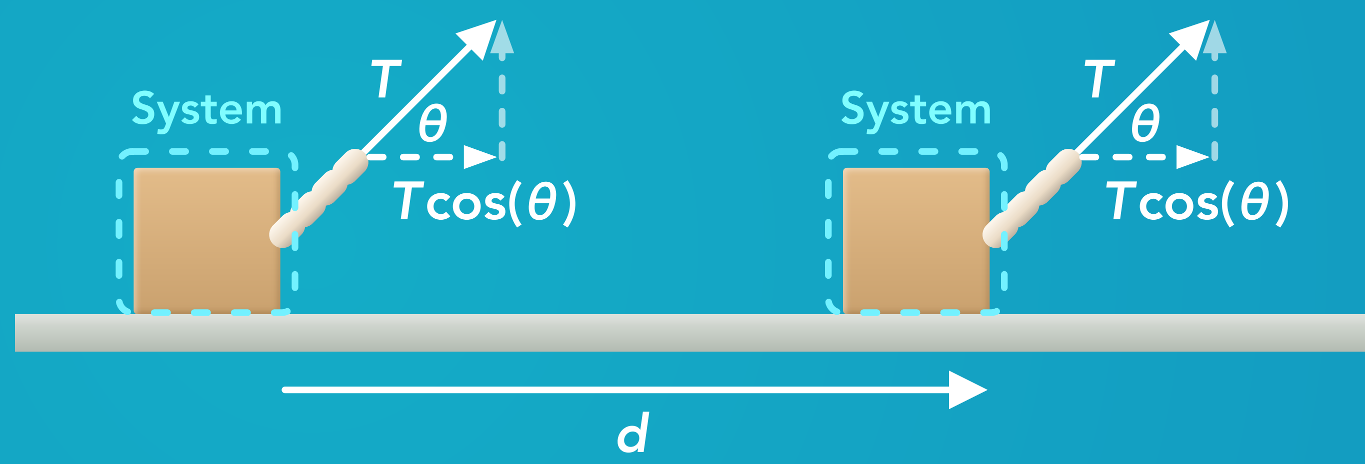
$F_{\parallel}$ and d are in the same direction,
the work done on the system is positive

$\Delta E_{system} = W = F_{\parallel} d = Td$



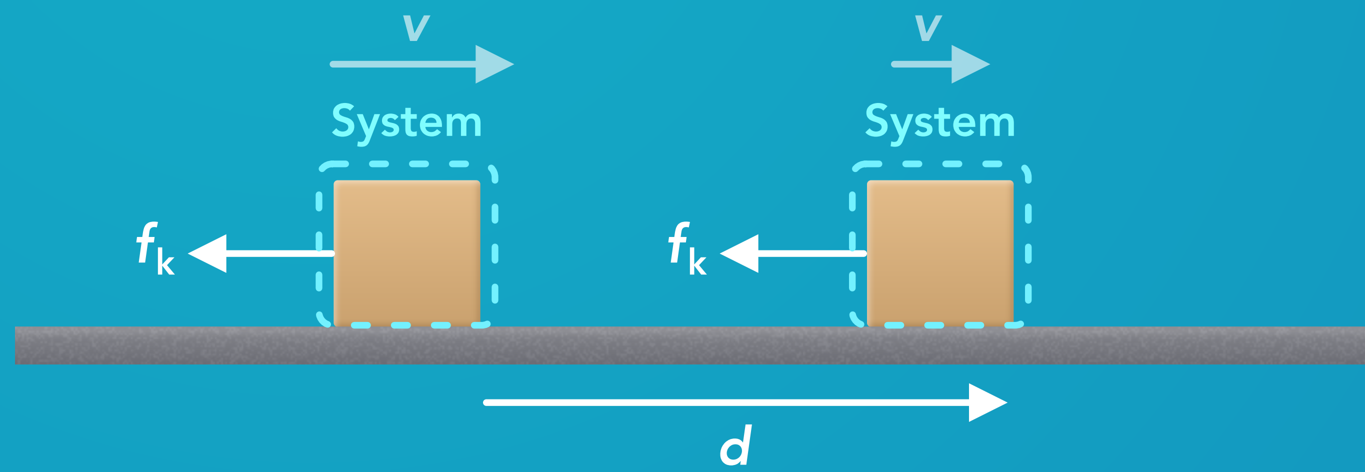
$F_{\parallel}$ and d are in the same direction,
the work done on the system is positive

$\Delta E_{system} = W = F_{\parallel} d = T\cos(\theta)\, d$



$F_{\parallel}$ and d are in opposite directions,
the work done on the system is negative

$\Delta E_{system} = W = F_{\parallel} d = -f_k d$

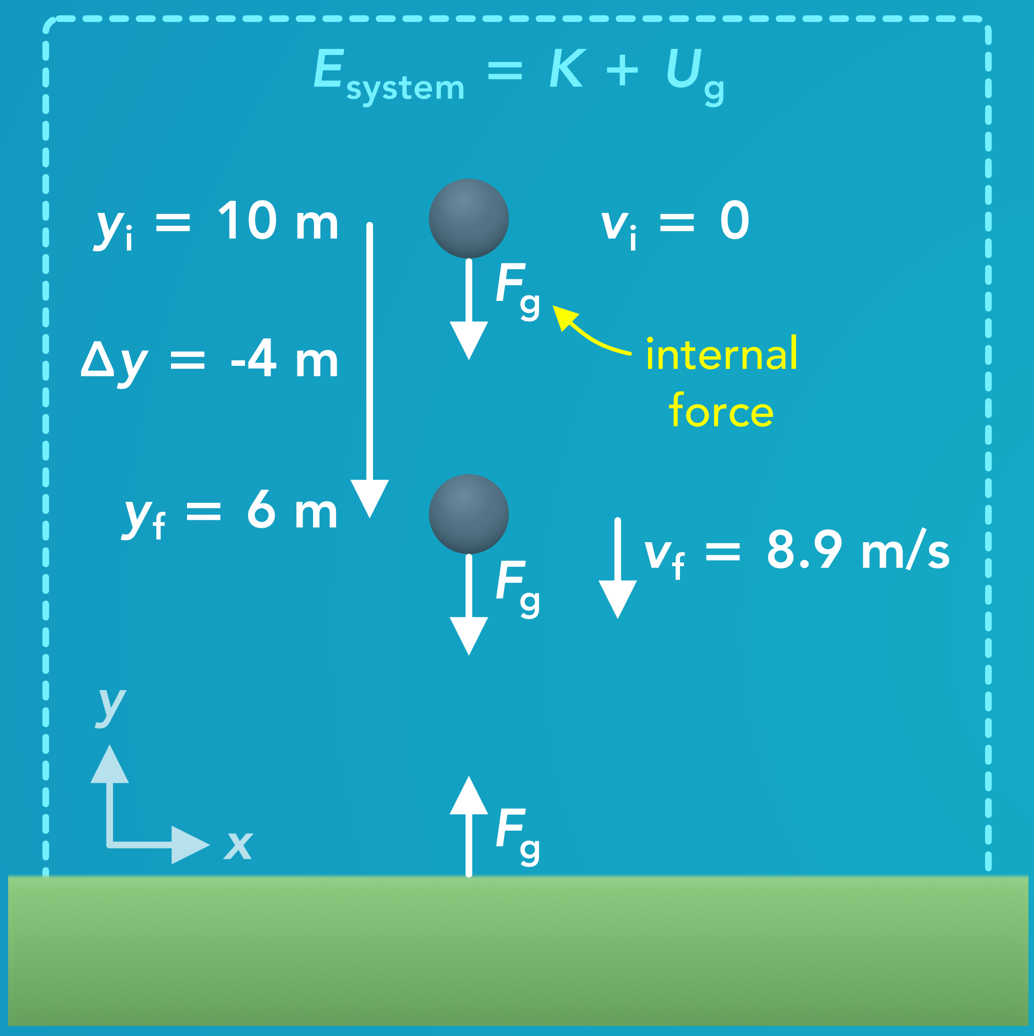


- **Internal forces** are any forces acting between two objects that are inside the system (internal objects). Internal forces can convert energy between different types within the system, but they do not change the total amount of energy in the system.
- **External forces** are any forces acting between an object outside of the system (external object) and an object inside the system (internal object). This does not include forces between two objects that are outside the system.
- **Work is only done on a system by external forces**. Internal forces do not do work on a system.

Example: Two different ways to analyze the same scenario by choosing different systems

If the earth is included in the system, the gravitational force between the ball and the earth is an internal force which converts gravitational potential energy into kinetic energy within the system but it does not do work on the system, and the total energy in the system is conserved.

Ball-earth system (isolated)



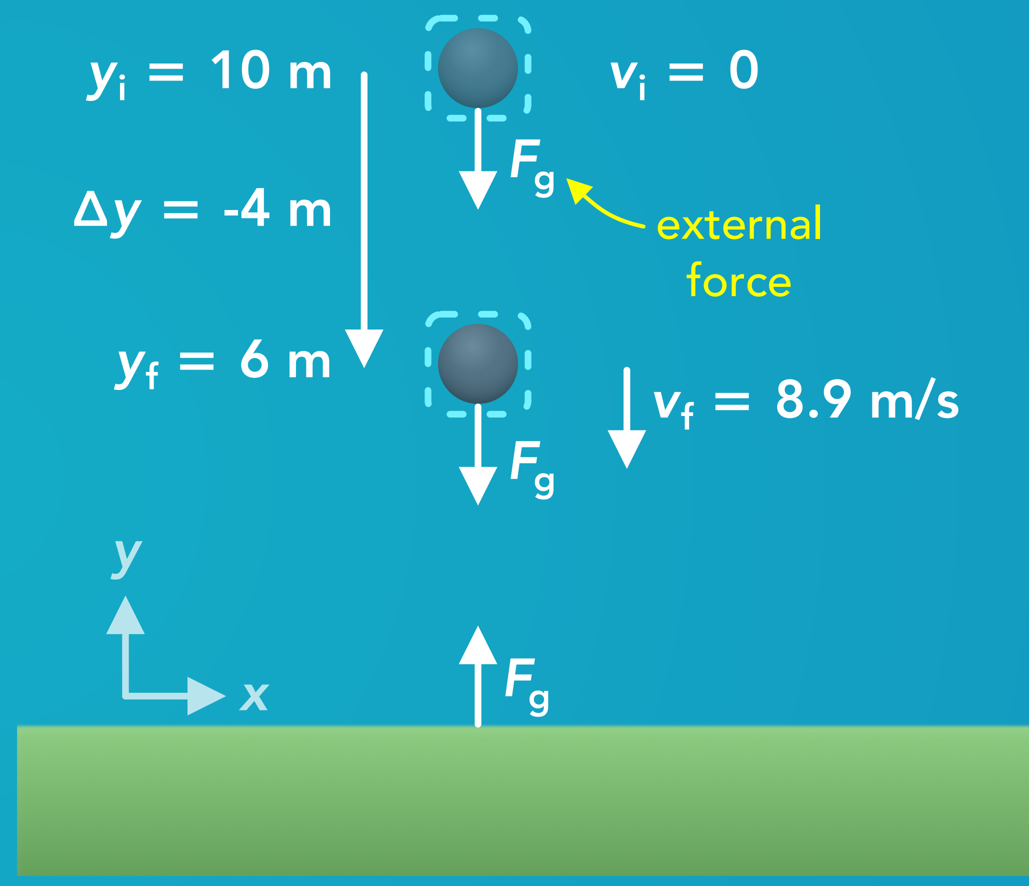
$$\Delta E_{system} = 0$$
$$E_{system\ i} = E_{system\ f}$$
$$K_i + U_{gi} = K_f + U_{gf}$$
$$K_f - K_i = -(U_{gf} - U_{gi})$$
$$\Delta K = -\Delta U_g$$
$$\Delta K = -mg\Delta y$$
$$\Delta K = -mg(\text{-4 m})$$
$$\Delta K = mg(4\text{ m})$$

The change in the kinetic energy of the ball-earth system is equal to the negative of the change in gravitational potential energy of the ball-earth system

If the earth is not included in the system, the gravitational force acting on the ball is an external force which does work on the ball system, changing its kinetic energy. The ball system does not have gravitational potential energy because the earth is not included in the system.

Ball system (not isolated)

$E_{system} = K$



$$\Delta E_{system} = W$$
$$\Delta K = F_{\parallel} d$$
$$\Delta K = (-mg)(\text{-4 m})$$
$$\Delta K = mg(4\text{ m})$$

The change in the kinetic energy of the ball system is equal to the work done on the ball system

Power

- **Power** is the rate of energy transfer or energy conversion over time.
- The SI unit for power is a watt (W) which should not be confused with the variable for work (*W*). 1 watt is equal to 1 joule/second (J/s).

Variables		SI Unit
P	power	$W = \frac{J}{s}$
E	energy	J
W	work	J
F	force	N
v	velocity	$\frac{m}{s}$

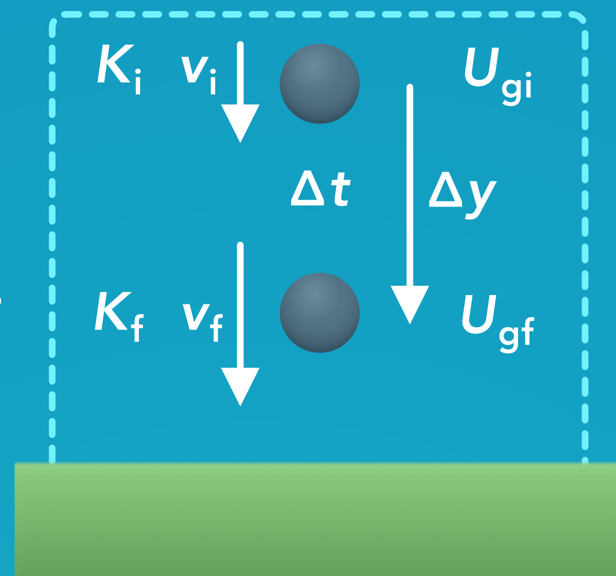
If energy is being converted from one type of energy to another, the power is the change in one type of energy divided by the period of time

Power

$$P = \frac{\Delta E}{\Delta t}$$

Example:

$$P = \frac{\Delta K}{\Delta t} = \frac{-\Delta U_g}{\Delta t}$$



If work is being done on a system, the power is the amount of work done divided by the period of time, which is also equal to the parallel force component multiplied by the velocity of the system

Power

$$P = \frac{W}{\Delta t} = \frac{F_{\parallel} d}{\Delta t} = F_{\parallel} v$$

$F_{\parallel}$: component of force parallel to v
v : velocity of the system
F must be an external force