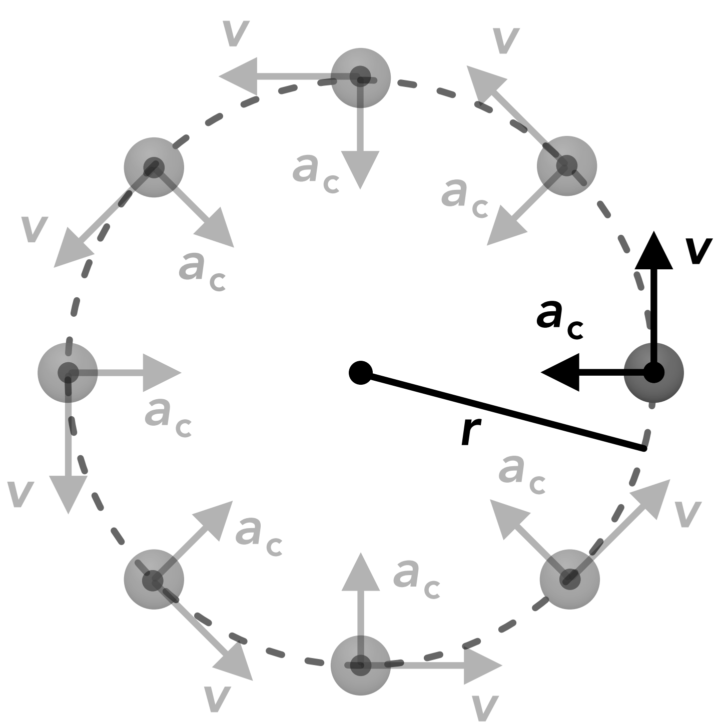
STUDY GUIDE

CENTRIPETAL ACCELERATION AND FORCE

Centripetal Acceleration

Variables		SI Unit
a_c	centripetal acceleration	$\frac{m}{s^2}$
v	velocity	$\frac{m}{s}$
r	radius	m

- When an object is in circular motion, the direction of the velocity is always changing (it is always tangent to the circular path).
- The acceleration that is causing the direction of the velocity to change is called the **centripetal acceleration**.
- The direction of the centripetal acceleration always points towards the center of the circle ("centripetal" means "towards the center").
- Centripetal acceleration is always perpendicular to the velocity, so centripetal acceleration changes the direction of the velocity, but it does not change the magnitude of the velocity (the speed).
- The magnitude of the centripetal acceleration is given by the equation below, and it depends on the speed and the radius of the circular path.



Centripetal acceleration

$$a_c = \frac{v^2}{r}$$

$\vec{v}$ is always tangent to the circle

$\vec{a}_c$ always points towards the center of the circle

$\vec{a}_c$ and $\vec{v}$ are always perpendicular

We can also write the equation in terms of the angular speed, period or frequency by substituting for the speed

$$a_c = \frac{v^2}{r} = \omega^2 r = (2\pi f)^2 r = \left(\frac{2\pi}{T}\right)^2 r$$

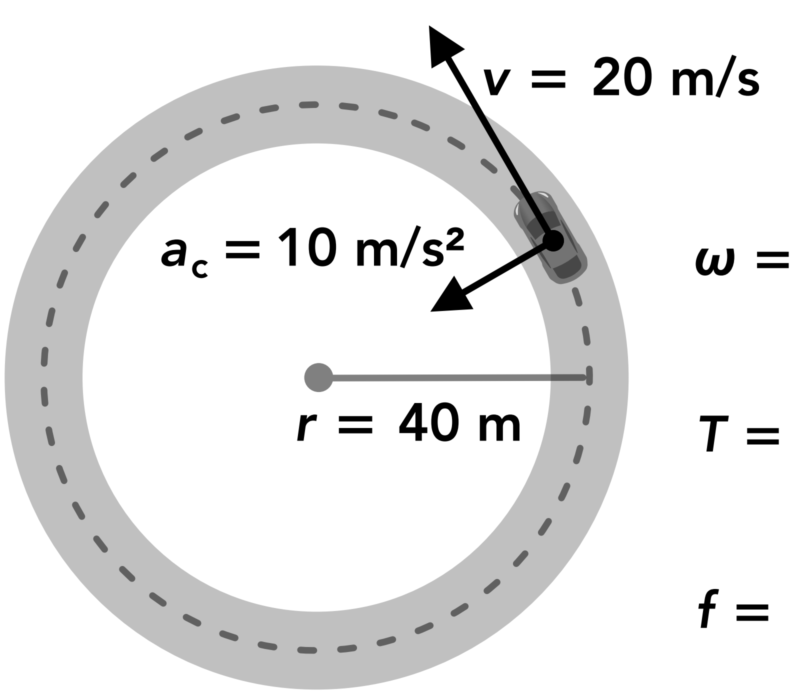
$$v = r\omega \quad v = 2\pi r f \quad v = \frac{2\pi r}{T}$$

ω : angular speed (rad/s)

f : frequency (Hz = circles/s)

T : period (s)

Example: Calculating the magnitude of the centripetal acceleration in uniform circular motion



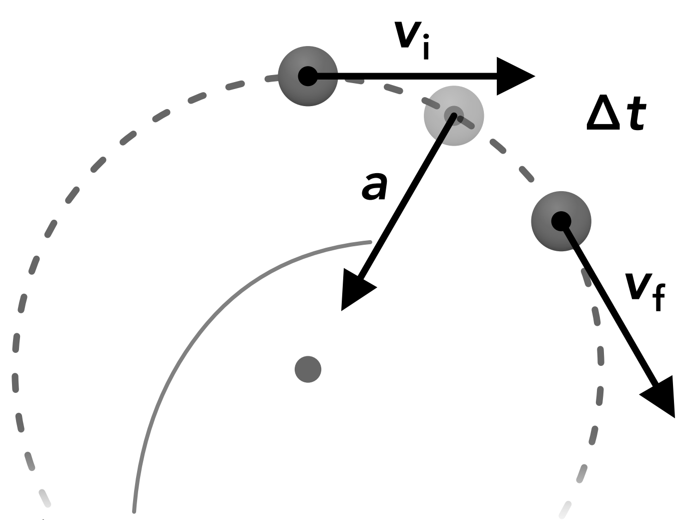
$v = 20$ m/s: $a_c = \frac{v^2}{r} = \frac{(20 \text{ m/s})^2}{40 \text{ m}} = 10 \text{ m/s}^2$

$\omega = 0.5$ rad/s: $a_c = \omega^2 r = (0.5 \text{ rad/s})^2 (40 \text{ m}) = 10 \text{ m/s}^2$

$T = 12.57$ s: $a_c = \left(\frac{2\pi}{T}\right)^2 r = \left(\frac{2\pi}{12.57 \text{ s}}\right)^2 (40 \text{ m}) = 10 \text{ m/s}^2$

$f = 0.0796$ Hz: $a_c = (2\pi f)^2 r = (2\pi(0.0796 \text{ Hz}))^2 (40 \text{ m}) = 10 \text{ m/s}^2$

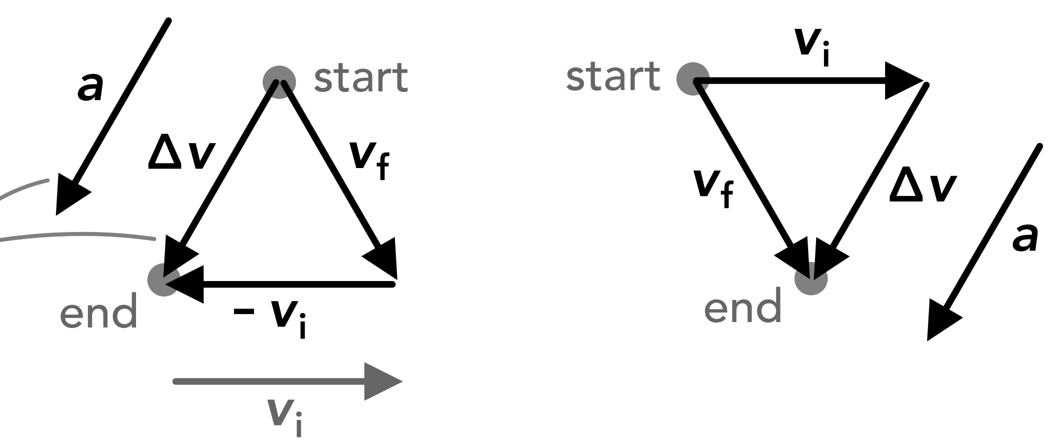
- To understand why the centripetal acceleration points towards the center of the circle, remember that velocity and acceleration are vectors which have a magnitude and a direction.
- Acceleration is the change in the velocity vector over time, which can be a change to the magnitude of the velocity (the speed) or the direction of the velocity.
- If we look at the velocity vector of an object in circular motion at two points in time, $\vec{v}_i$ and $\vec{v}_f$, we can find the change in velocity vector $\Delta\vec{v}$ using the tip-to-tail method. Two different versions are shown below, and remember that subtracting a vector is the same as adding the negative of that vector (which has the opposite direction).
- The acceleration vector $\vec{a}$ has the same direction as the change in velocity vector $\Delta\vec{v}$. These are the average vectors during this period of time, so we can draw the acceleration vector halfway between the initial and final times and see that it points towards the center of the circle.



$\vec{a}$ is the average acceleration vector during this period of time, so we can draw $\vec{a}$ in the middle of the initial and final points

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$
$$\Delta\vec{v} = \vec{v}_f + (-\vec{v}_i)$$

$$\vec{v}_f = \vec{v}_i + \Delta\vec{v}$$



$\vec{a}$ is in the same direction as $\Delta\vec{v}$

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

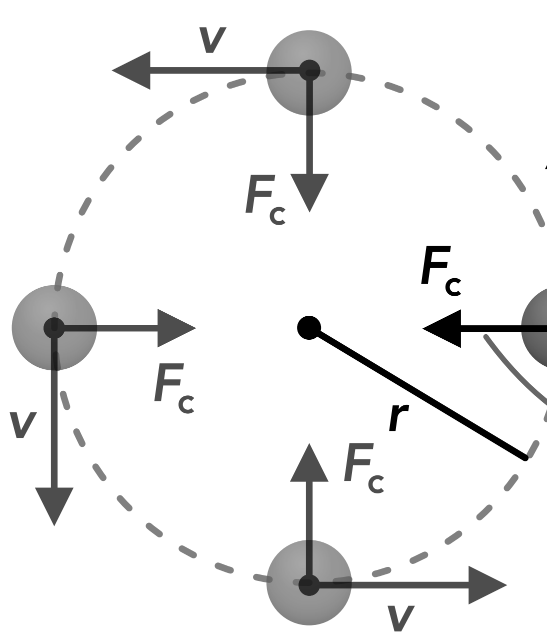
Centripetal Force

Variables		SI Unit
F_c	centripetal force	N
a_c	centripetal acceleration	$\frac{m}{s^2}$
m	mass	kg
v	velocity	$\frac{m}{s}$
r	radius	m

- When an object is in circular motion, there is an acceleration that points towards the center of the circle that is changing the direction of the velocity, which we call the centripetal acceleration.
- If there is an acceleration, Newton's 2nd law of motion ($\sum \vec{F} = m\vec{a}$) says that there must be a net force acting on the object that points in the same direction as the acceleration.
- The net force that causes a centripetal acceleration in circular motion is called the **centripetal force**.
- The direction of the centripetal force is towards the center of the circle (the same as the centripetal acceleration). We usually say this is the positive direction in Newton's 2nd law.
- The magnitude of the centripetal force is the mass multiplied by the centripetal acceleration (Newton's 2nd law).

Centripetal force

$$F_c = m\frac{v^2}{r}$$



Centripetal force causes centripetal acceleration

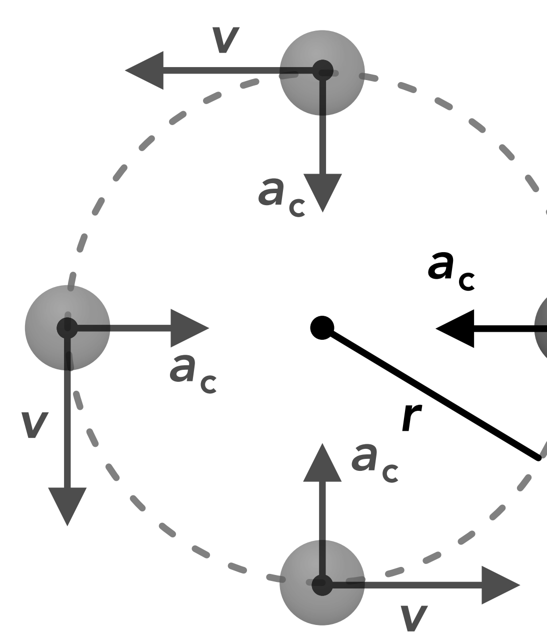
$$\sum \vec{F} = m\vec{a}$$

$$\vec{F}_c = m\vec{a}_c$$

$$F_c = m\frac{v^2}{r}$$

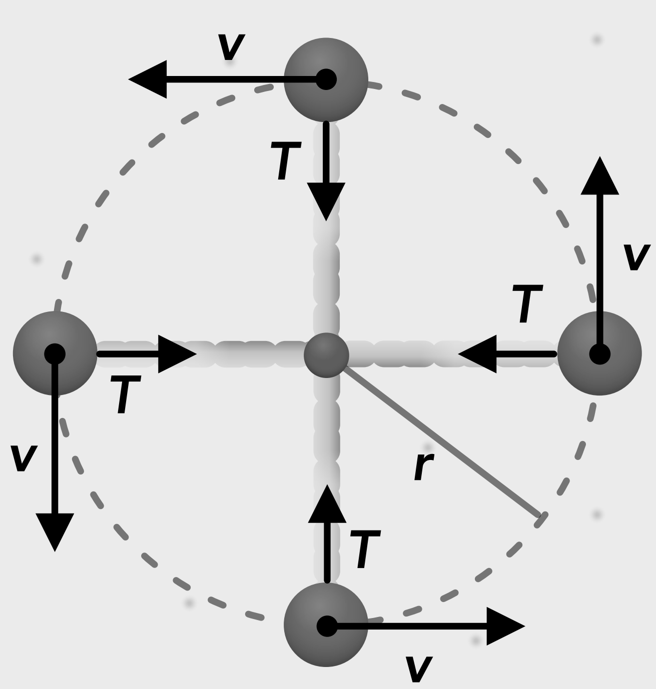
Centripetal acceleration

$$a_c = \frac{v^2}{r}$$

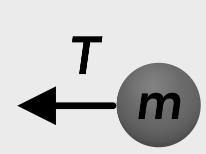


- The centripetal force could be a single force, a component of a force, or the sum of multiple forces or components of forces. In all of those cases, the net force acting towards the center of the circle is called the centripetal force.
- We usually apply Newton's 2nd law ($\sum \vec{F} = m\vec{a}$) in the radial direction and say the positive direction is towards the center of the circle, so the net force (the centripetal force) and the acceleration are both positive.

Example 1: A ball is attached to a string and is swinging around in uniform circular motion in space (assuming no gravity). The tension force from the string is the only force acting on the ball in the radial direction, so the tension force is the centripetal force.



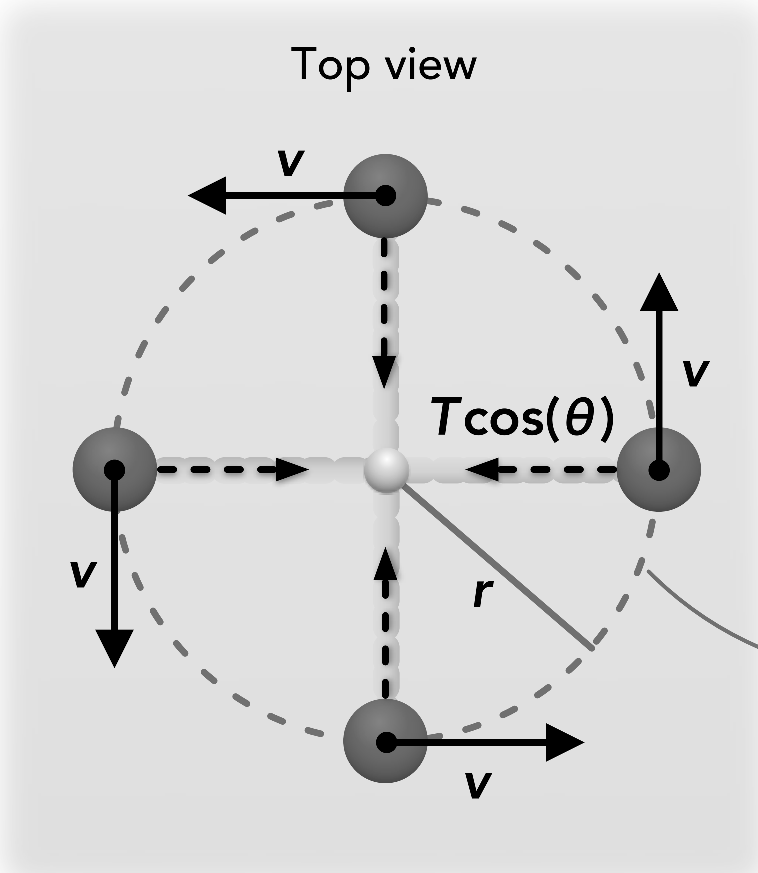
In the radial direction (towards the center of the circle is positive):

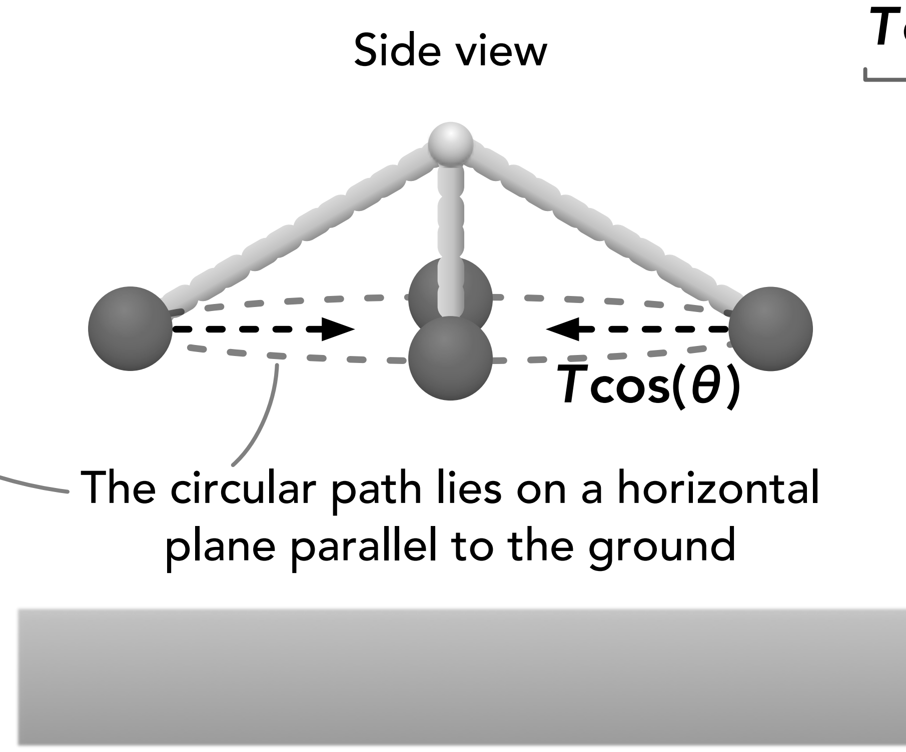


$$\sum F = ma$$

$$\underbrace{T}_{F_c} = m\underbrace{\frac{v^2}{r}}_{a_c}$$

Example 2: A ball is attached to a string and swings around in uniform circular motion. The circle is horizontal, parallel to the ground, and gravity causes the ball to pull the string down at an angle. The horizontal component of the tension force is the centripetal force, and it always points towards the center of the circle.

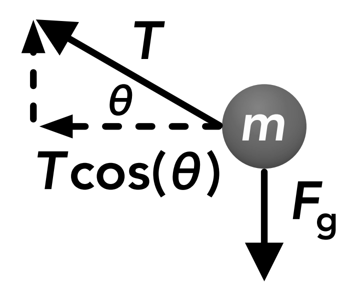




In the radial direction (towards the center of the circle is positive):

$$\sum F = ma$$

$$\underbrace{T\cos(\theta)}_{F_c} = m\underbrace{\frac{v^2}{r}}_{a_c}$$

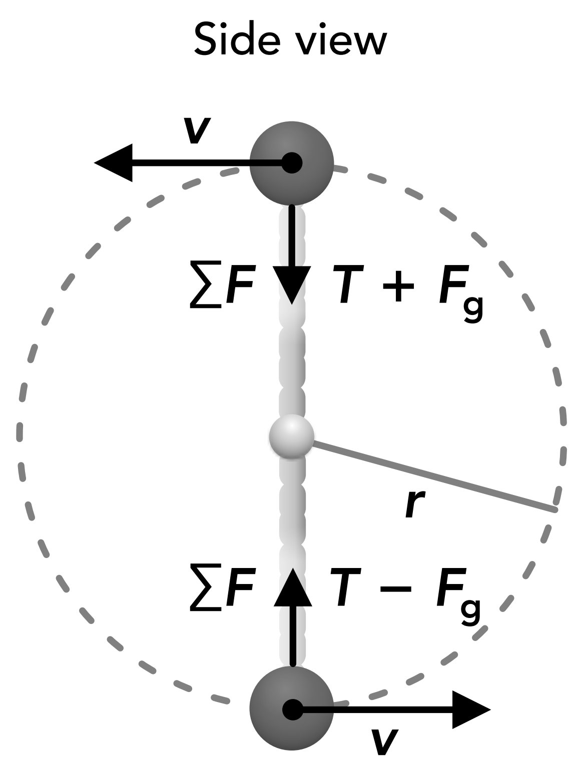


Example 3: A ball is attached to a string and swings around vertically in uniform circular motion. At each point in time there is a tension force and a gravitational force acting on the ball. Both forces contribute to the centripetal force (net force). The centripetal force (net force) is constant and the gravitational force does not change, so the tension force must change as the ball moves around.

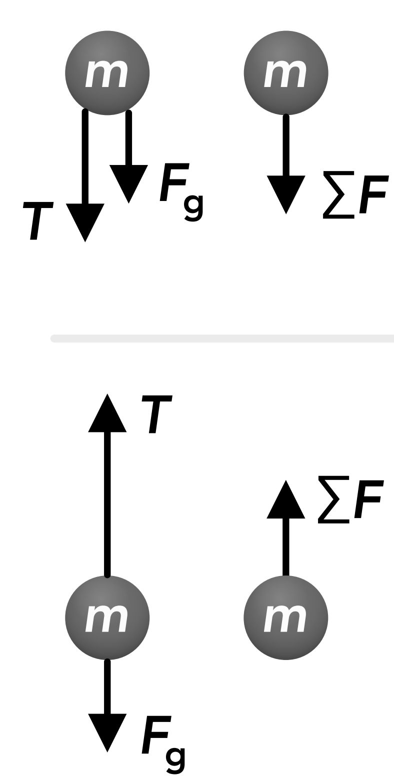
$\sum F$ (F_c) is the same at each point

F_g is the same at each point

T is greater when the ball is at the bottom of the circle



In the radial direction (towards the center of the circle is positive):



$$\sum F = ma$$

$$\underbrace{T + F_g}_{F_c} = m\underbrace{\frac{v^2}{r}}_{a_c}$$

$$\sum F = ma$$

$$\underbrace{T - F_g}_{F_c} = m\underbrace{\frac{v^2}{r}}_{a_c}$$

- A common point of confusion when working with circular motion is the concept of "centrifugal force". This is a "fictitious force" which is a force that does not actually exist. When the circular motion of an object is viewed in a rotating reference frame, in which the object appears to be stationary, it may seem that a force is pushing or pulling the object away from the center of the circle, which we call a centrifugal force. This imaginary force only arises because of the rotating reference frame.
- The only real force (or forces) acting on the object is a centripetal force, a net force that acts inwards towards the center of the circle.
- According to Newton's 1st law of motion an object will maintain its velocity (continue moving in a straight line at a constant speed) unless acted on by a net force. In circular motion we call that net force the centripetal force. If the centripetal force was removed, the object would travel in a straight line tangent to the circle because of its inertia, not because there is a force pulling or pushing it away from the circle.