

12: Probability of Single Events

Probability and Sample Space:

1. Probability is the measure of chance or the likelihood of an event happening.
2. In the study of probability, an **experiment** is an investigation from which the results are observed or recorded. An **outcome** is a possible result of an experiment.
3. A **sample space** is the set of all possible outcomes in the experiment. It is usually denoted by the letter S . Sample space can be written using the set notation, $\{ \}$.

For example:

- (a) Experiment 1: Tossing a coin
Possible outcomes are *head or tail*.
Sample Space, $S = \{head, tail\}$.
- (b) Experiment 2: Tossing a die
Possible outcomes are the numbers 1, 2, 3, 4, 5, and 6.
Sample Space, $S = \{1, 2, 3, 4, 5, 6\}$.
- (c) Experiment 3: Picking a card from a stack of six cards with letters that spell the word 'WINNER'.
Possible outcomes are $W, I, N, E, \text{ and } R$.
Sample Space, $S = \{W, I, N_1, N_2, E, R\}$. Note there are 2 cards with the letter 'N'.

Probability of Single Events:

4. In an experiment, an event is a result that we are interested in. In general, the probability of an event A , written as $P(A)$, is defined as:

$$P(A) = \frac{\text{Number of outcomes favourable to the occurrence of event } A}{\text{Total number of equally likely outcomes}} = \frac{n(A)}{n(S)}$$

5. For any event E to occur, $0 \leq P(E) \leq 1$. Note that,
 - $P(E) = 0$ if and only if the event E is an impossible event, i.e. it will **never** occur.
 - $P(E) = 1$ if and only if the event E is a certain event, i.e. it will **definitely** occur.

6. For two events A and B ,

- if events A and B are equally likely to occur, then $P(A) = P(B)$,
- if event A is more likely to occur than event B , then $P(A) > P(B)$.

Complementary Events:

7. The complement of an event A is the event that A does not occur and is denoted by A' . Two events are called complementary events if they are the only two possible outcomes.

8. In general, if the probability of an event A is written as $P(A)$, then the probability that event A would not occur is $P(A') = 1 - P(A)$.

For example, if the probability of drawing a red marble from a bag is $\frac{2}{5}$, then the probability of drawing a marble which is not red from the same bag will be $\frac{3}{5}$.

This is because:

$$\begin{aligned}P(\text{drawing a marble which is not red}) &= 1 - P(\text{drawing a red marble}) \\ &= 1 - \frac{2}{5} \\ &= \frac{3}{5}\end{aligned}$$

Continue on the next page.

Example Questions (Advanced):

1. A bag contains 25 balls of which x are black balls and the rest are white balls. After 5 black balls are removed from the bag, the probability of drawing a black ball from the bag is decreased by $\frac{1}{10}$. Calculate the value of x .

Solution:

$$P(\text{drawing a black ball before}) = \frac{x}{25}$$

$$\begin{aligned} P(\text{drawing a black ball after}) &= \frac{x-5}{25-5} \\ &= \frac{x-5}{20} \end{aligned}$$

An equation in x :

$$\frac{x-5}{20} + \frac{1}{10} = \frac{x}{25}$$

$$\frac{x-5}{20} + \frac{2}{20} = \frac{x}{25}$$

$$\frac{x-5+2}{20} = \frac{x}{25}$$

$$\frac{x-3}{20} = \frac{x}{25}$$

$$25(x - 3) = 20(x)$$

$$25x - 75 = 20x$$

$$5x = 75$$

$$\therefore x = 15$$

2. Rachel spins a biased 5-sided spinner, numbered 1 to 5. The probability of each number is shown in the table.

Number	1	2	3	4	5
Probability	$\frac{1}{3}$	x	$\frac{1}{4}$	$\frac{1}{6}$	x

- Find the value of x .
- Rachel spins the spinner once. Find the probability that the number is
 - 2 or 3,
 - odd,
 - not more than 4.
- If Rachel spins the spinner 60 times, find the expected number of times that the spinner will show 3 or 4.

Solution:

$$P(\text{spinning the number } 1, 2, 3, 4, 5) = 1$$

$$\begin{aligned} P(\text{spinning the number } 1, 3, 4) &= \frac{1}{3} + \frac{1}{4} + \frac{1}{6} \\ &= \frac{3}{4} \end{aligned}$$

$$\begin{aligned} P(\text{spinning the number } 2, 5) &= 1 - \frac{3}{4} \\ &= \frac{1}{4} \end{aligned}$$

An equation in x :

$$\begin{aligned} 2x &= \frac{1}{4} \\ \text{(a) } x &= \frac{1}{8} \end{aligned}$$

$$\begin{aligned} \text{(b)(i) } P(\text{spinning the number } 2, 3) &= \frac{1}{8} + \frac{1}{4} \\ &= \frac{3}{8} \end{aligned}$$

$$\begin{aligned} \text{(b)(ii) } P(\text{spinning an odd number}) &= P(\text{spinning the number } 1, 3, 5) \\ &= \frac{1}{3} + \frac{1}{4} + \frac{1}{8} \\ &= \frac{17}{24} \end{aligned}$$

$$\begin{aligned} \text{(b)(iii) } P(\text{spinning a number not more than } 4) &= P(\text{spinning the number } 1, 2, 3, 4) \\ &= \frac{1}{3} + \frac{1}{8} + \frac{1}{4} + \frac{1}{6} \\ &= \frac{7}{8} \end{aligned}$$

$$\begin{aligned} \text{(c) } P(\text{spinning the number } 3, 4) &= \frac{1}{4} + \frac{1}{6} \\ &= \frac{5}{12} \end{aligned}$$

There's a 5 in 12 chance of spinning the number 3 or 4.

∴ There's a 25 in 60 chance of spinning the number 3 or 4.

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Version 2