STUDY GUIDE

CONSERVATION OF MOMENTUM

Law of Conservation of Momentum

- When a scenario involves multiple objects we can consider them as a system.
- If there is **no external force** acting on the system we call it an **isolated sytem**.
- **The law of conservation of momentum**: the total momentum within an isolated system is conserved (it's constant over time) regardless of the internal interactions between the objects in the system. The momentum of each object is not conserved, only the total momentum of the system.
- This does not mean that the total momentum within a system is zero (although it can be), it means the change in momentum is zero between two times.
- If there is a net external force on the system, the system is not isolated and an impulse is exerted on the objects in the system, changing the total momentum.

Variables		SI Unit
p	momentum	$\frac{kg\cdot m}{s}$
m	mass	kg
v	velocity	$\frac{m}{s}$
J	impulse	$\frac{kg\cdot m}{s}$
F	force	N
t	time	s

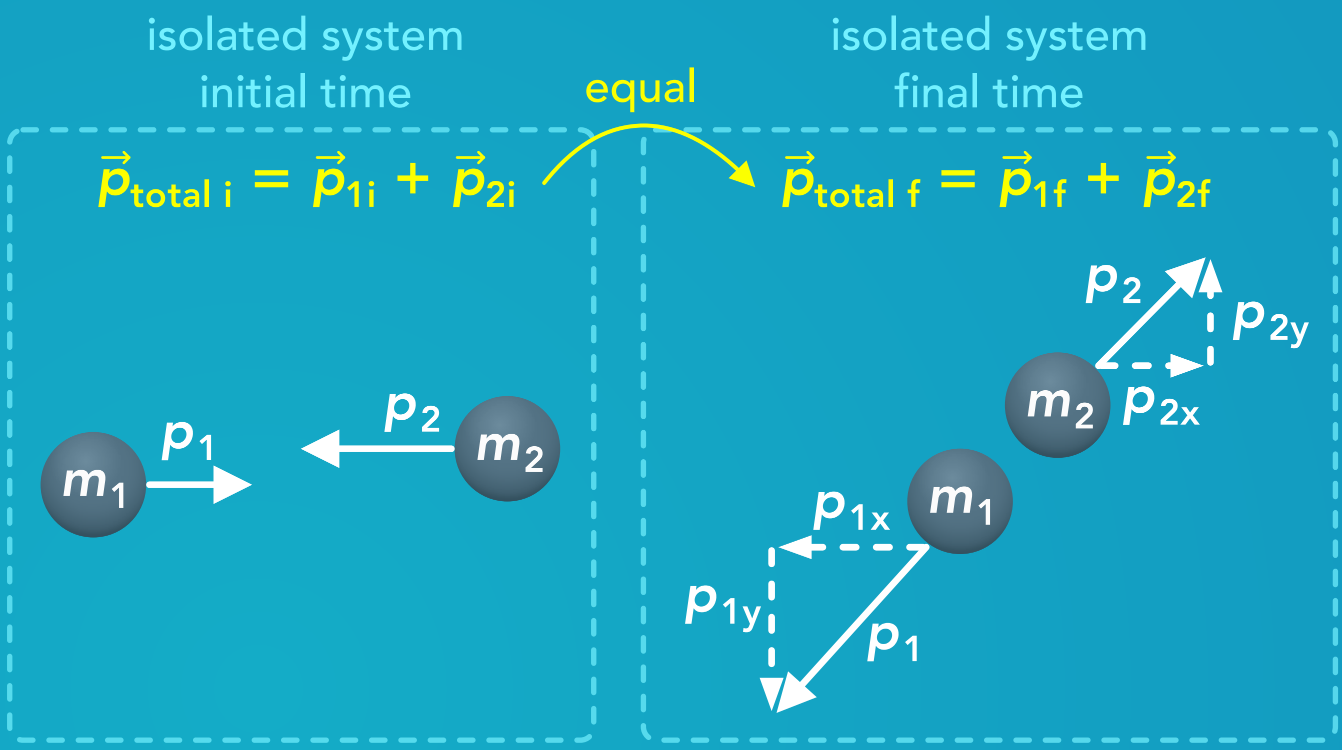
Law of conservation of momentum (universe and isolated systems)

$$\Delta\vec{p}_{total} = 0 \quad , \quad \vec{p}_{total\ i} = \vec{p}_{total\ f}$$

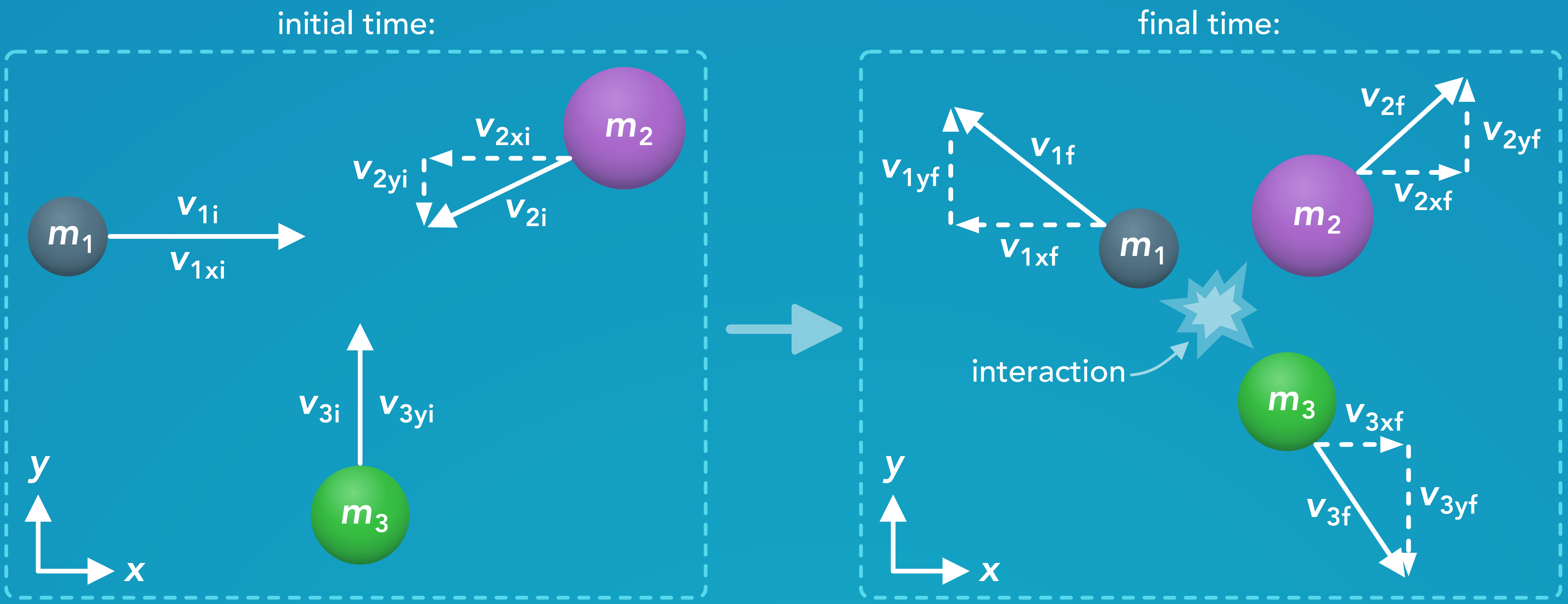
$$\Delta p_{x\ total} = 0 \quad , \quad p_{xi\ total} = p_{xf\ total}$$

$$\Delta p_{y\ total} = 0 \quad , \quad p_{yi\ total} = p_{yf\ total}$$

Isolated system: there is no net external force acting on the system



Momentum is a vector so momentum is conserved in the *x* and *y* directions



$$\vec{p}_{\text{total i}} = \vec{p}_{\text{total f}}$$

x momentum components:

$$p_{xi\ \text{total}} = p_{xf\ \text{total}}$$

the initial *x* momentum components for each object → $p_{1xi} + p_{2xi} + p_{3xi} = p_{1xf} + p_{2xf} + p_{3xf}$ ← the final *x* momentum components for each object

$$m_1 v_{1xi} + m_2 v_{2xi} + m_3 v_{3xi} = m_1 v_{1xf} + m_2 v_{2xf} + m_3 v_{3xf}$$

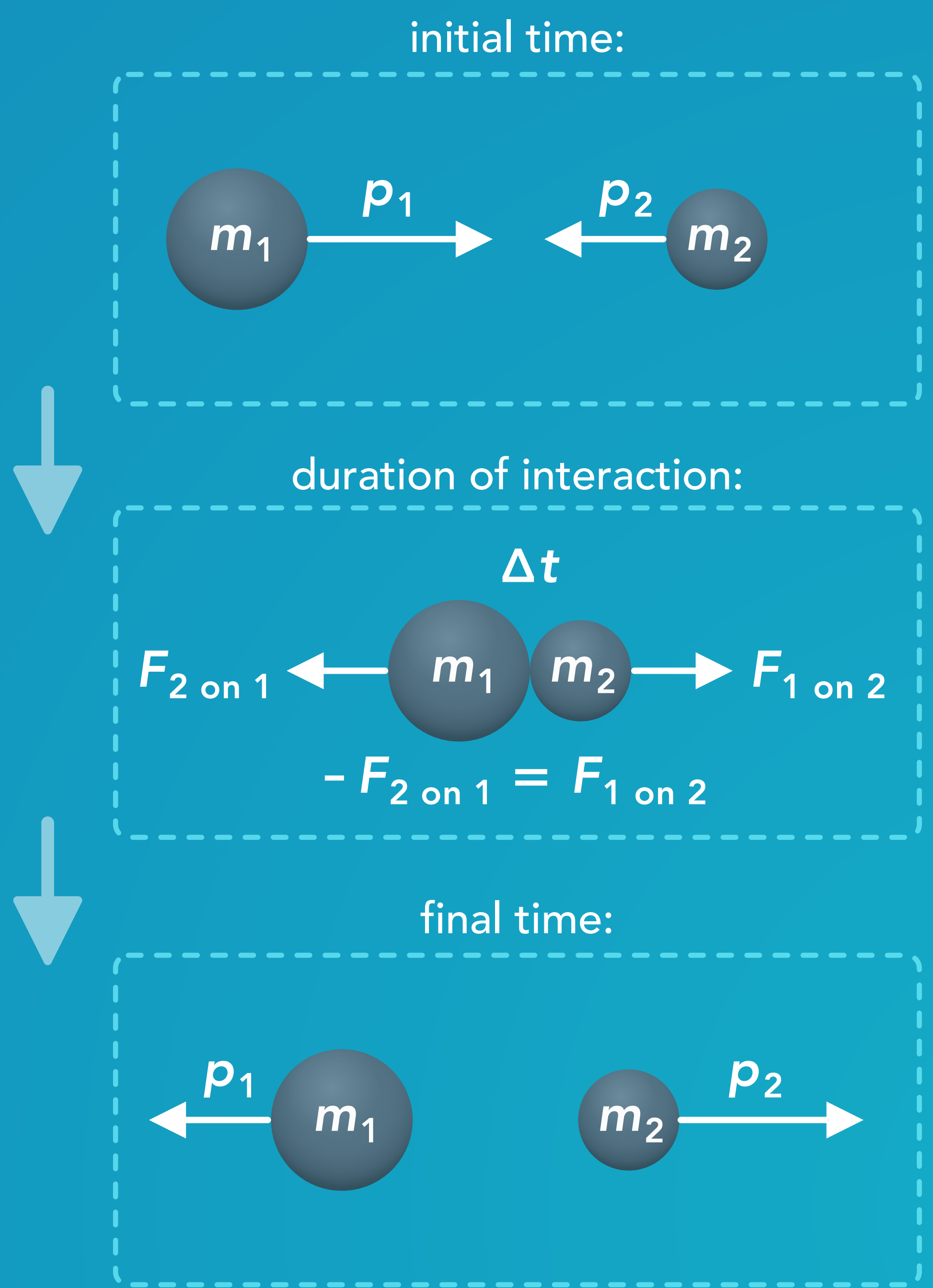
y momentum components:

$$p_{yi\ \text{total}} = p_{yf\ \text{total}}$$

the initial *y* momentum components for each object → $p_{1yi} + p_{2yi} + p_{3yi} = p_{1yf} + p_{2yf} + p_{3yf}$ ← the final *y* momentum components for each object

$$m_1 v_{1yi} + m_2 v_{2yi} + m_3 v_{3yi} = m_1 v_{1yf} + m_2 v_{2yf} + m_3 v_{3yf}$$

- When two objects **inside** a system interact, the forces they exert on each other (internal forces) are a pair of forces which are equal in magnitude and opposite in direction (Newton's 3rd law of motion).
- If the force acting on one object has the same magnitude and is exerted for the same duration as the force on the other object, the impulse exerted on each object is the same but in opposite directions. This means the net impulse on the pair of objects is zero, so **the total change in momentum is zero**.



$\vec{p}_{\text{total i}} = \vec{p}_{1i} + \vec{p}_{2i}$

the impulse exerted on each object is the same (but in opposite directions)

$J_1 = -F_{2\text{ on }1}\Delta t$

$J_2 = F_{1\text{ on }2}\Delta t$

$J_{\text{total}} = J_1 + J_2 = 0 \longrightarrow \Delta p_{\text{total}} = 0$

the total change in momentum of the two objects is zero

$\vec{p}_{\text{total f}} = \vec{p}_{1f} + \vec{p}_{2f}$

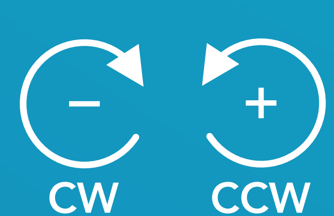
- **The law of conservation of angular momentum**: the total angular momentum of an isolated system is conserved (it's constant over time) regardless of the interactions within the system.
- For rotation, an **isolated system** means there is **no net external torque** acting on the system.
- Angular momentum is a **vector** which has the same direction as the angular velocity (either clockwise or counterclockwise).

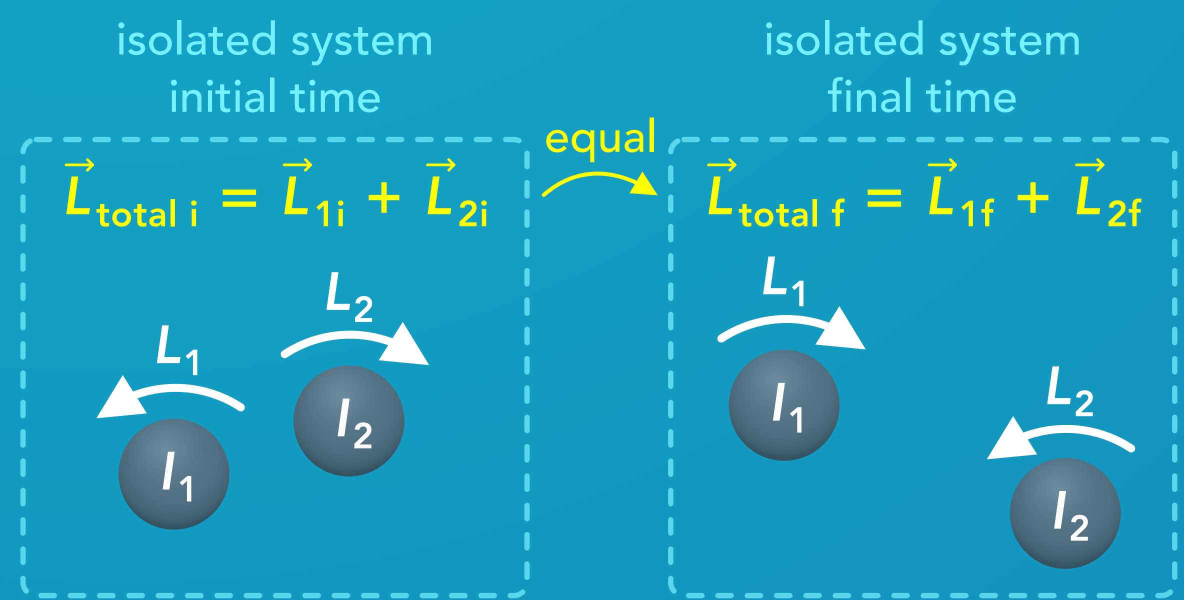
Variables		SI Unit
L	angular momentum	$\frac{kg \cdot m^2}{s}$
I	moment of inertia	$kg \cdot m^2$
ω	angular velocity	$\frac{rad}{s}$

Law of conservation of angular momentum (universe and isolated systems)

$$\Delta \vec{L}_{\text{total}} = 0 \quad , \quad \vec{L}_{\text{total i}} = \vec{L}_{\text{total f}}$$

Isolated system: there is no net external torque acting on the system

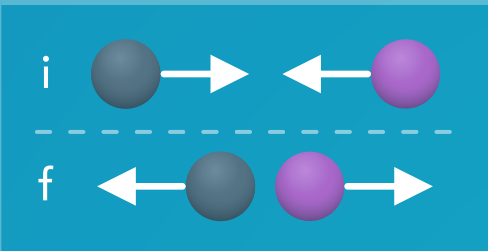
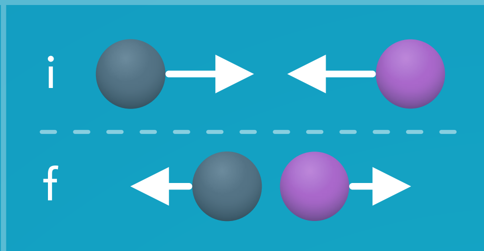
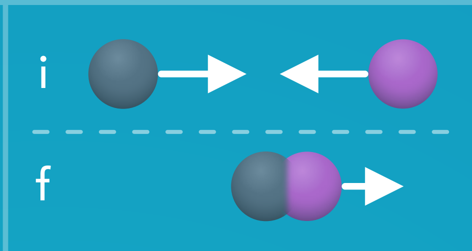
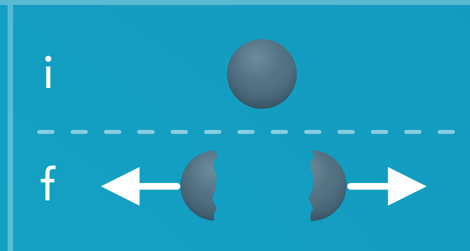




Types of Collisions & Events

Variables		SI Unit
p	momentum	$\frac{\text{kg}\cdot\text{m}}{\text{s}}$
K	kinetic energy	J

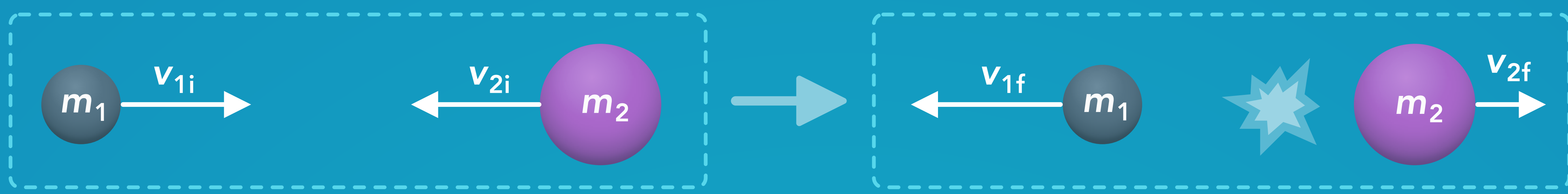
- There are several types of “events” which can be studied using the law of conservation of momentum such as collisions and explosions.
- If the system is defined so that all of the relevant objects are included within the system and there are no external forces acting on the system, then **the total momentum in the system is conserved** and we can use that law to analyze the individual objects before and after the event.
- For an elastic collision the total kinetic energy of the system is also conserved.

Elastic collision (perfectly elastic)	Inelastic collision (partially elastic)	Perfectly inelastic collision	Explosion
			
momentum is conserved $\vec{p}_{\text{total i}} = \vec{p}_{\text{total f}}$	momentum is conserved $\vec{p}_{\text{total i}} = \vec{p}_{\text{total f}}$	momentum is conserved $\vec{p}_{\text{total i}} = \vec{p}_{\text{total f}}$	momentum is conserved $\vec{p}_{\text{total i}} = \vec{p}_{\text{total f}}$
kinetic energy is conserved $K_{\text{total i}} = K_{\text{total f}}$	kinetic energy is NOT conserved $K_{\text{total i}} \neq K_{\text{total f}}$	kinetic energy is NOT conserved $K_{\text{total i}} \neq K_{\text{total f}}$	kinetic energy is NOT conserved $K_{\text{total i}} \neq K_{\text{total f}}$

Elastic collision (perfectly elastic collision)

- An **elastic collision** (a perfectly elastic collision) is when two or more objects collide with each other and then move away from each other (or in the same direction at different speeds).
- **The total kinetic energy of the system is conserved** in a perfectly elastic collision.

initial time: final time:



$$\vec{p}_{\text{total i}} = \vec{p}_{\text{total f}}$$

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \longleftarrow \text{(equation 1)}$$

system of two equations can be used to solve for unknown variables

$$K_{\text{total i}} = K_{\text{total f}}$$

$$K_{1i} + K_{2i} = K_{1f} + K_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \longleftarrow \text{(equation 2)}$$

By combining the two equations above (conservation of momentum and conservation of kinetic energy) using substitution and some algebra, we get an equation that we can use when two objects collide elastically and we don't know either of the final velocities. This can then be used with the conservation of momentum as a simpler set of two equations to solve.

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \longleftarrow \text{(equation 1)}$$

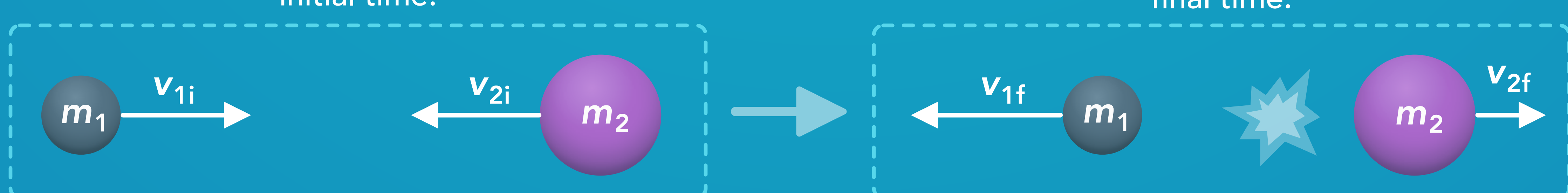
$$v_{1i} + v_{1f} = v_{2i} + v_{2f} \quad \longleftarrow \text{(equation 3)}$$

if both final velocities are unknown (elastic collision only)

Inelastic collision (partially elastic collision)

- An **inelastic collision** (a partially elastic collision) is when two or more objects collide with each other and then move away from each other (or in the same direction at different speeds).
- **The total kinetic energy of the system is not conserved** in an inelastic collision. Some of the initial kinetic energy is converted into thermal energy, sound energy, light energy or energy that deforms the objects.

initial time: final time:



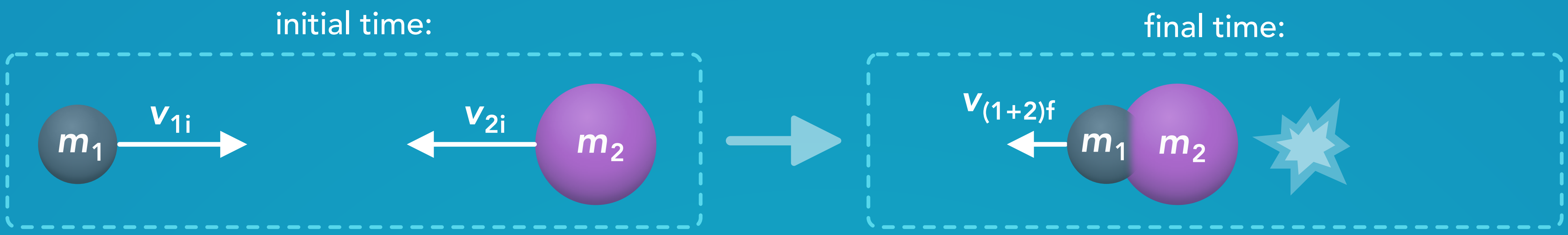
$$\vec{p}_{\text{total i}} = \vec{p}_{\text{total f}}$$

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Perfectly inelastic collision

- A **perfectly inelastic collision** is when two or more objects collide with each other and stick together, so the objects move together with the **same final velocity** after the collision and can be treated as a single object.
- **The total kinetic energy of the system is not conserved** in a perfectly inelastic collision. Some or all of the initial kinetic energy is converted into thermal energy, sound energy, light energy or energy that deforms the objects.

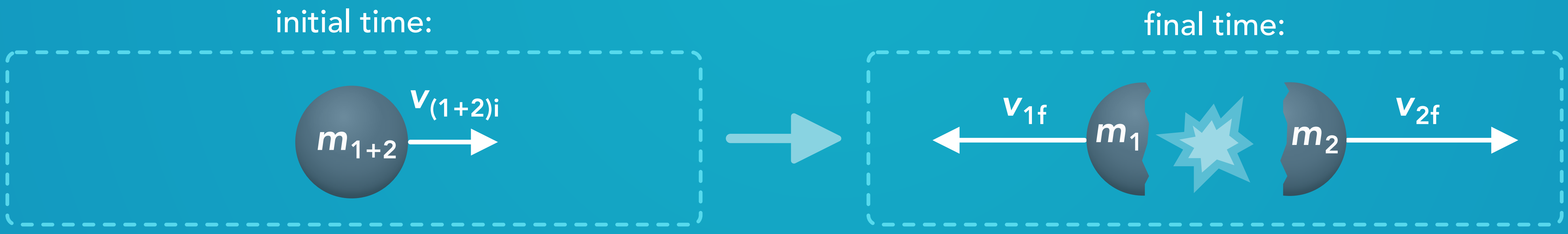


$$\vec{p}_{\text{total i}} = \vec{p}_{\text{total f}}$$
$$p_{1i} + p_{2i} = p_{(1+2)f}$$
$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_{(1+2)f}$$

the two objects stick together and move as one object

Explosion

- An **explosion** is when one object breaks apart into smaller pieces or a group of objects start together and then move away from each other. The object or group of objects may have some velocity before the explosion.
- **The total kinetic energy of the system is not conserved** in an explosion. Since kinetic energy is a scalar quantity and not a vector quantity it does not depend on direction and all of the kinetic energies will be positive. We can imagine an event where the initial kinetic energy is zero but there are final kinetic energies after the explosion.



$$\vec{p}_{\text{total i}} = \vec{p}_{\text{total f}}$$
$$p_{(1+2)i} = p_{1f} + p_{2f}$$
$$(m_1 + m_2) v_{(1+2)i} = m_1 v_{1f} + m_2 v_{2f}$$

the pieces or group start together as one object