SPRING FORCE & HOOKE'S LAW

Spring Force and Hooke's Law

Variables		SI Unit
F_{sp}	spring force	N
Δx	displacement	m
k	spring constant	$\frac{N}{m}$

- A spring **changes length when a force is applied to both ends**. If the forces pull the ends away from each other the spring gets longer. If the forces push the ends together the spring gets shorter.
- In the real world there are different types of springs with different behaviors, and all materials actually behave similar to springs. But we usually start out by working with "ideal springs".
- An **ideal spring** is…
  - massless: the spring itself has no mass, no inertia and no weight
  - frictionless: there are no friction forces acting on or within the spring itself
  - linearly elastic / follows Hooke's law: the change in length is linearly proportional to the applied force

- **Hooke's Law** states that the magnitude of the force required to stretch or compress a spring by a displacement of Δx is linearly proportional to that the displacement, $F_{sp} = k\Delta x$, where k is the **spring constant** or **stiffness** of the spring.

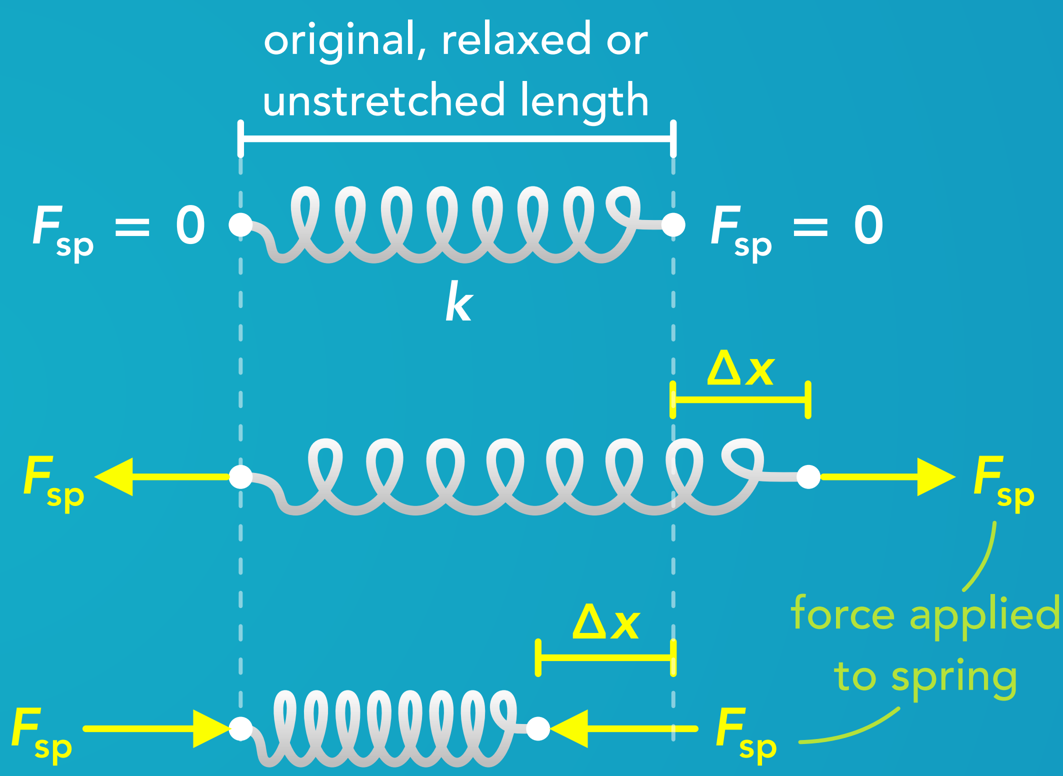
Spring force (Hooke's Law)

$$F_{sp} = k\Delta x$$

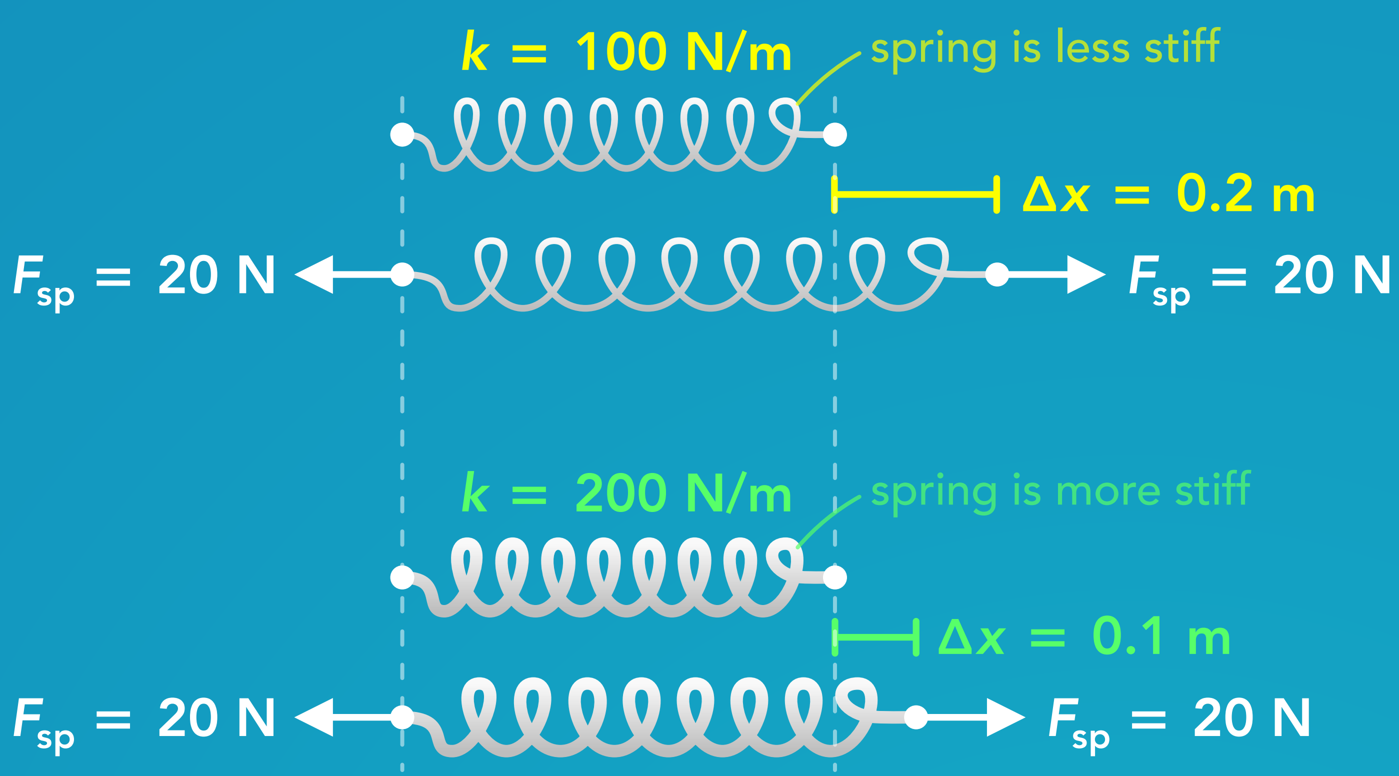
When no force is applied the spring is at its original length, relaxed length or unstretched length

When a tension (pulling) force is applied to both ends the spring gets longer by a change of Δx

When a compression (pushing) force is applied to both ends the spring gets shorter by a change of Δx



- The **spring constant** ***k*** is a value that represents the stiffness of a particular spring. A spring that is more stiff has a higher spring constant and requires more force to cause the same displacement as a spring that is less stiff and has a lower spring constant.
- The spring constant has a unit of Newtons/meter (N/m) given by units of force and displacement in Hooke's law.



$F_{sp} = k\Delta x$

$(20\text{ N}) = (100\text{ N/m})\Delta x$

$\Delta x = 0.2\text{ m}$

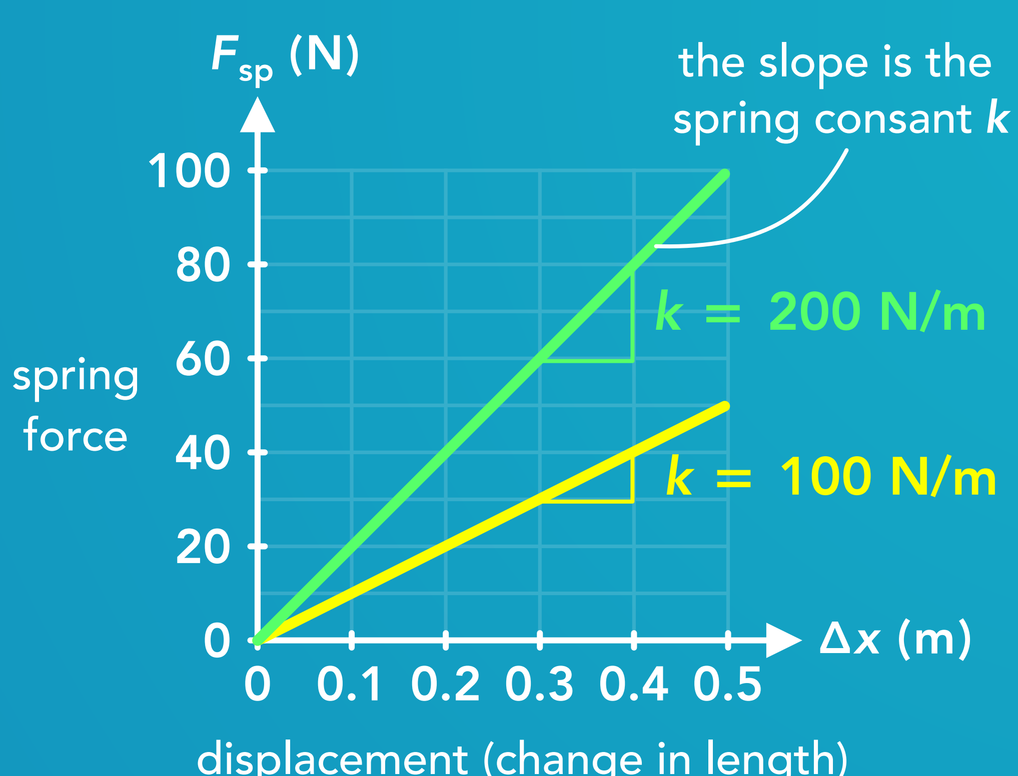
$F_{sp} = k\Delta x$

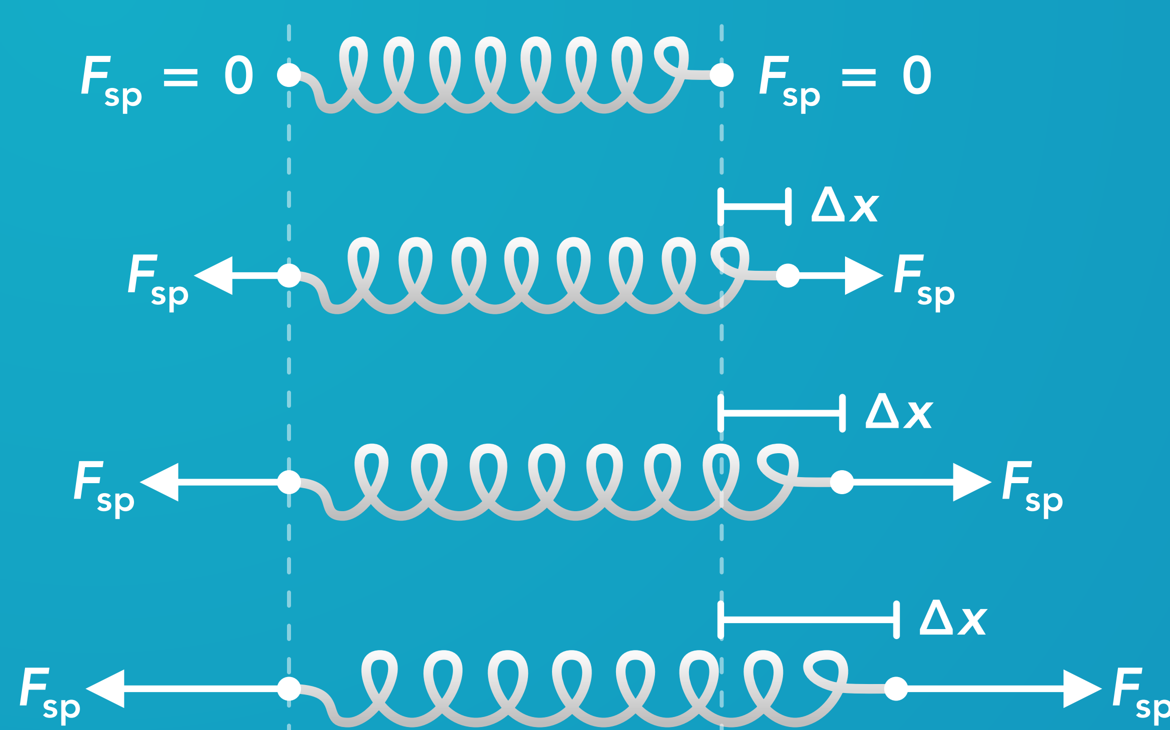
$(20\text{ N}) = (200\text{ N/m})\Delta x$

$\Delta x = 0.1\text{ m}$

- Since the change in length Δx is linearly proportional to the spring force F_{sp}, a graph of the spring force vs the displacement is a straight line.
- If the spring force is on the vertical axis and the displacement is on the horizontal axis, **the slope of the graph is the spring constant *k***. If the axes are flipped the slope is $1/k$.

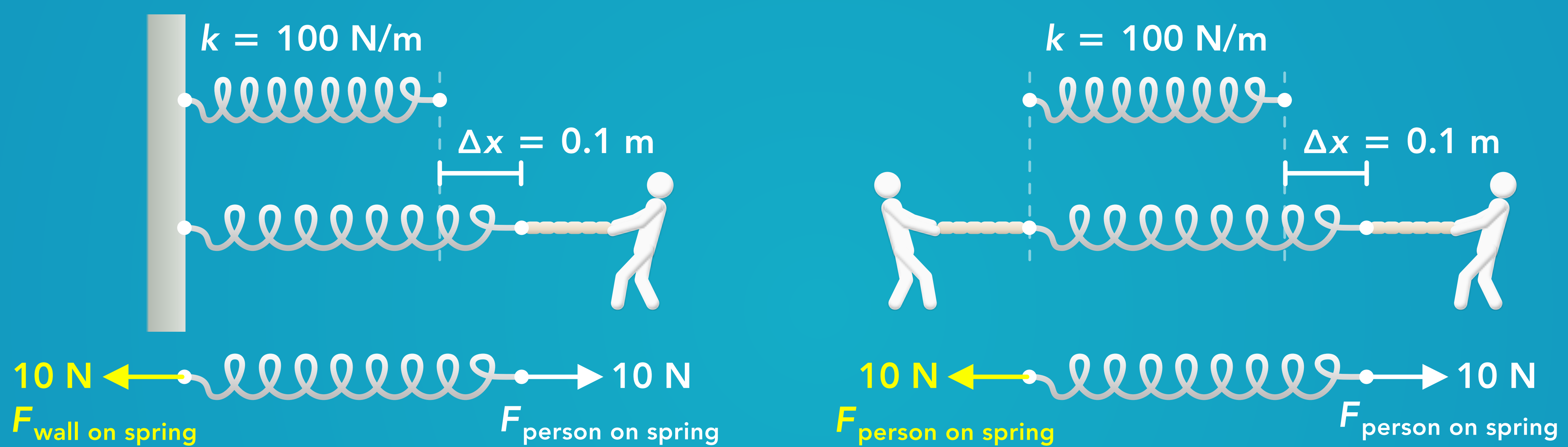
Hooke's Law: $F_{sp} = k\Delta x$





- A source of common confusion is the direction and magnitude of the spring force.
- A "spring force" is not a fundamental type of force like the gravitational force. When we use the term "spring force" we either mean a force exerted on the spring by an object, or the force exerted on an object by the spring.
- First, the forces acting on each end of a spring are equal in magnitude and opposite in direction. Not because they are a pair of equal and opposite forces as described in Newton's 3rd law of motion, but because we're treating the spring as ideal and **we're assuming the net force acting on the spring is zero**. In cases where the the spring is in static equilibrium and not moving (and therefore not accelerating) this must be true according to Newton's 2nd law of motion, $F_{net} = ma$. Even in cases where one or both ends of the spring are accelerating and the forces acting on the ends are changing, an ideal spring has no mass and **we assume it instantaneously transmits forces from one end to the other**. This is the same thing that happens for an ideal rope when working with tension forces, so you can think of the "spring force" on an object like a tension force acting on the object.
- Even when one end of the spring is fixed to a wall or a non-moving object, the wall still exerts a force on the spring just like if it were being pulled or pushed by a person or some other more "visible" force. If the wall was not exerting this force, the net force would not be zero and the spring would accelerate. Again, this is the same thing that happens with the tension force in a rope.

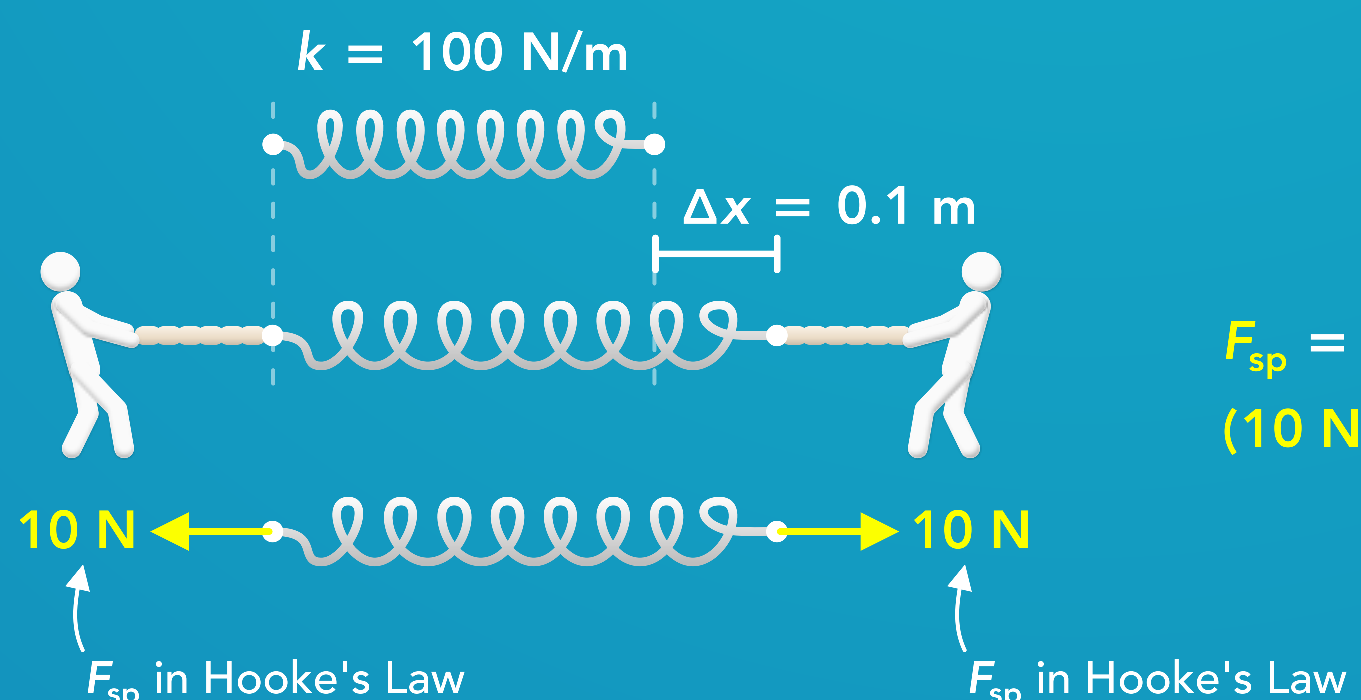
In both cases the spring is in static equilibrium (not moving) so the net force acting on the spring is zero. The wall exerts a force on the spring just like if it were pulled or pushed by a person.



$$\sum F_x = ma_x$$
$$10\text{ N} - 10\text{ N} = m(0\text{ m/s}^2)$$

$$\sum F_x = ma_x$$
$$10\text{ N} - 10\text{ N} = m(0\text{ m/s}^2)$$

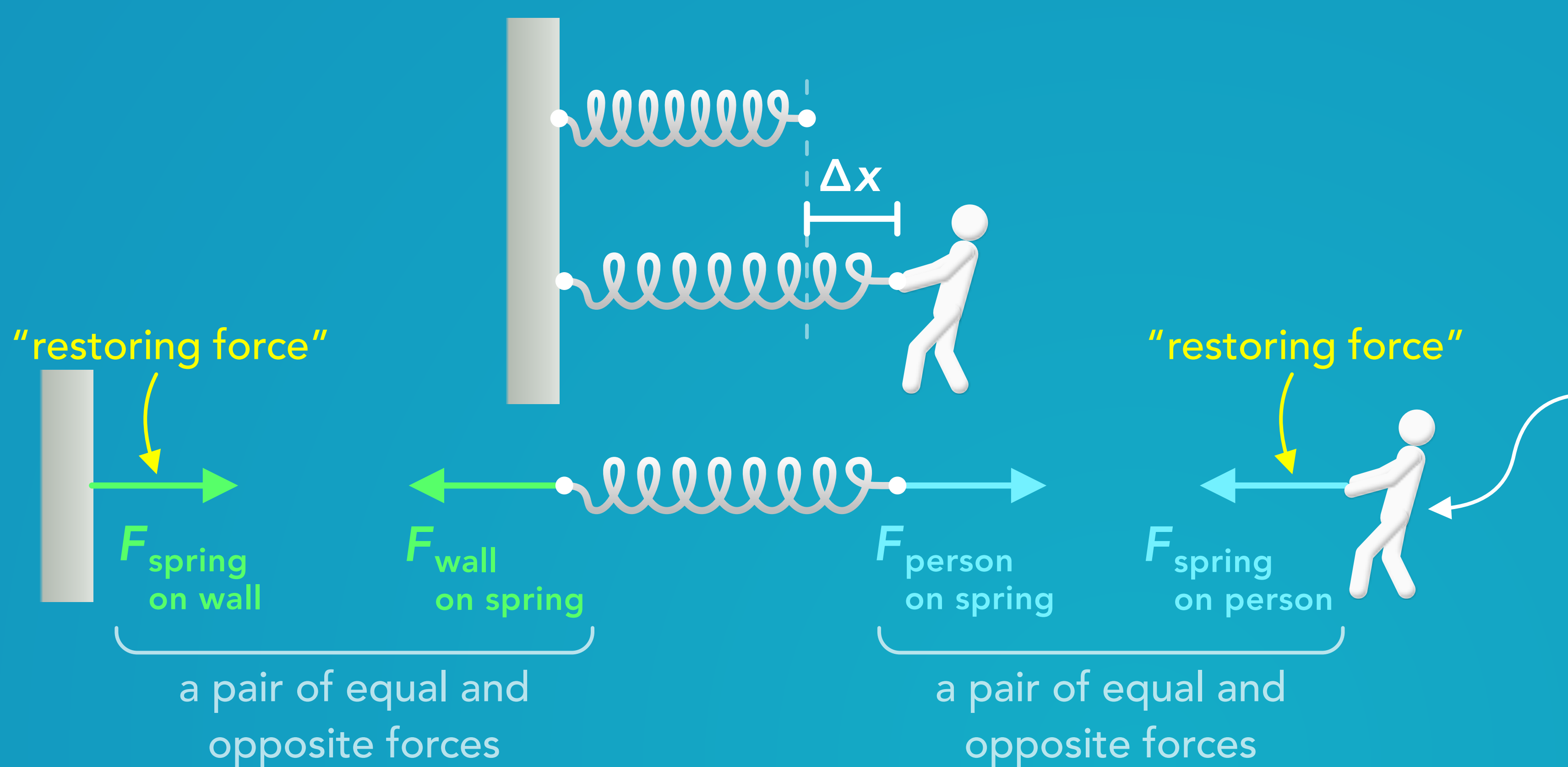
- Second, in the context of Hooke's Law the spring force F_{sp} refers to **the magnitude of the force acting on each end of a spring** (they're the same). We don't double the force or add the forces from each end together.



$$F_{sp} = k\,\Delta x$$
$$(10\text{ N}) = (100\text{ N/m})(0.1\text{ m})$$

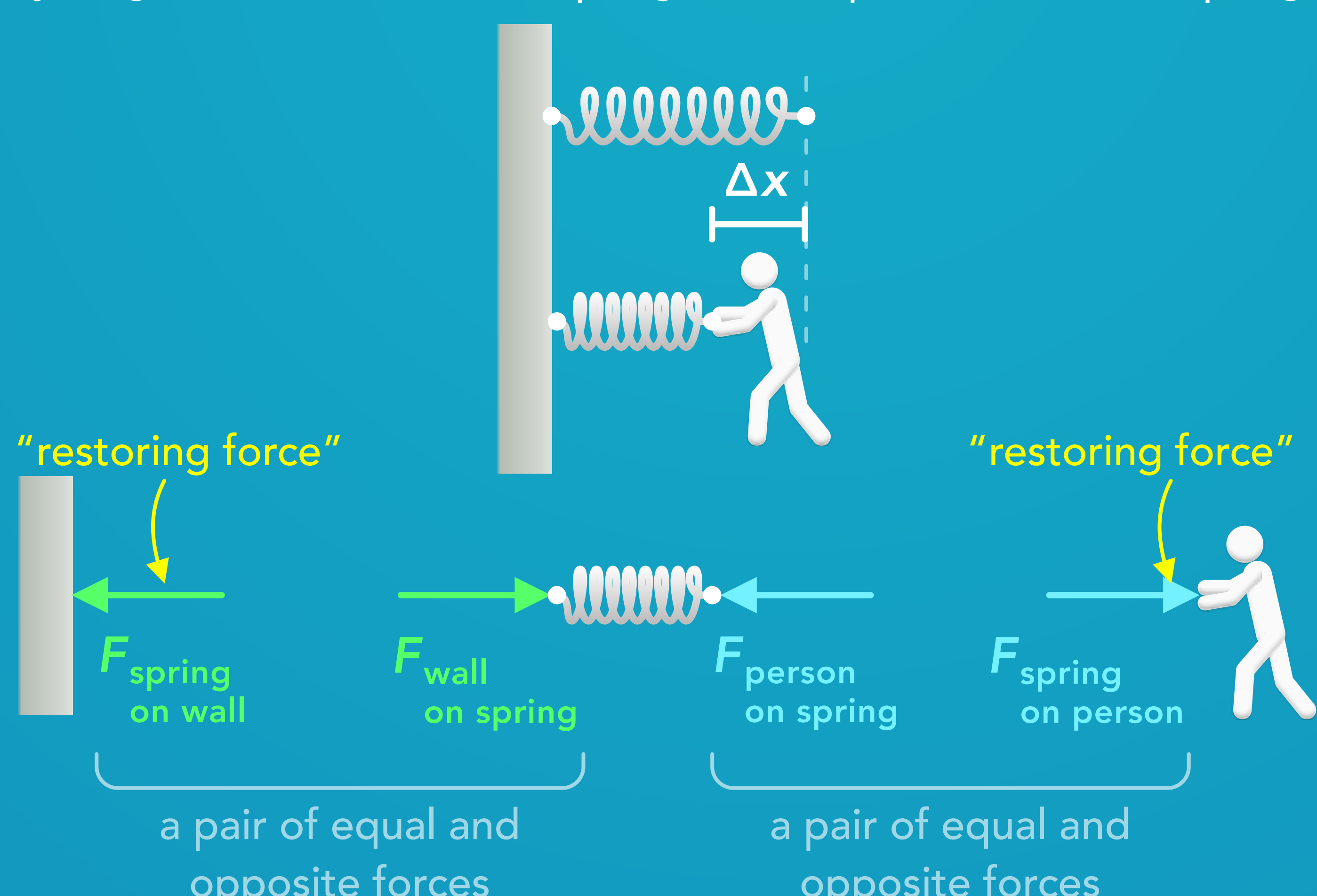
- Third, when we use the term "spring force" we need to be specific about which object the force is exerted on and which object is causing the force. When a spring is attached to an object the spring and the object exert contact forces on each other. The force exerted on the spring by the object is equal and opposite to the force exerted on the object by the spring (these are a pair of forces as described in Newton's 3rd law of motion).
- When a spring changes length, **it also exerts a force on the objects it's in contact with** (again this is just from Newton's 3rd law of motion). The force exerted by the spring on an object is called the **restoring force** because this force is trying to restore the spring to its original length.
- In most cases, we're focused on an object that is in contact with a spring, not the spring itself. In those scenarios we usually call the force exerted on the object by the spring the "spring force" $\boldsymbol{F}_{sp}$.
- It's also important to clearly label the forces in a free body diagram so we know what a force is acting on and what is causing the force. Remember, the free body diagram for an object only shows the forces acting on that object.

Free body diagrams of the wall, the spring and the person when the spring is stretched

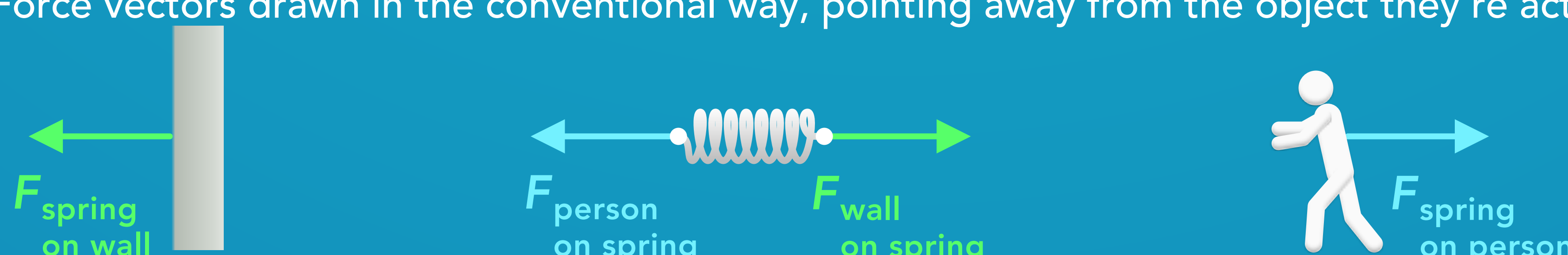


We're usually focused on the object in contact with the spring, not the spring itself. A free body diagram of the object shows the "restoring force" exerted on the object by the spring, which is equal and opposite to the force exerted on the spring by the object, which is the force in Hooke's Law.

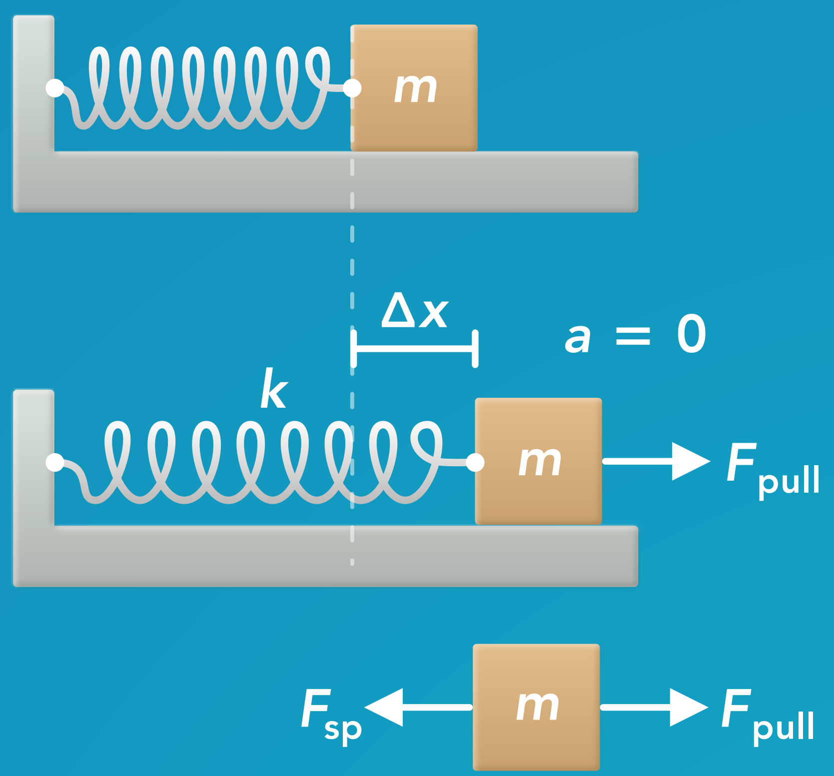
Free body diagrams of the wall, the spring and the person when the spring is compressed



Force vectors drawn in the conventional way, pointing away from the object they're acting on:



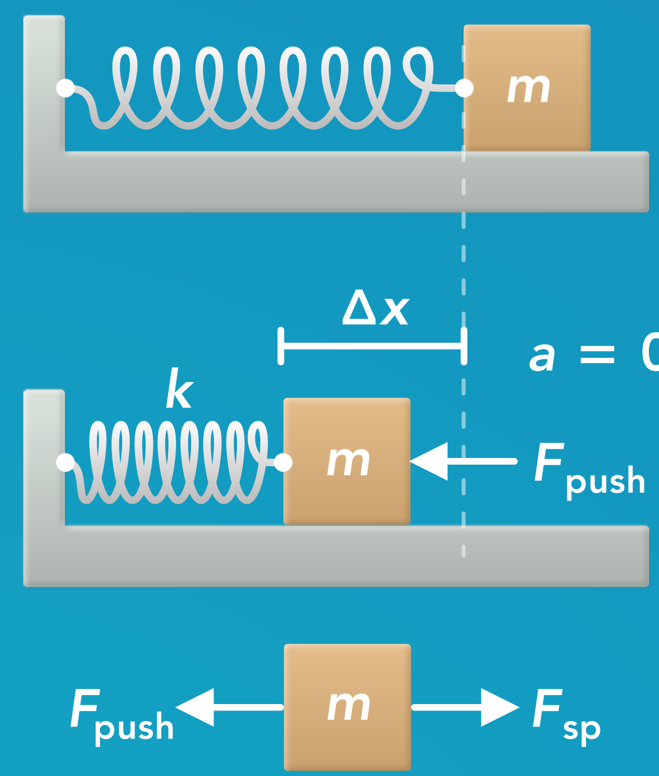
Examples of free body diagrams and Newton's 2nd law involving spring forces



$$\sum F_x = ma_x$$

$$F_{pull} - F_{sp} = m(0)$$

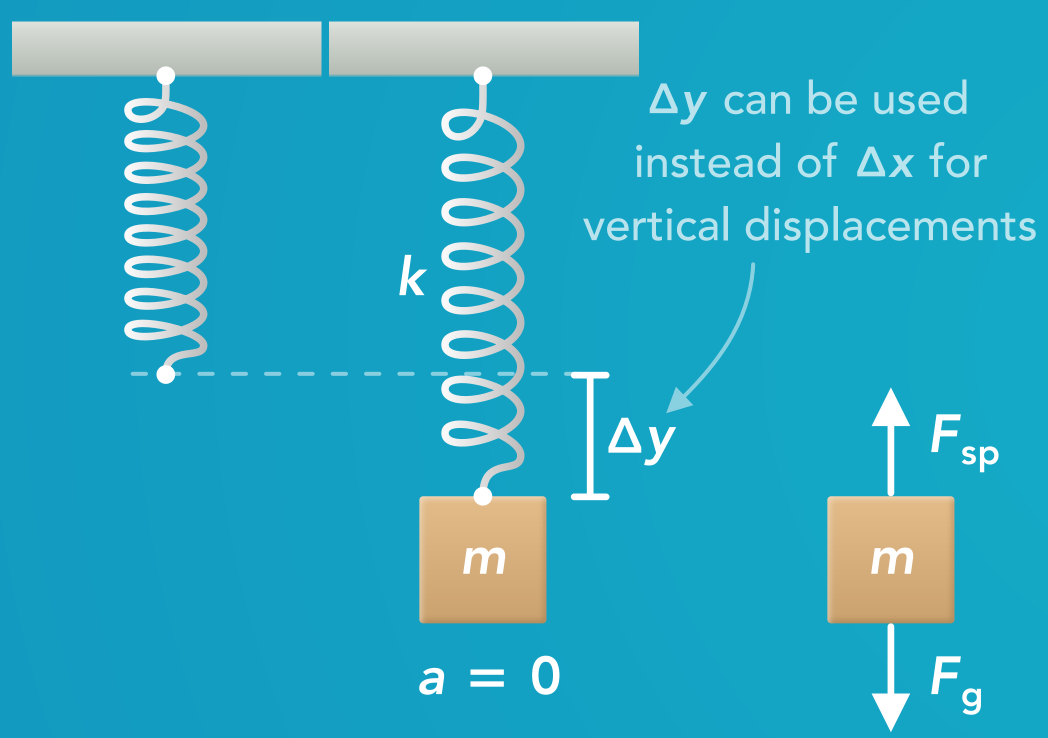
$F_{sp} = k\,\Delta x$ $\quad F_{pull} - (k\Delta x) = 0$



$$\sum F_x = ma_x$$

$$F_{sp} - F_{push} = m(0)$$

$F_{sp} = k\,\Delta x$ $\quad (k\Delta x) - F_{push} = 0$

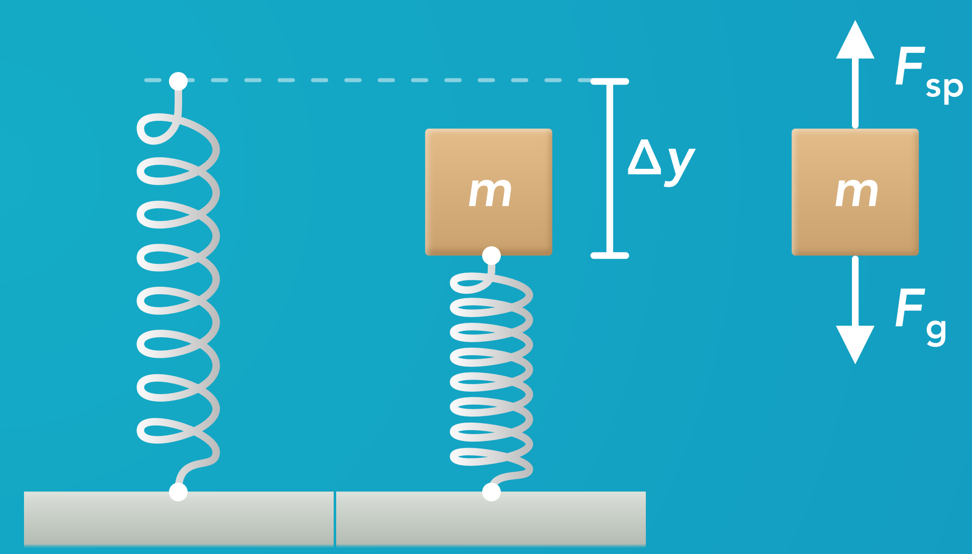


$$\sum F_y = ma_y$$

$$F_{sp} - F_g = m(0)$$

$F_{sp} = k\,\Delta y$ $\quad (k\Delta y) - (mg) = 0$

$F_g = mg$



$$\sum F_y = ma_y$$

$$F_{sp} - F_g = m(0)$$

$F_{sp} = k\,\Delta y$ $\quad (k\Delta y) - (mg) = 0$

$F_g = mg$