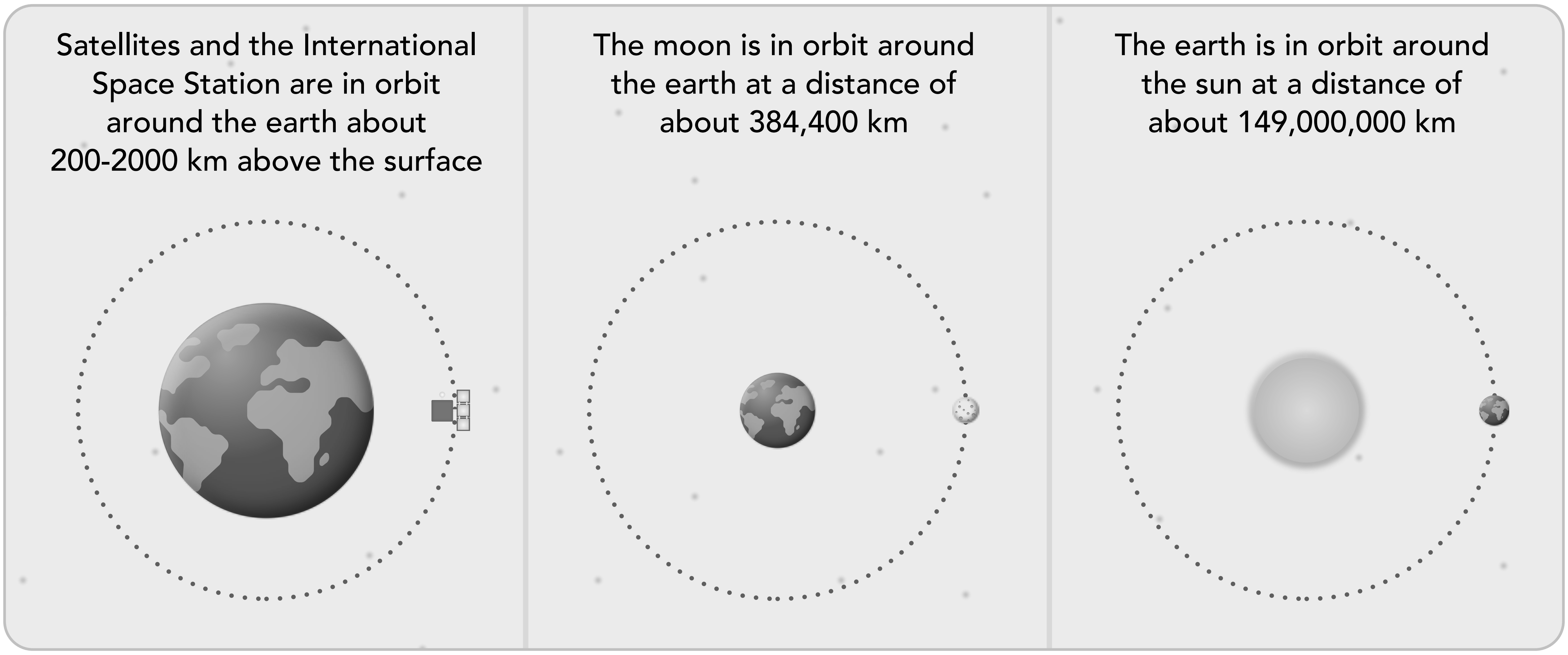
STUDY GUIDE

ORBITAL MOTION

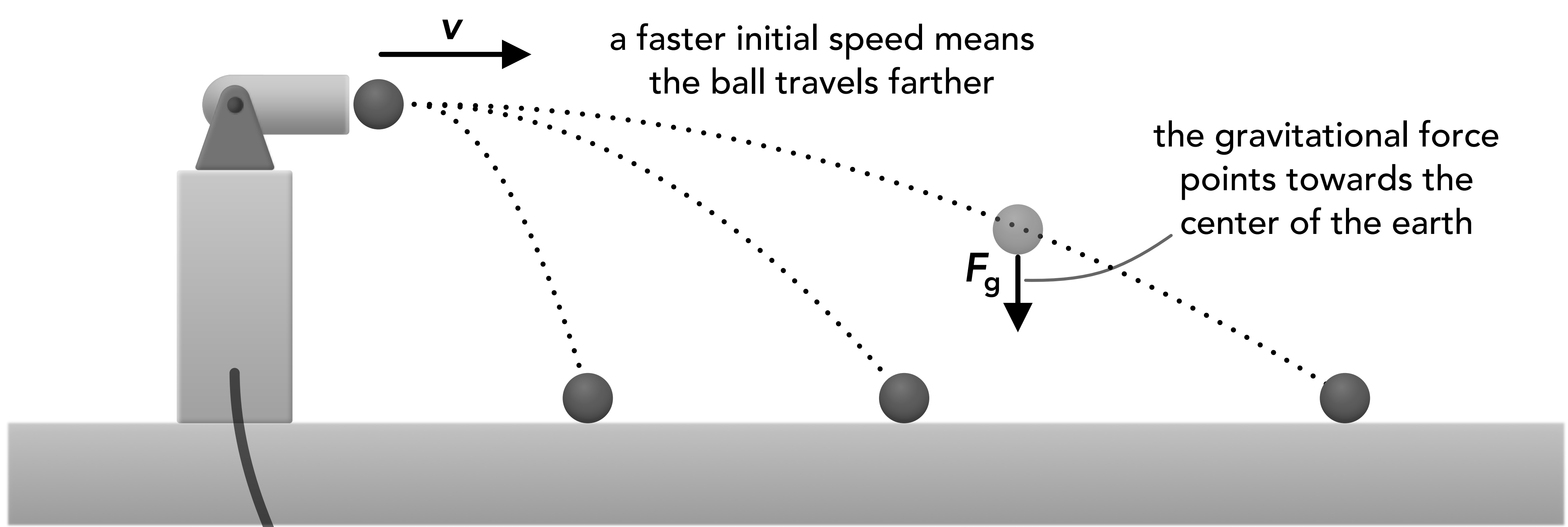
Orbital Motion

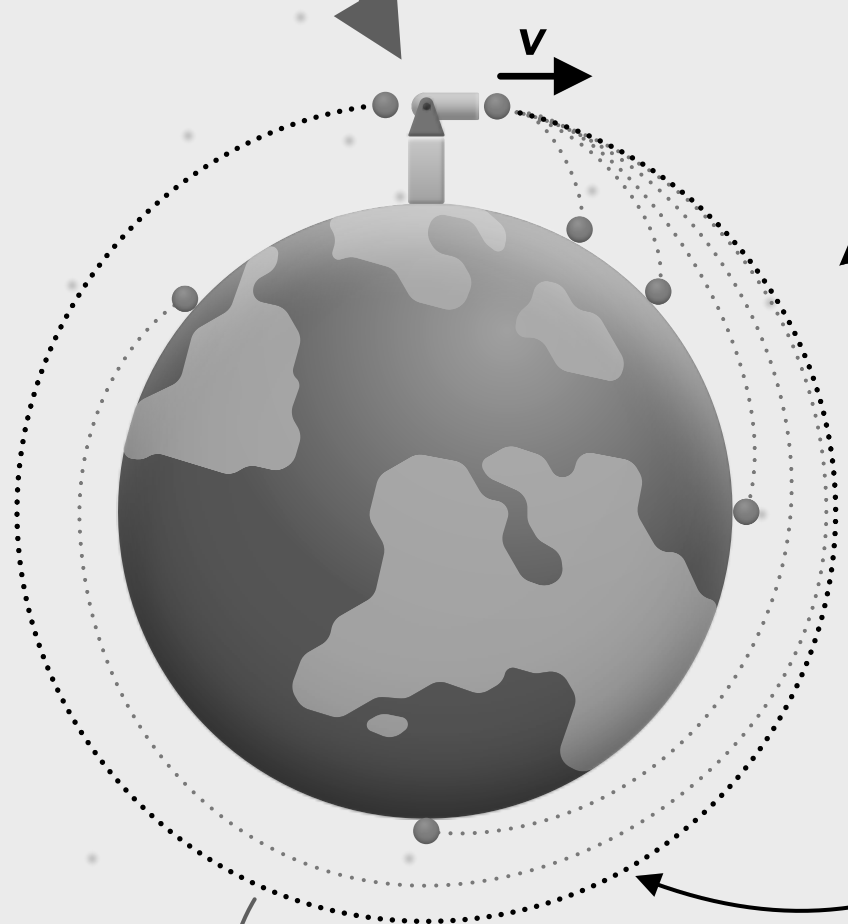
- **Orbital motion** is when an object follows a circular or elliptical path (an "orbit") around another object, where the only force acting on the object is gravity.
- Technically the two objects are both in orbit around a common point called a "barycenter" but we'll focus on a simplified version for now, where one of the objects has much more mass than the other and the barycenter is approximately at the center of the larger object.



- An object in orbital motion around a planet is actually in **projectile motion** or **free fall**.
- The only force acting on the object is the **gravitational force** from the planet which always points towards the center of the planet. It may seem like some force is required to keep the object moving, but we know from Newton's 1st law of motion that an object in motion will continue moving on its own unless a net force is applied to stop it from moving.
- We're only going to focus on the continuous orbital motion itself, not the cause of the initial velocity that started the orbital motion or changes to the orbital motion.

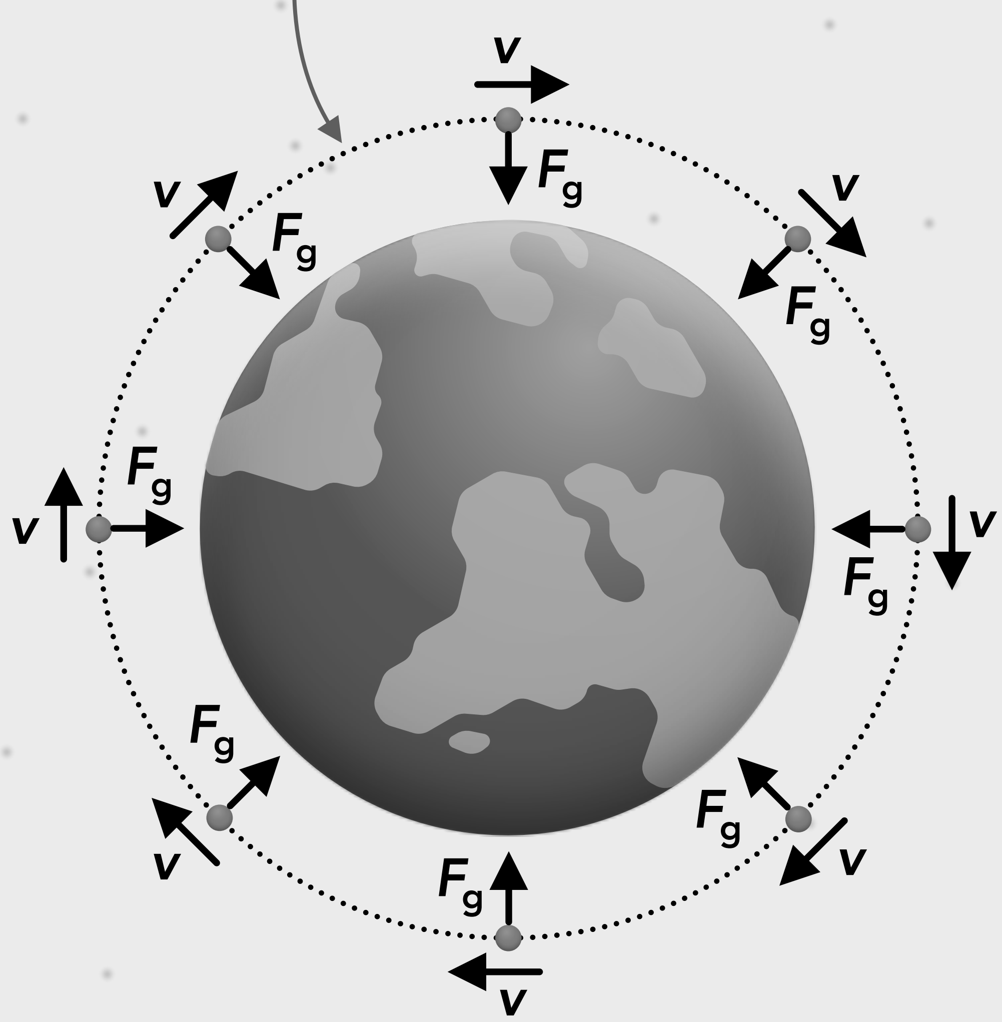
In projectile motion (or free fall) the only force acting on the object is the gravitational force (ignoring air resistance)





As the speed and the range increase to a larger scale, the curvature of the earth becomes relevant and the ground begins to “fall away” below the path of the ball.

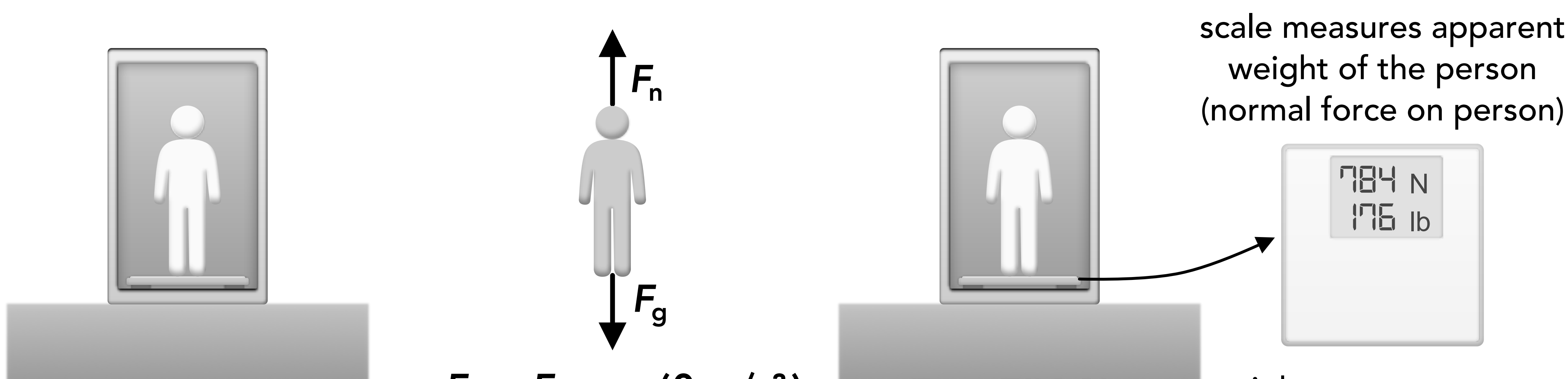
If the initial speed is fast enough (~ 7,900 m/s) the ball will never hit the ground, the projectile motion completely circles the earth and the ball ends up where it started. At this velocity the ball is in orbital motion around the earth.



Once the ball is in orbit its speed is constant and the only force acting on it is the gravitational force (assuming no air resistance). The gravitational force is perpendicular to the velocity and changes its direction, but no force is required for the ball to keep moving.

- Since an object in orbital motion is in projectile motion or free fall, and the only force acting on it is gravity, **the object has no apparent weight** and it experiences “weightlessness”.
- Gravity is still acting on an object in orbital motion. For example, the International Space Station (ISS) is in orbit around the earth at an altitude of about 400 km. At that distance from the earth, the acceleration due to gravity g is still about 8.7 m/s² or 89% of the acceleration due to gravity at the surface of the earth.
- An astronaut in the ISS orbiting the earth will feel like they’re falling, because they are. It’s the same thing as being in an elevator that is in free fall where you appear to be weightless. The difference is that an object in orbital motion is also moving sideways very fast, and the direction of the gravitational force keeps rotating.

Your apparent weight is the normal force supporting you from below, which is equal to the actual weight if the net force and acceleration are zero

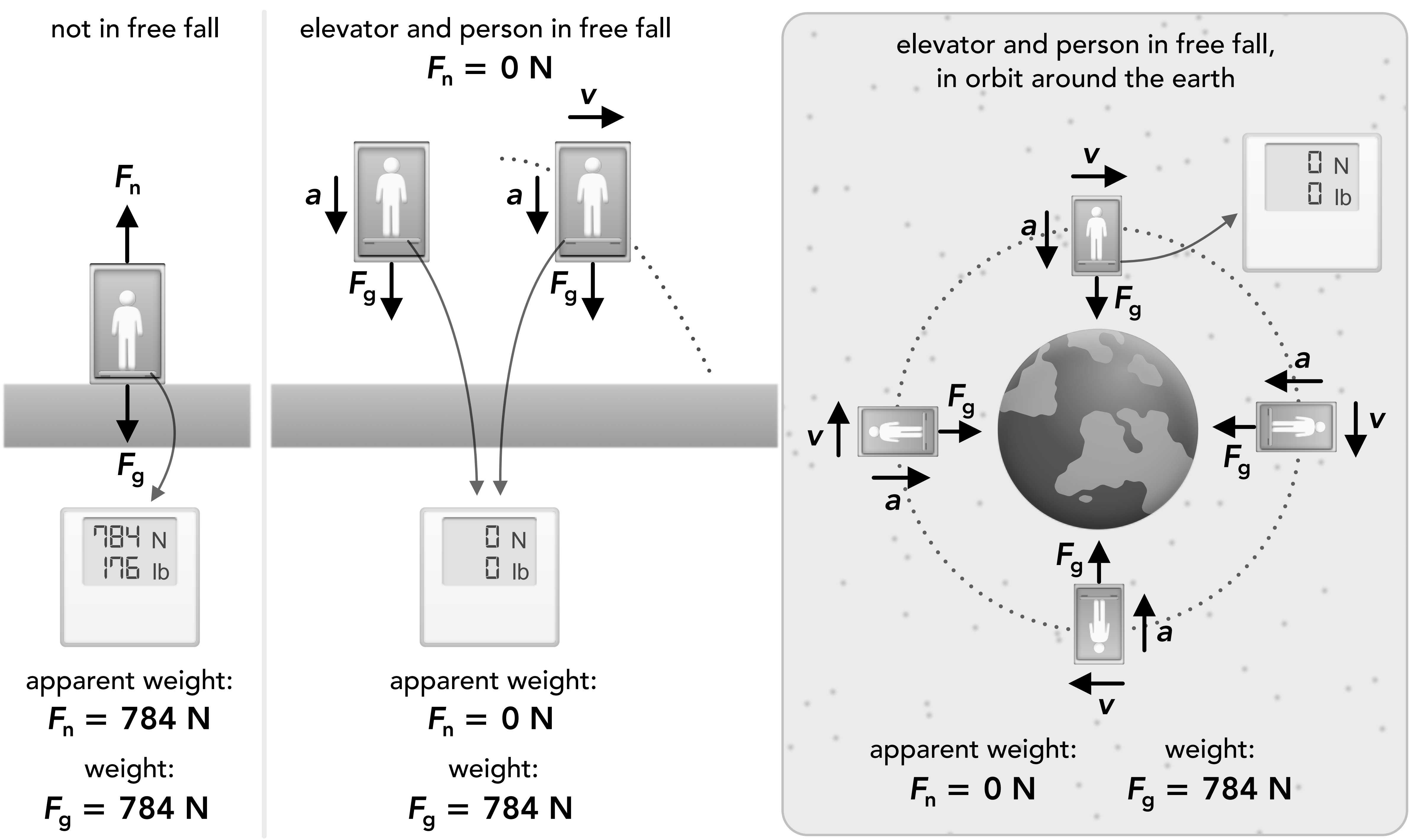


person:
$m = 80$ kg
$F_g = mg = 784$ N

$F_n - F_g = m(0 \text{ m/s}^2)$
$F_n = F_g$
$F_n = 784$ N

$F_g = 784$ N ⟵ weight
$F_n = 784$ N ⟵ apparent weight

When you’re in free fall and the only force acting on you is the gravitational force, you have no apparent weight and you experience “weightlessness”, even if you have some velocity. This is the case for an object in orbital motion.

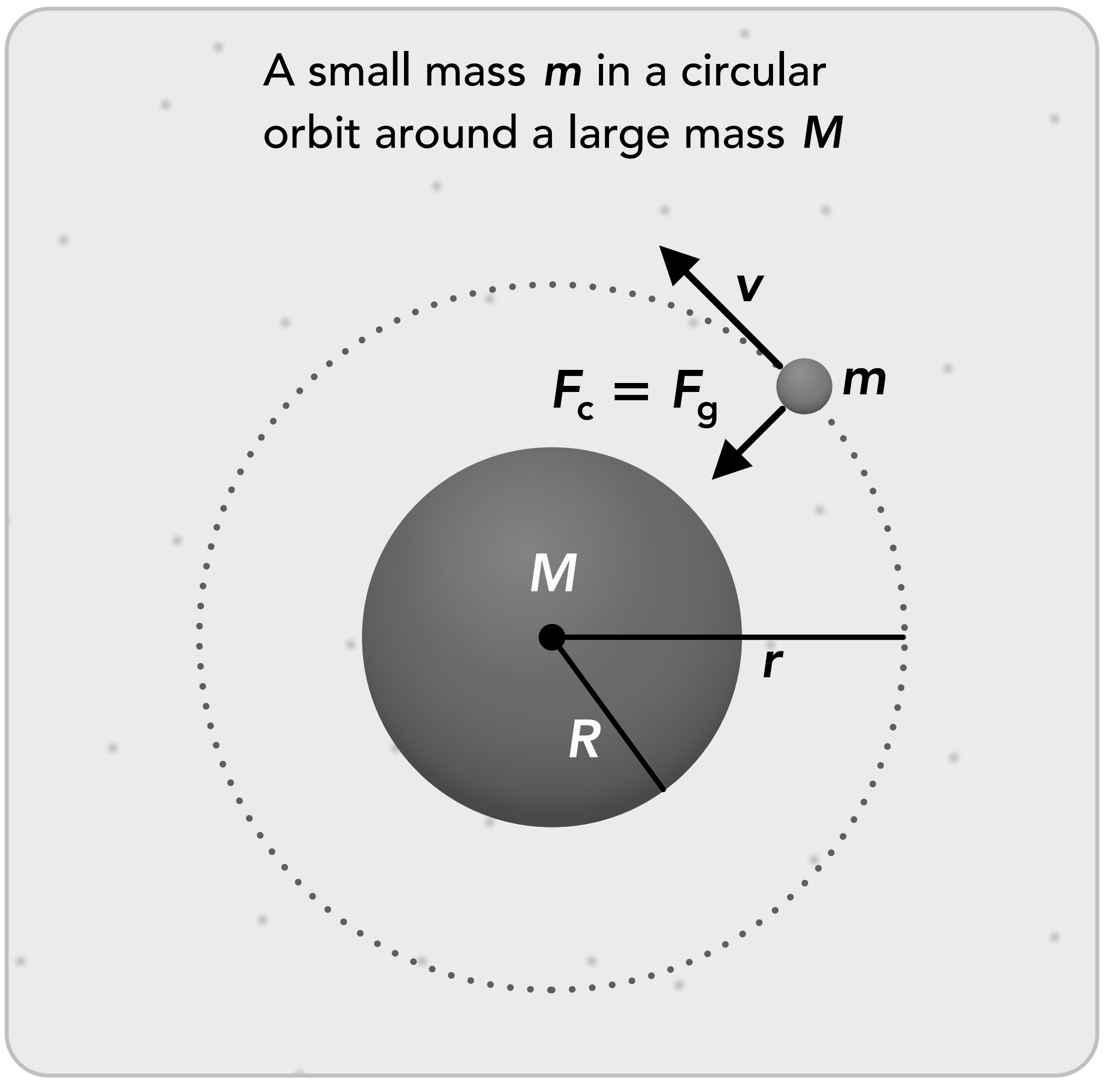


Circular Orbits

Constants		Unit	Name
G	6.67×10^{-11}	$\frac{m^3}{kg \cdot s^2}$	gravitational constant

Variables		SI Unit
M	planet mass	kg
m	object mass	kg
R	planet radius	m
r	orbital radius	m
v	orbital speed	$\frac{m}{s}$
T	orbital period	s
F_g	gravitational force	N
F_c	centripetal force	N

- The path of an object in orbit around a planet can be a circular orbit or an elliptical orbit. The same laws which are covered in the elliptical orbit section also apply to circular orbits.
- An object in a **circular orbit** is in **uniform circular motion** around the planet. Remember that an object in circular motion must have a centripetal force acting on it which always points towards the center of the circle.
- For circular orbits, **the centripetal force is the gravitational force** and the centripetal acceleration is the gravitational acceleration at that distance from the center of the planet. This means we can equate some concepts from uniform circular motion and Newton's law of universal gravitation.



The centripetal force for a circular orbit is the gravitational force acting on the small mass so we can combine circular motion and gravitation

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r}$$

Orbital speed

$$v = \sqrt{\frac{GM}{r}}$$

$$T = \frac{2\pi r}{v}$$

Orbital period (Kepler's 3rd law)

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Orbital Motion Energy

Variables		SI Unit
E	total energy	J
K	kinetic enerrgy	J
U_g	potential energy	J

- Using the equation above for the orbital speed we can derive an equation for the kinetic energy K of an object in a circular orbit.
- The gravitational potential energy U_g of the object in orbit is just the gravitational potential energy of the two-mass system, regardless of the motion of either mass (so this is not specific to orbital motion).
- The total energy of an object in a circular orbit is the sum of kinetic energy and the potential energy.

Kinetic energy of object in a circular orbit

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

Gravitational potential energy of two-mass system

$$U_g = -\frac{GMm}{r}$$

Total energy of object in a circular orbit

$$E = K + U_g = -\frac{GMm}{2r}$$