

Variables and Kinematic Equations

Variables		SI Unit
t	time	s
x	horizontal position	m
y	vertical position	m
v	velocity	$\frac{\text{m}}{\text{s}}$
a	acceleration	$\frac{\text{m}}{\text{s}^2}$

Variable
 v_x
Subscript
"horizontal velocity"

Subscripts		
i	0	initial
f	-	final
x		horizontal
y		vertical

delta
 $\Delta = \text{final} - \text{initial}$

$$\Delta x = x_f - x_i$$

or

$$\Delta x = x - x_0$$

Horizontal motion



Displacement:

$$\Delta x = x_f - x_i$$

Velocity:

$$v_x = \frac{\Delta x}{\Delta t}$$

Velocity
(rearranged):

$$x_f = x_i + v_x \Delta t$$

Acceleration:

$$a_x = \frac{\Delta v_x}{\Delta t}$$

Acceleration
(rearranged):

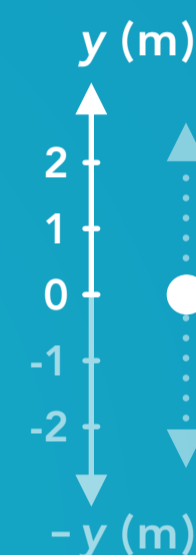
$$v_{xf} = v_{xi} + a_x \Delta t$$

Kinematic equations for
constant acceleration:

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

$$v_{xf}^2 = v_{xi}^2 + 2 a_x (x_f - x_i)$$

Vertical motion



$$\Delta y = y_f - y_i$$

$$v_y = \frac{\Delta y}{\Delta t}$$

$$y_f = y_i + v_y \Delta t$$

$$a_y = \frac{\Delta v_y}{\Delta t}$$

$$v_{yf} = v_{yi} + a_y \Delta t$$

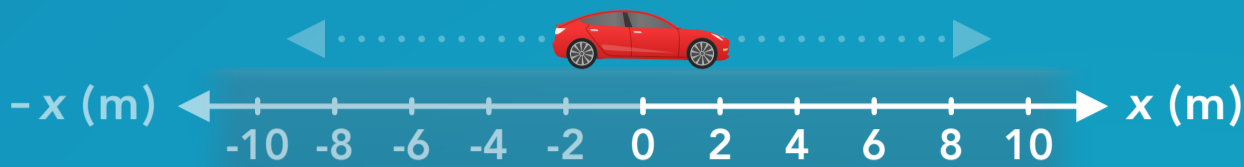
$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

$$v_{yf}^2 = v_{yi}^2 + 2 a_y (y_f - y_i)$$

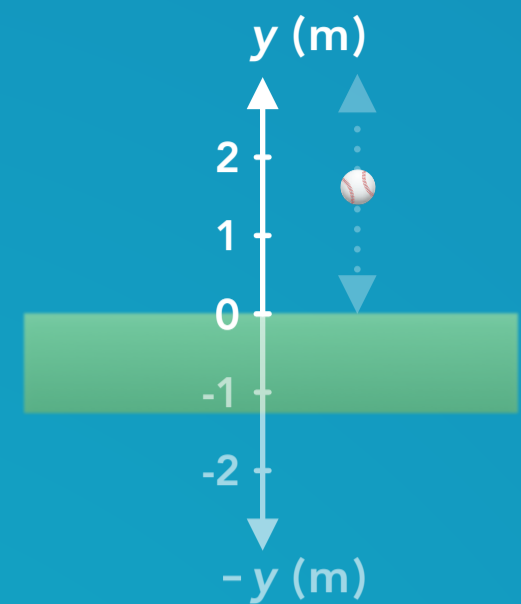
- The kinematic equations for horizontal motion and vertical motion are the same, but we use different variables and subscripts to represent the different directions. Any equation can be rearranged algebraically.
- " Δ " and " ${}_f - {}_i$ " are interchangeable. " t " and " Δt " are often used interchangeably.

1D Motion (Linear Motion)

Horizontal motion



Vertical motion



- **1-dimensional motion (1D motion or linear motion)** is a category of motion where an object only moves along a straight line, in either direction.
- The **x** axis is typically used to describe horizontal motion and the **y** axis is typically used to describe vertical motion.

Average Speed

Average speed

$$v_{\text{avg}} = \frac{\text{total distance}}{\text{total time}}$$

$$v_{\text{avg}} = \frac{300 \text{ km}}{5 \text{ hours}} = 60 \text{ km/h}$$

$$\frac{150 \text{ km}}{50 \text{ km/h}} = 3 \text{ hours}$$

$$\frac{150 \text{ km}}{75 \text{ km/h}} = 2 \text{ hours}$$



$$v_{\text{avg}} \neq \frac{v_1 + v_2}{2}$$

$$v_{\text{avg}} \neq \frac{(50 \text{ km/h}) + (75 \text{ km/h})}{2} = 62.5 \text{ km/h}$$

- The **average speed** of an object over some period of time is the total distance traveled divided by the total amount of time. The average speed is not the average or mean of the different speeds during that period.
- Average speed is the "time-weighted-average speed" where each of the different speeds is weighted based on the amount of time spent traveling at that speed (instead of the amount of distance traveled at that speed).

Scalar and Vector Quantities

Scalars

distance

5 m

2 km

speed

8 m/s

60 km/h

Vectors

displacement

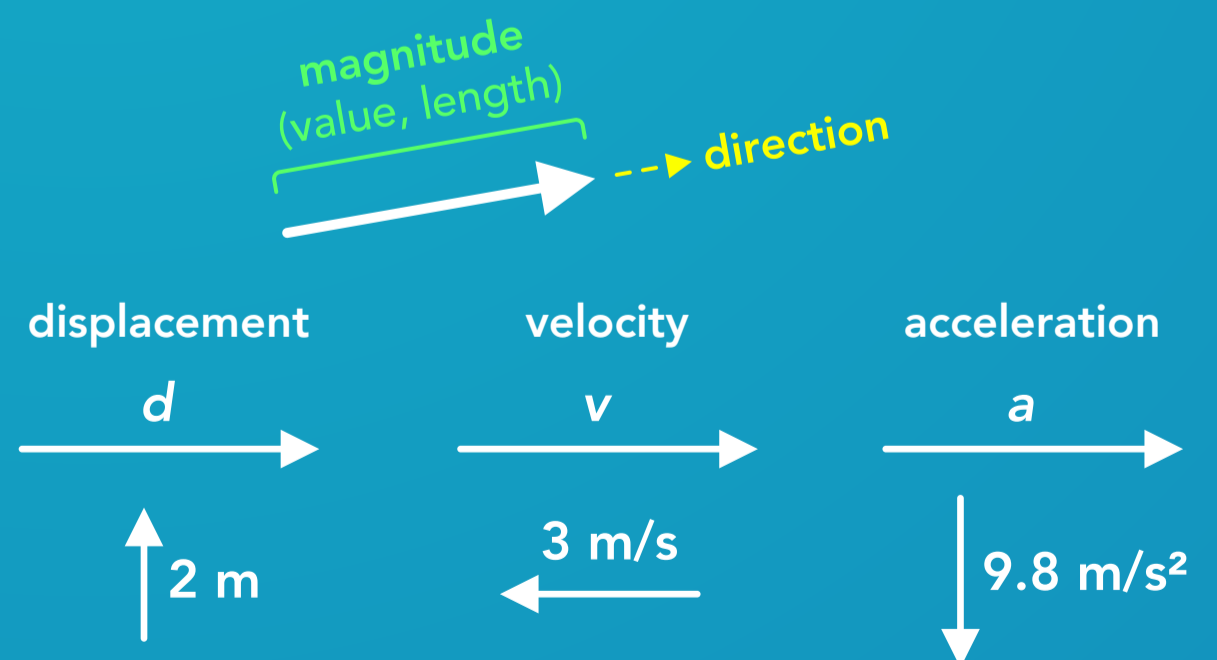
5 m in the north direction

2 km in the +x direction

velocity

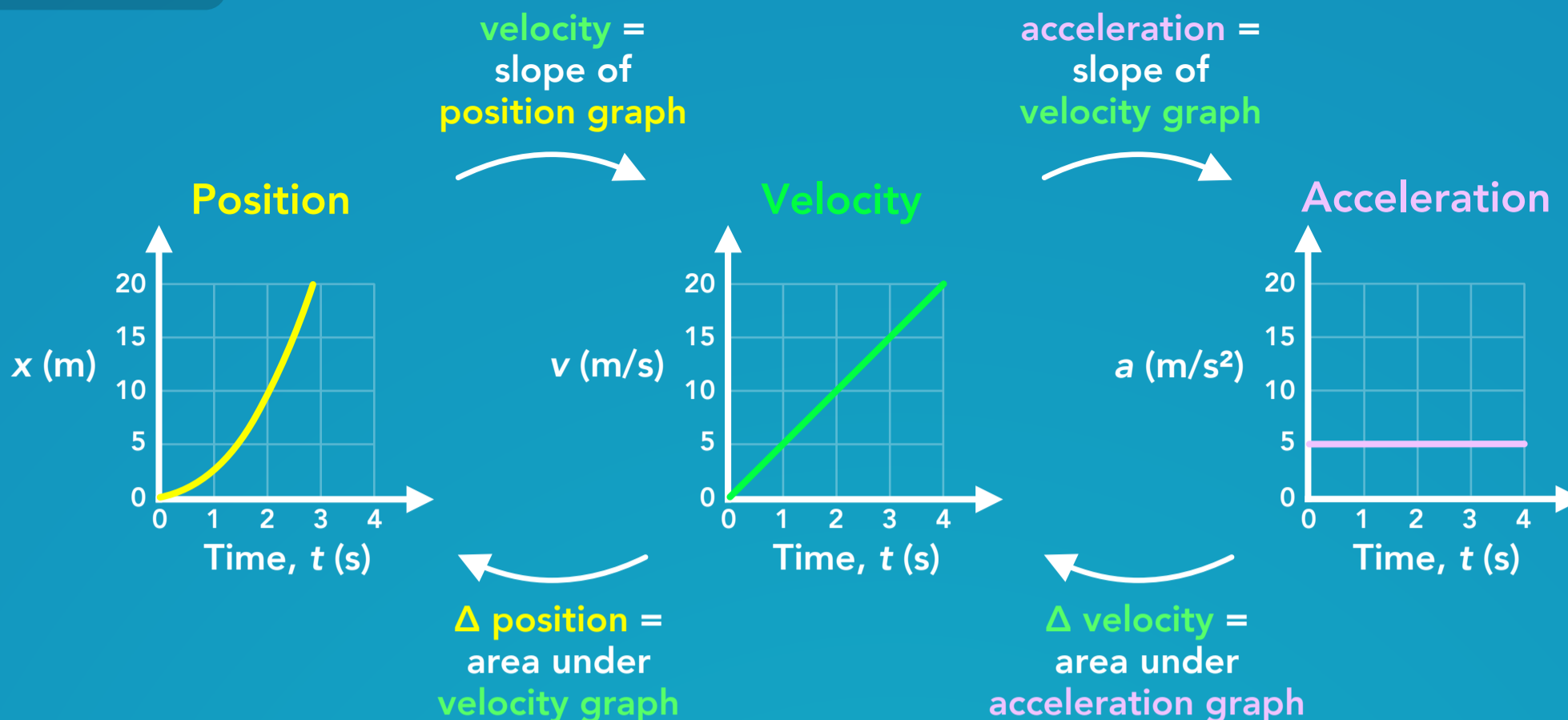
8 m/s to the left

60 km/h in the east direction



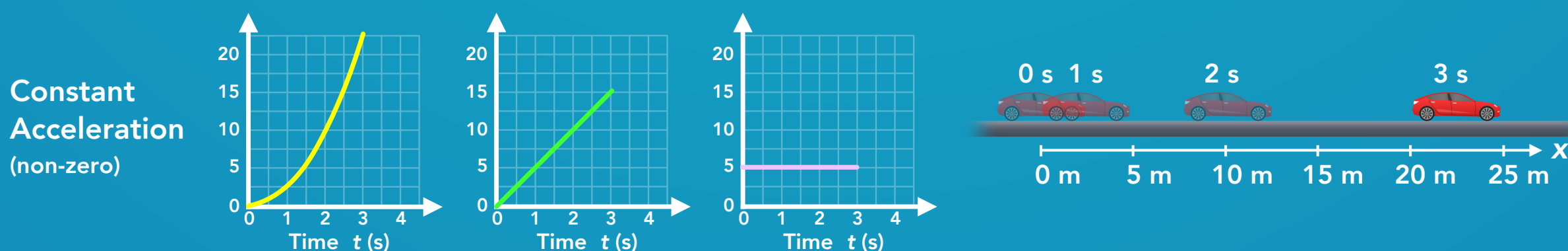
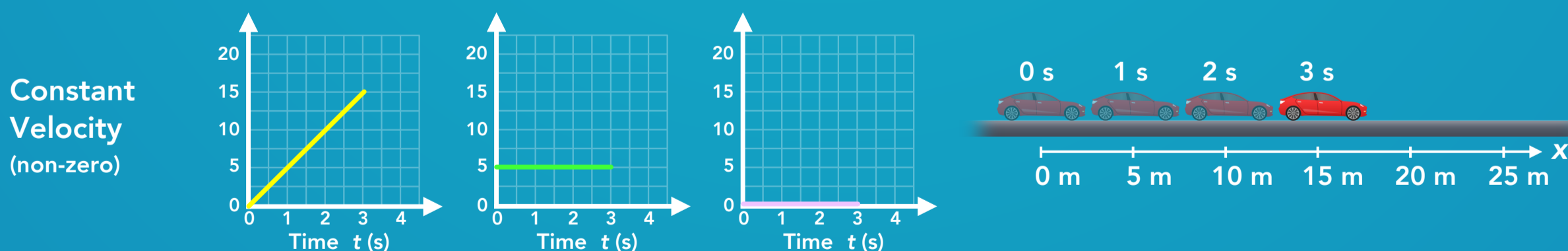
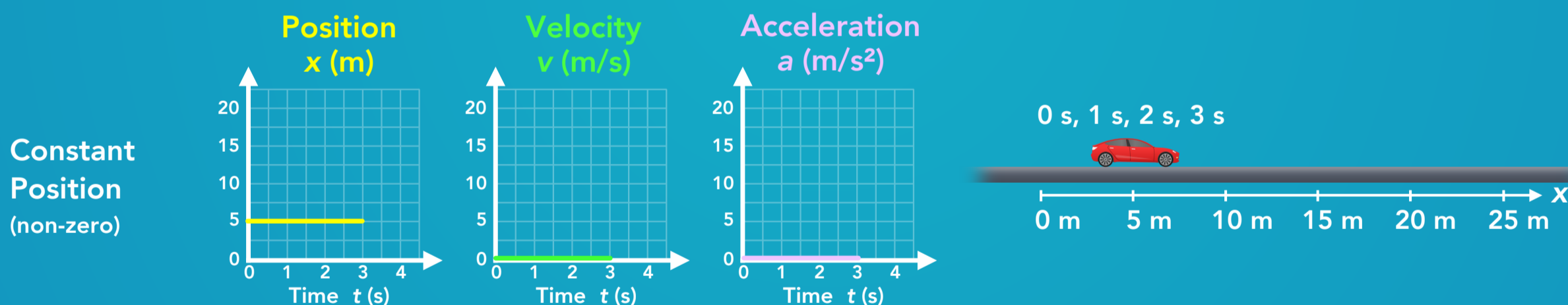
- A **scalar** quantity includes only a **magnitude** (a value) and no direction. Distance and speed are scalars.
- A **vector** quantity includes both a **magnitude** and a **direction**. Displacement and velocity are vectors.
- When a vector quantity is represented using an arrow, the length of the arrow represents the magnitude of the vector and the arrow points in the direction of the vector.

Motion Graphs



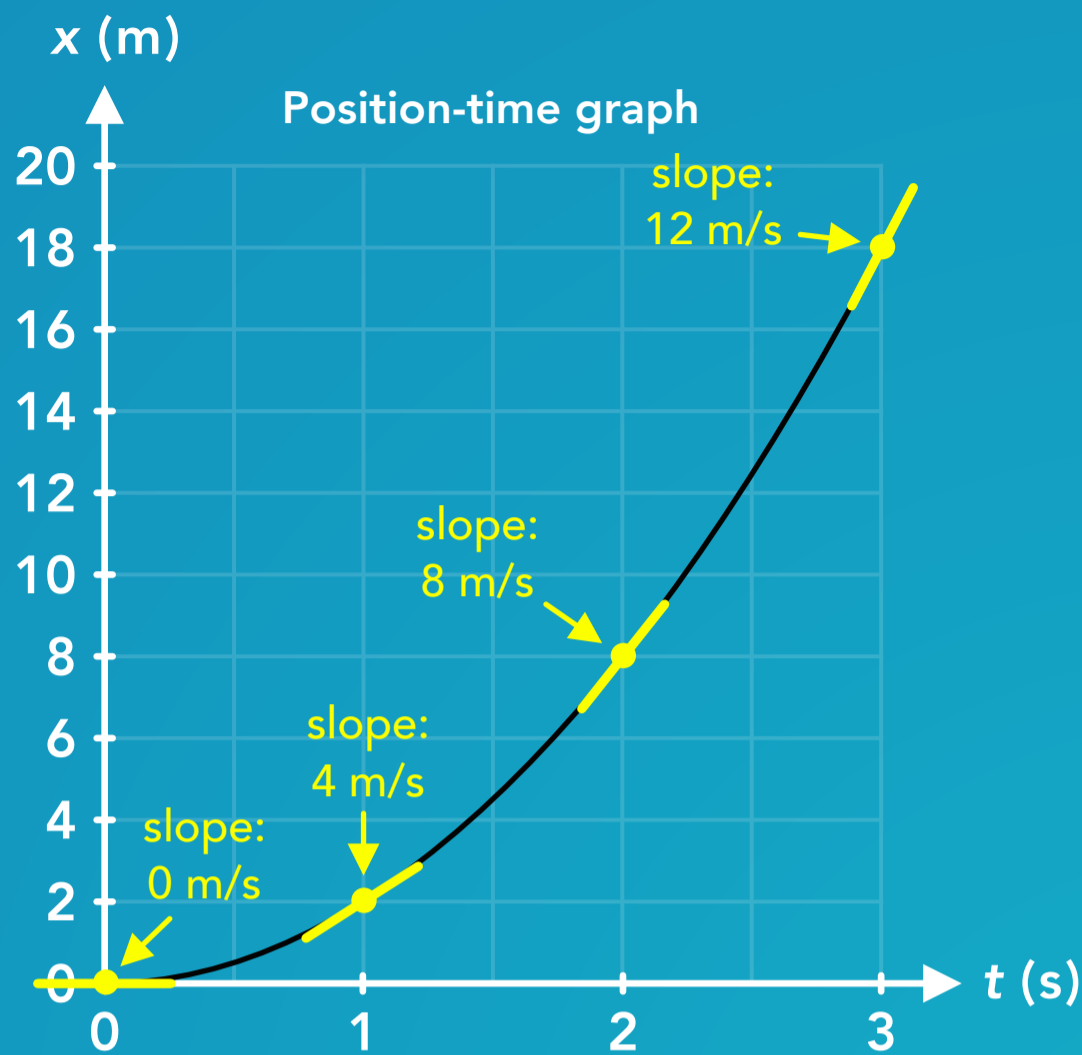
- A **motion graph** shows an object's position, velocity or acceleration over time.
- The instantaneous slope of the position graph (the slope at a single point or instant in time) is the instantaneous velocity of the object at that time. Using calculus, velocity is the derivative of position with respect to time.
- The area under the curve of the velocity graph for a period of time (the area between the graph line and the horizontal axis between two time points) is the displacement or the change in position of the object during that time (areas above the horizontal axis are positive and areas below the horizontal axis are negative). Using calculus, the change in position is the integral of velocity with respect to time.
- The instantaneous slope of the velocity graph is the instantaneous acceleration of the object.
- The area under the curve of the acceleration graph for a period of time is the change in velocity during that time.

Examples of motion graphs:

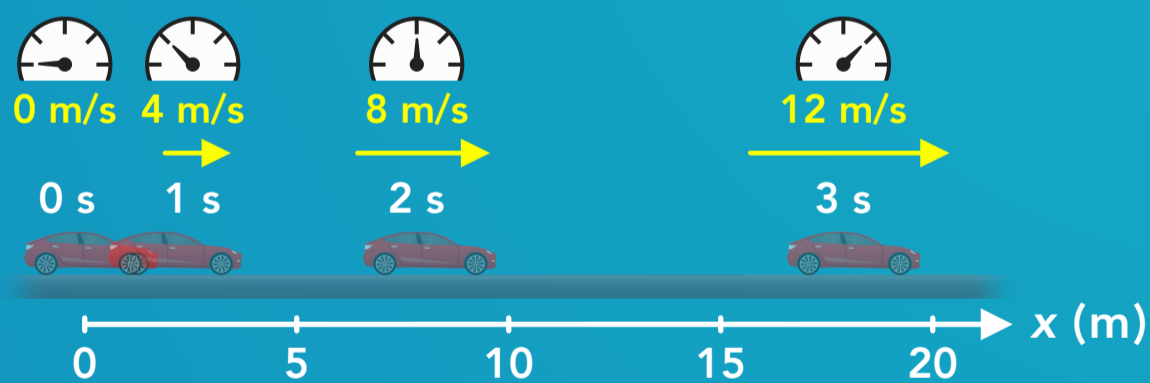


Instantaneous vs Average Velocity

Instantaneous velocity

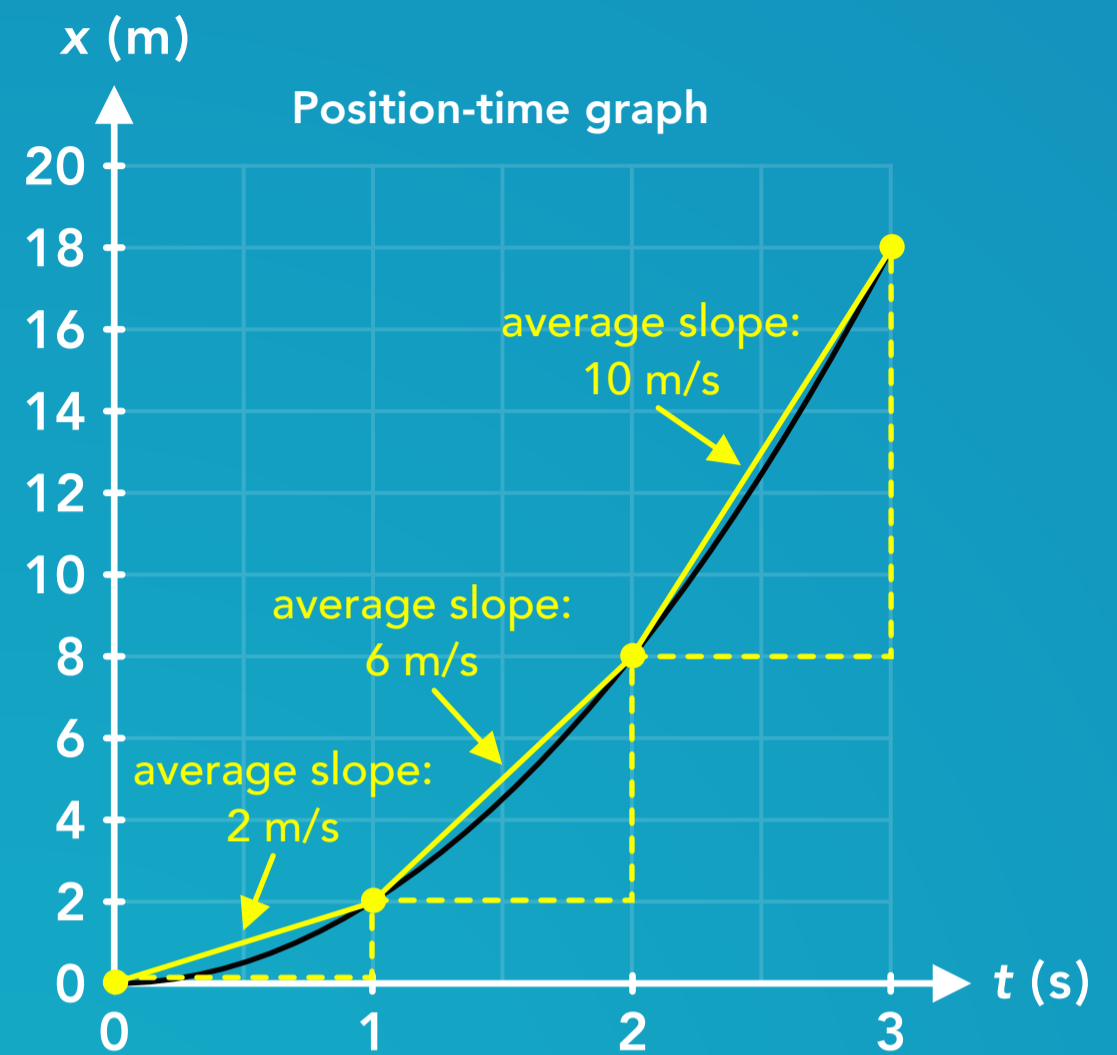


t	x	Instantaneous v
0 s	0 m	0
1 s	2 m	4
2 s	8 m	8
3 s	18 m	12

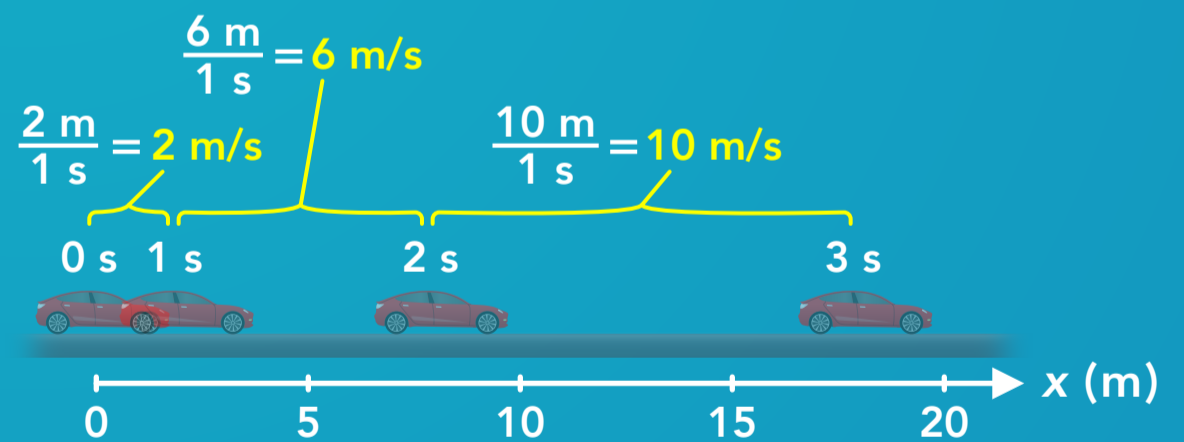


- **Instantaneous velocity** (or instantaneous speed) is the velocity of an object at a single instant in time.
- It is represented as the instantaneous slope of the position-time graph at a single point.

Average velocity



t	x	Average v
0 s	0 m	$(2 \text{ m} - 0 \text{ m}) / (1 \text{ s}) = 2 \text{ m/s}$ $(8 \text{ m} - 2 \text{ m}) / (1 \text{ s}) = 6 \text{ m/s}$ $(18 \text{ m} - 8 \text{ m}) / (1 \text{ s}) = 10 \text{ m/s}$
1 s	2 m	
2 s	8 m	
3 s	18 m	



- The **average velocity** (or average speed) of an object is the displacement divided by a period of time.
- It is represented as the average slope of the position-time graph for a period of time (between two points).