lec # 17 :-

Example 9.1 — Homework.

p 480

Exercise 481 - The first 15-20 Questions

Closures of a Relation:-

R → It is not Reflexive.

we want to make it Reflexive.

if we add some additional tuples → we will be able to make it reflexive

The process of inducing The given property is Known as closure.

Closure with respect to Reflexive.

$R = \{\underline{(1,1)}, (1,2), (2,1), (3,2)\}$

$A = \{1, 2, 3\}$.

R is not Reflexive. $\underbrace{(2,2) \quad (3,3)}_{\text{missing}}$.

$\Delta = \{ (a,a) \mid a \in A\}$

$(R \cup \Delta)$ is reflexive.

A is the set Based on which R is defined.

$\Delta = \{(1,1), (2,2), (3,3)\}$.

$R = \{(1,1), (1,2), (2,1), (3,2)\}$

$R \cup \Delta = \{ (1,1), (1,2), (2,1), (3,2)$

$$\underbrace{R \cup \Delta}_{\Downarrow} = \{(1,1),(1,2),(2,1),(3,2),(2,2),(3,3)\}.$$

reflexive.

$$\underbrace{R \cup \Delta}_{\Downarrow} \qquad \Delta = \{(a,a) \mid a \in A\}.$$

reflexive.

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Downarrow$

Observation: Δ = Identity matrix. Δ

Ex 483:- $R = \{(a,b) \mid a < b\}$

$A = \mathbb{Z}$.

$\Delta = \{(a,a) \mid a \in A\}$.

$\Delta = \{(a,a) \mid a \in \mathbb{Z}\}$.

$\Delta = \{(a,b) \mid a = b\}$.

$R \cup \Delta = \{(a,b) \mid a < b \vee a = b\}$

$R \cup \Delta = \{(a,b) \mid a \leq b\}$.

$(-\infty,-\infty)$
$\vdots$
$(0,0)$
$(1,1)$
$(2,2)$
$\vdots$
$(+\infty,+\infty)$.

→ Closure with respect to Symmetry.

$R = \{(1,1), \underbrace{(1,2)}_{\times}, (2,2), \underbrace{(2,3)}_{\checkmark}, \underbrace{(3,1)}_{\times}, \underbrace{(3,2)}_{\checkmark}\}$.

✓ $(2,1)$ is absent.
✓ $(1,3)$ " ".

$R^{-1} = \{(1,1),(2,1),(2,2),(3,2),(1,3)\}$

$R^{-1} = \{(1,1), (2,1), (2,2), (3,2), (1,3)$
$(2,3)\}$.

$R \cup R^{-1} = \{(1,1), (1,2), (2,1), (2,2),$
$(2,3), (3,2), (3,1), (1,3)\}$.

$R \cup R^{-1}$ for symmetric Closure.

Ex2 483:- $R = \{(a,b) \mid a > b\}$ $A = Z^+$

$R = \{(b,a) \mid (a,b) \in R\}$.

symmetric Closure?

$R^{-1} = \{(b,a) \mid b > a\}$ $A = Z^+$.

$b > a$

$R \cup R^{-1} = \{(a,b) \mid a > b$ or $a < b\}$. $\Rightarrow$ $a < b$.

$= \{(a,b) \mid a \neq b\}$.

. Closure with Respect to transitive.

Tricky = ?

$R = \{(1,3), (1,4), (2,1), (3,2)\}$

$A = \{1,2,3,4\}$.

$(1,2)$.
$(2,3)$
$(2,4)$
$(3,1)$
missing elements.

$R' = \{(1,3), (1,4), (2,1), (3,2),$
$(1,2), (2,3), (2,4), (3,1)\}$. X

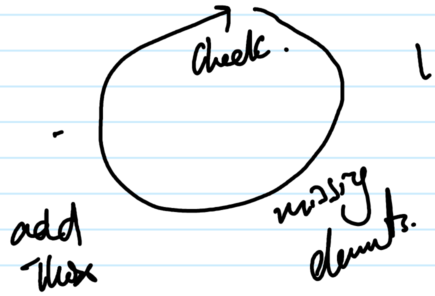
$(3,4)$ is absent.

observation:- In R → missing elements.

we add those missing to R

→ additional missing elements.

iterative. (when R is transitive).



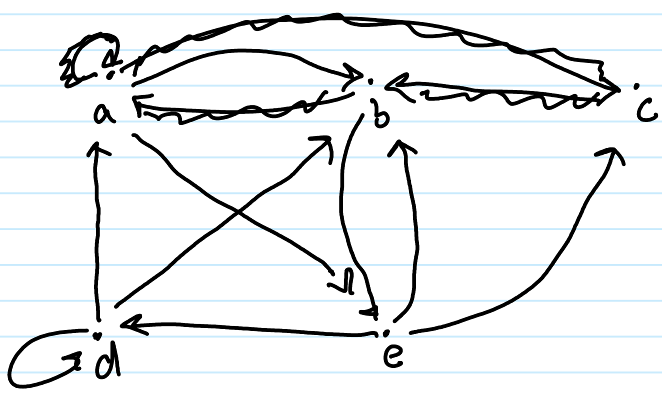
looping through the relation.

To understand algorithms for transitive closure.

→ we need to learn additional notions.

PATH :- A directed Graph G.
if there is a sequence of edges.
$(a, x_1), (x_0, x_1), (x_1, x_2) \ldots (x_n, b)$.
we say there is a path b/w a and b.

Ex 3
484.



a b d e.
⇓
$(a,b)\ (b,d)\ (d,e)$.
X.

a, e, c, d, b
$(a,e), (e,c), (c,a)$
(d,b) X.

$\begin{pmatrix} \overset{1}{a} & \overset{2}{c} & \overset{3}{b} & \overset{4}{a} & \overset{5}{a} \end{pmatrix}$ ✓
$= \overset{1}{(a,c)}, \overset{2}{(c,b)}, \overset{3}{(b,a)}, \overset{4}{(a,a)}$

length of a path. = Number of edges in the path
= 4.
= Number of Vertices − 1

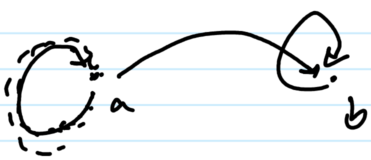
$= \text{Number of Vertices} - 1$

P485: theorem: R is defined on A.
$\exists$ a path of length n from a to b.
if $(a,b) \in R^n$.

$R^n = R^{n-1} \circ R$.
$\downarrow$
Composite operat

$(a,b) \in R^2$ $n=2$ a to b

$R^{n-1} = R^{n-2} \circ R^1$
$\vdots$



$R = \{(a,a)\ (b,b)\}$.

$R^2 = R^1 \circ R^1$

$R^2 = \{(a,a),(b,b)\}$.

R^2 = Composite.
$R \circ R$

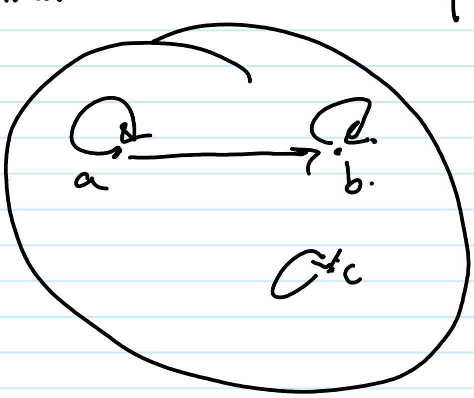
aaa $\Rightarrow$ $(a,a)\ (a,a)$.
bbb $\Rightarrow$ $(b,b)\ (b,b)$.

aab.
$(a,a)\ (a,b)$.

$(a,b) \in R^2$ $\exists$ a path of length 2.
then $(a,b) \in R^2$

abb $\Rightarrow$ $(a,b)(b,b)$

485: Connectivity Relation R^*
consists of pairs (a,b)
there is atleast one path from a to b in R.

$R =$ 

$R^* = \{(a,a),(a,b),(c,c)\}$.

R^1

$$R^* = \bigcup_{i=1}^{\infty} R^n$$

$$\bigcup \begin{cases} R^1 \\ R^2 = R^1 \circ R^1 \\ R^3 = R^2 \circ R^1 \\ \vdots \\ R^\infty = R^{\infty-1} \circ R^1 \end{cases}$$

1) Do Exercise. p491. (15–20) Questions.

2). $\underset{2}{\overset{+}{\text{Email}}}$ – $\underset{1}{\overset{*}{\text{Slack}}}$ – $\underset{3.}{\overset{-}{\text{Slate}}}$

3). R^* application in the next lecture.

4). Equivalence Relation. (Special type of Relation).

5) Questions in online Sessions.