lec 18 :- week -9.

Closure.
→ Relexive.
→ symmetric.
→ transitive.
→ path. R^*.

R^{*1} = Connectivity Relation.
It Consists of pair (a,b)
such that ∃ a path btw a & b
in R.

Ex 4 / 485) $R = \{(a,b) \mid a \text{ has met } b\}$

A Set of people in The world.

what is R^n $n \geq 1$.
what is R^*.

$$\begin{tikzcd}
 & \text{Solution:- } R^2 = R^1 \underset{\text{Composite}}{\circ} R^1 \arrow[dl] \arrow[dr] & \\
(a,b) \in R^1 \arrow[d, Rightarrow] & & (b,c). \in R^1 \\
(a,c) \in R^2 & &
\end{tikzcd}$$

$R = R^1$

$\underbrace{(a,c)}_{a \text{ has met } c} \in R^1$ $\qquad$ $\underbrace{(c,b) \in R^1}_{c \text{ has met } b.}$

$(a,b) \in R^2$

$R = \{(a,b) \mid a \text{ has met } b\}$.

$(a,b) \in R^2$ = when ∃ a person c Such That
c has met a and c has met b.

$R^3 = ?$ (a_1, a_4).

(a_1,a_2) (a_2,a_3) (a_3,a_4).

(a_1, a_2) (a_2, a_3) (a_3, a_4).

$R^4 = ?$ (a_1, a_5). (a_2, a_3, a_4).

_ _ _ _

⋮

$R^n = ?$ $(n-1)$ persons.

$(a_1, a_2) (a_2, a_3), (a_3, a_4) \dots (a_n, a_{n+1})$.

$R^* =$ ∃ a path btw a & b.

$(a,b) \in R^*$.

$(a,b) \in R^* \Rightarrow$ there exist 0 - +ve of people. between a & b.

Ex 6
486:

$R = \{(a,b) \mid$ a and b has a common border $\}$.

$A = \{$ Set of states in US $\}$.

R^n

R^*

$R^2 =$ $(a,b) \in R^2$ if ∃ state c such that a & c has a common border & c & b has also a common border.

$R^3 = ?$ two state c_1, c_2. between.

$(a,b) \in R^3$

a has a border with c_1

c_1 has a border with c_2

c_2 " " " a b.

c2 " " " a b.

R^n

$(a,b) \in R^n$ ⟶ .

(n-1) states in between.

$R^* = ?$

$(a,b) \in R^*$ if they has a Common border or if ∃ exist +ve # of states in between.

Connectivity Relation Can be Used the determine transitive Closure.

→ skip details.

Equivelence Relation:-

1) Reflexive
2) Symmetric
3) Transitive.

Ex1 :- $R = \{(a,b) \mid a=b \text{ or } a=-b\}$.
494

$A = R$.

Reflexive:- $\forall a \in A \ (a,a) \in R$.

$\forall a \in R \quad \underbrace{a=a}_{\checkmark} \ \underbrace{\text{or}} \ a=-a$.

Reflexive holds.

Symmetric:- $\forall a,b \in A$ if $(a,b) \in R \to (b,a) \in R$

$\forall a,b \in R$ if $(a=b \text{ or } a=-b) \to$
$(b=a \text{ or } b=-a)$

this holds also.

this holds also.

Transitive: $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall a,b,c \in \mathbb{R}$ if $a=b$ or $a=-b$ $\wedge$
$b=c$ or $b=-c$ $\rightarrow$
$a=c$ or $a=c$

that the Relation is Equivalence.

Ex6 495 $R = \{(a,b) \mid a \text{ divides } b\}$

Equivalence Relation? $A = \mathbb{Z}$.

1) Reflexive $\forall a \in A$ $(a,a) \in R$.
$\forall a \in \mathbb{Z}$ a divides a. Holds.

2) Symmetric $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$
$\forall a,b \in \mathbb{Z}$ if a divides b $\rightarrow$ b divides a.
does not hold.

this is not Equivalence Relation.

$(5,10)$ $\frac{10}{5} = 2$.
$\frac{5}{10} \neq$ whole #

Ex7: 495 $R = \{(a,b) \mid |a-b| < 1\}$.

$A = \mathbb{R}$.

Equivalence Relation.

1) Reflexive. $\forall a \in A$ $(a,a) \in R$.
$\forall a \in \mathbb{R}$ $|a-a| < 1$.
true Hold.

$|\overset{\downarrow}{a}-\overset{\downarrow}{b}|$.
+ve Value.

$|3-5| = |-2|$
$= 2$.

$|100-100| < 1$.

2) Symmetric $\forall a,b \in R$ if $(a,b) \in R$
$\rightarrow (b,a) \in R$.

$\forall a,b \in \mathbb{R}$ if $|a-b| < 1 \rightarrow |b-a| < 1$

2,5

$|2-5| \not< 1$

U $\to (b,a) \in R$.

$\forall a,b \in \mathbb{R}$ if $\underbrace{|a-b| < 1}_{T} \to |b-a| < 1$.

Holds.

3) Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R$

$\to (a,c) \in R$.

$\forall a,b,c \in \mathbb{R}$ if $|a-b| < 1 \wedge |b-c| < 1$

$\to \boxed{|a-c| < 1.}$

$|0.5-1.4| < 1 \wedge |1.4-1.9| < 1$

$\to |0.5-1.9| \not< 1$

Roo not Holds

is not Equivalence Relation.

Ex3: 495

$R = \{(a,b) \mid \boxed{a \equiv b} \bmod m\}$.

$m \geq 1$. $A = \mathbb{Z}$.

$\equiv \left\{ \begin{array}{l} \boxed{8} \bmod \boxed{5} = 3. \\ \boxed{13} \bmod 5 = 3 \end{array} \right.$

$5 \overline{)13}$ quotient 2, 10, remainder 3.

$8 \equiv 13 \bmod 5$.

$a \equiv b \bmod m \Rightarrow \boxed{a-b = km}$

$R = \{(a,b) \mid a-b = km\}$.

reflexive: $\forall a \in A \ (a,a) \in R$.

2,5

$|2-5| \not< 1$.

$|2-2.5| < 1$.

$|2.5-2| < 1$

$a = 0.5$

$b = 1.4$

$c = 1.9$.

Remainder.

$\Uparrow$

$8 \bmod 5$.

$= 3$.

$5 \overline{)8}$ quotient 1, 5, remainder $\boxed{3}$

Remainder.

≥ 0.

$\forall a \in Z \quad a - a = km$ Holds.

$0 = \underset{=0}{k}\, m.$

Symmetric:- $\forall a,b \in A$ if $(a,b) \in R$

$\rightarrow (b,a) \in R.$

$\forall a,b \in Z$ if $a - b = km \rightarrow$

$\underbrace{b - a} = -km.$

$a - b = km$

$-a + b = -km.$

$b - a = -km.$

$\overset{\overset{a}{\uparrow}}{(8,} \overset{\overset{b}{|}}{12)} \quad m = 4$

$8 - 12 = k\overset{4}{\cancel{m}}. \Rightarrow k = -1.$

$12 - 8 = k4 \Rightarrow k = 1.$

Transitive $\forall abc \in A$ if $(a,b) \in R \wedge (b,c) \in R$

$\rightarrow (a,c) \in R$

$\forall abc \in Z$ if $a - b = k_1 m \wedge b - c = k_2 m$

$\rightarrow a - c = k_1 k_2 m.$

Equivalence class:-

$[a] = \{ s \mid (a,s) \in R \}.$

$R = \{ (\underline{1},1), (\underline{1},2), (\underline{2},1), (3,3), (2,2) \}.$

$[1] = \{1,2\} = [2].$

$[2] = \{1,2\}.$

$[3] = \{3\}.$