

NESTED QUANTIFIERS.

$$\forall x P(x) = P(1) \wedge P(2) \wedge P(3) \wedge \dots \wedge P(N) \quad x \in \{1, 2, 3, \dots, N\}$$

$$\exists x P(x) = P(1) \vee P(2) \vee P(3) \vee \dots \vee P(N)$$

$$\begin{aligned} \forall x \forall y P(x,y) & \quad x,y \in \{1, 2, 3, \dots, N\} \\ &= \forall x [P(x,1) \wedge P(x,2) \wedge \dots \wedge P(x,N)] \\ &= [P(1,1) \wedge P(1,2) \wedge P(1,3) \wedge \dots \wedge P(1,N)] \wedge \\ & \quad [P(2,1) \wedge P(2,2) \wedge P(2,3) \wedge \dots \wedge P(2,N)] \wedge \\ & \quad [P(3,1) \wedge P(3,2) \wedge P(3,3) \wedge \dots \wedge P(3,N)] \wedge \\ & \quad \vdots \\ & \quad [P(N,1) \wedge P(N,2) \wedge P(N,3) \wedge \dots \wedge P(N,N)] \end{aligned}$$

$$\begin{aligned} \forall x \exists y P(x,y) &= \forall x [P(x,1) \vee P(x,2) \vee \dots \vee P(x,N)] \\ & \quad [P(1,1) \vee P(1,2) \vee P(1,3) \vee \dots \vee P(1,N)] \wedge \\ & \quad [P(2,1) \vee P(2,2) \vee P(2,3) \vee \dots \vee P(2,N)] \wedge \\ & \quad \vdots \\ & \quad [P(N,1) \vee P(N,2) \vee P(N,3) \vee \dots \vee P(N,N)] \end{aligned}$$

$$\begin{aligned} \exists x \forall y P(x,y) &= \exists x [P(x,1) \wedge P(x,2) \wedge \dots \wedge P(x,N)] \\ & \quad [P(1,1) \wedge P(1,2) \wedge P(1,3) \wedge \dots \wedge P(1,N)] \vee \\ & \quad [P(2,1) \wedge P(2,2) \wedge P(2,3) \wedge \dots \wedge P(2,N)] \vee \\ & \quad \vdots \\ & \quad [P(N,1) \wedge P(N,2) \wedge P(N,3) \wedge \dots \wedge P(N,N)] \end{aligned}$$

$$\exists x \exists y P(x,y) = \text{logical expression.}$$

Q: Compute the logical expression. $\forall x \exists y P(x,y) \quad x,y \in \{1,2,3\}$

$$\forall x \exists y P(x,y) = \exists x \forall y \neg P(x,y) \quad \text{① - } \forall x \neg P(x) = \exists x \neg P(x)$$

$$\text{② - } \neg \exists x P(x) = \forall x \neg P(x)$$

$$= \exists x \forall y \neg P(x,y)$$

$$= \exists x [\neg P(x,1) \wedge \neg P(x,2) \wedge \neg P(x,3)]$$

$$= [\neg P(1,1) \wedge \neg P(1,2) \wedge \neg P(1,3)] \vee$$

$$[\neg P(1,1) \wedge \neg P(2,2) \wedge \neg P(3,3)] \vee$$

$$[\neg P(3,1) \wedge \neg P(3,2) \wedge \neg P(3,3)].$$

Nested Quantifiers. to write its logical expression.

① Negation with quantifiers.

→ Apply the negation on quantifiers.

→ Simplified expressions.

② To compute the complete expressions.

Homework:- $\forall x \exists y \forall z P(x,y,z)$ $x,y,z \in \{1,2\}$.
check with your friends.

① Computational Aspect.

② Meaning or Semantics.

Ex1 P47 $\forall x \forall y (x+y = y+x)$ $x,y \in \mathbb{R}$.

det $P(x,y) = x+y = y+x$

$\forall x \forall y P(x,y) = \text{True}$

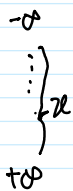
$-\frac{4}{5} + \frac{1}{3} = \frac{1}{3} - \frac{4}{5}$

$-\infty \dots \dots \dots +\infty$
fractions.

$\forall x \exists y P(x,y) = x+y=0$
 $= \text{True}$

$\forall x \exists y P(x,y) = \text{True or False}$
 $x,y \in \mathbb{R}$.

$x=5$
 $y=-5$



Ex4 P48 $\exists y \forall x P(x,y)$ $P(x,y) = x+y=0$ $x,y \in \mathbb{R}$.

$= \text{False}$

observations
Change the order of Quantifiers
the truth value may change.

one value of y
any value of x

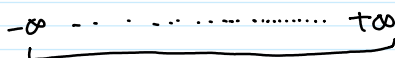
$\exists y \forall x P(x,y) = \text{False}$
 $\forall x \exists y P(x,y) = \text{True}$

Ex5 P49 $Q(x,y,z) = x+y=z$ $x,y,z \in \mathbb{R}$.

① $\forall x \forall y \exists z Q(x,y,z) = ?$

② $\exists z \forall x \forall y Q(x,y,z)$.

T



$1, 2 = 3$

$1, -0.5 = 0.5$

$z = 5$

? F

$$1 \cdot 2 = 3$$

$$1 - 0.5 = 0.5$$

$$z = 5$$

$$x + y = z$$

$$\begin{array}{l} 2 + 3 = 5 \\ 6 - 1 = 5 \\ 5 + 0 = 5 \\ 6 + 1 \neq 5 \end{array}$$

Ex 9
PS 1

Translate into English.

$$\forall x (C(x) \vee \exists y (C(y) \wedge F(x, y)))$$

$C(x)$ = x has a Computer

$F(x, y)$ = x and y are friends

for all x, x is a student in your school,
x has a Computer or there exist y,
y is a student in your school,
y has a Computer and
x and y are friends.

$x, y \in$ Set of students in
your school.

Ex 10
PS 1

Translate into English

$$\exists x \forall y \forall z (F(x, y) \wedge F(x, z) \wedge (x \neq z)) \rightarrow \neg F(y, z)$$

$F(a, b)$ = a and b are friends

$x, y, z \in$ All students
in your school.

There exists x, such that for all y, z, where
x, y, z are students in your school,

if x and y are friends and y and z are also friends
and x and z are not the same, then y and z are not
friends.