Friday, March 19, 2021 9:32 AM

NESTED QUANTIFIERS.

$\forall_x P(x) = P(1) \wedge P(2) \wedge P(3) \wedge \cdots \wedge P(N) \quad x \in \{1,2,3,\cdots N\}$

$\exists_x P(x) = P(1) \vee P(2) \vee P(3) \vee \cdots \vee P(N)$.

$\forall_x \forall_y P(x,y) \quad x,y \in \{1,2,3,\cdots N\}$

$= \forall_x [P(x,1) \wedge P(x,2) \wedge \cdots \wedge P(x,N)]$.

$= [P(1,1) \wedge P(1,2) \wedge P(1,3) \wedge \cdots \wedge P(1,N)] \wedge$
$[P(2,1) \wedge P(2,2) \wedge P(2,3) \wedge \cdots \wedge P(2,N)] \wedge$
$[P(3,1) \wedge P(3,2) \wedge P(3,3) \wedge \cdots \wedge P(3,N)] \wedge$
$\vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots$
$[P(N,1) \wedge P(N,2) \wedge P(N,3) \wedge \cdots \wedge P(N,N)]$

$\forall_x \exists_y P(x,y) = \forall_x [P(x,1) \vee P(x,2) \vee \cdots \vee P(x,N)]$
$[P(1,1) \vee P(1,2) \vee P(1,3) \vee \cdots \vee P(1,N)] \wedge$
$[P(2,1) \vee P(2,2) \vee P(2,3) \vee \cdots \vee P(2,N)] \wedge$
$\vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots$
$[P(N,1) \vee P(N,2) \vee P(N,3) \vee \cdots \vee P(N,N)]$

$\exists_x \forall_y P(x,y) = \exists_x [P(x,1) \wedge P(x,2) \wedge \cdots \wedge P(x,N)]$
$[P(1,1) \wedge P(1,2) \wedge P(1,3) \wedge \cdots \wedge P(1,N)] \vee$
$[P(2,1) \wedge P(2,2) \wedge P(2,3) \wedge \cdots \wedge P(2,N)] \vee$
$\vdots \qquad\qquad\qquad\qquad\qquad\qquad \vdots$
$[P(N,1) \wedge P(N,2) \wedge P(N,3) \wedge \cdots \wedge P(N,N)]$

$\exists_x \exists_y P(x,y) =$ Logical Expression.

Q: Compute the logical expression. $\neg \forall_x \exists_y P(x,y) \quad x,y \in \{1,2,3\}$

$\neg \forall_x \underbrace{(\exists_y P(x,y))}_{P(x)} = \exists_x \neg \exists_y P(x,y)$

① $\neg \forall_x P(x) = \exists_x \neg P(x)$

② $\neg \exists_x P(x) = \forall_x \neg P(x)$

$= \exists_x \forall_y \neg P(x,y)$

$= \exists_x [\neg P(x,1) \wedge \neg P(x,2) \wedge \neg P(x,3)]$

$= [\neg P(1,1) \wedge \neg P(1,2) \wedge \neg P(1,3)] \vee$

$[\neg P(1,1) \wedge \neg P(2,2) \wedge \neg P(3,3)] \vee$
$[\neg P(3,1) \wedge \neg P(3,2) \wedge \neg P(3,3)]$.

Nested Quantifiers. to write its logical expression.

① Negation with quantifiers.
→ Apply the negation on quantifiers.
→ Simplified expressions.

② To compute the complete expressions.

Homework :- $\forall x \exists y \forall z\ P(x,y,z)$ $x,y,z \in \{1,2\}$.
check with your friends.

① Computational Aspect.
② Meaning or Semantics.

Ex1 P47 $\forall x \forall y\ (x+y = y+x)$ $x,y \in \mathbb{R}$.

let $P(x,y) = x+y = y+x$ $\forall x \forall y\ P(x,y) =$ T or F.

$-\frac{4}{5} + \frac{1}{3} = +\frac{1}{3} - \frac{4}{5}$ $-\infty \ldots\ldots +\infty$ fractions.

$\forall x \exists y\ P(x,y) = x+y=0$ $\forall x \exists y\ P(x,y) =$ T or F.

$=T$ $x,y \in \mathbb{R}$.

$x=5$, $y=-5$

$-\infty$ ⋮ x ⋮ $+\infty$

Ex4 P48 $\exists y \forall x\ P(x,y)$ $P(x,y) = x+y \geq 0$. $x,y \in \mathbb{R}$.

$=F$

$-\infty$ | one value of y
$+\infty$ | any value of x

Observations
{ Change the order of Quantifiers the truth value may change.

$\exists y \forall x\ P(x,y) = F$
$\forall x \exists y\ P(x,y) = T$

Ex5 P49 $Q(x,y,z) = x+y=z$ $x,y,z \in \mathbb{R}$.

① $\forall x \forall y \exists z\ Q(x,y,z) = ?$ ② $\exists z \forall x \forall y\ Q(x,y,z)$.

T ? F

$-\infty$ ..... $+\infty$

$1+2 = 3$

$1+-0.5 = 0.5$ $z=5$

1

$1, 2 = 3$

$1, -0.5 = 0.5$

$Z = 5$

$x + y = z$

$2+3 = 5$

$6 - 1 = 5$

$5 + 0 = 5$

$6+1 \neq 5$

Ex9 PS1) Translate into English.

$\forall x (C(x) \vee \exists y (C(y) \wedge F(x, y)))$.

$C(x)$: x has a Computer

$F(x, y)$: x and y are friends

$x, y \in$ Set of students in your school.

for all x, x is a student in your school,
x has a Computer or there exist y,
y is a student in your school,
y has a Computer and
x and y are friends.

Ex10 PS1 Translate into English

$\exists x \forall y \forall z (F(x, y) \wedge F(x, z) \wedge (x \neq z)) \rightarrow \neg F(y, z)$

$F(a, b)$: a and b are friends

$x, y, z \in$ All students in your school.

There exists x, such that for all y, z, where
x, y, z are students in your school,
if x and y are friends and y and z are also friends
and x and z are not the same, Then y and z are not friends.