Topic: Separable differential equations

Question: Find the solution to the separable differential equation.

$$\frac{dy}{dx} = \frac{\sec^2 x}{\tan^2 y}$$

Answer choices:

A $\tan y - y = C \tan x$

B $\tan y - y = \tan x + C$

C $y = \tan x + C$

D $y(\tan y - 1) = \tan x + C$

Solution: B

The first step here is to manipulate the equation so that all of the y variables are on the left, and all of the x variables are on the right.

$$\frac{dy}{dx} = \frac{\sec^2 x}{\tan^2 y}$$

$$dy = \frac{\sec^2 x}{\tan^2 y}\ dx$$

$$\tan^2 y\ dy = \sec^2 x\ dx$$

Now we'll integrate both sides, adding the constant of integration C to the right side when we integrate.

$$\int \tan^2 y\ dy = \int \sec^2 x\ dx$$

$$\int \left(\sec^2 y - 1\right)\ dy = \int \sec^2 x\ dx$$

Now we'll solve for y in order to find the solution to the separable differential equation.

$$\int \sec^2 y\ dy - \int 1\ dy = \int \sec^2 x\ dx$$

$$\tan y - y = \tan x + C$$

There's no way to solve this equation explicitly for y, so this is the implicit solution.

Topic: Separable differential equations

Question: Find the solution to the separable differential equation.

$$\frac{du}{dv} = \frac{3v\sqrt{1+u^2}}{u}$$

Answer choices:

A $u = \sqrt{\frac{3}{2}v^2} + C$

B $u = \sqrt{\frac{3}{2}v^2 + C}$

C $u = \sqrt{\left(\frac{3}{2}v^2 + C\right)^2 - \frac{3}{2}}$

D $u = \sqrt{\left(\frac{3}{2}v^2 + C\right)^2 - 1}$

Solution: D

The first step here is to manipulate the equation so that all of the u variables are on the left, and all of the v variables are on the right.

$$\frac{du}{dv} = \frac{3v\sqrt{1+u^2}}{u}$$

$$du = \frac{3v\sqrt{1+u^2}}{u}\ dv$$

$$u\ du = 3v\sqrt{1+u^2}\ dv$$

$$\frac{u}{\sqrt{1+u^2}}\ du = 3v\ dv$$

Now we'll integrate both sides, adding the constant of integration C to the right side when we integrate.

$$\int \frac{u}{\sqrt{1+u^2}}\ du = \int 3v\ dv$$

We'll have to use u-substitution to integrate the right side.

$$k = 1 + u^2$$

$$\frac{dk}{du} = 2u$$

$$dk = 2u\ du$$

$$du = \frac{dk}{2u}$$

Making this substitution into the integral gives

$$\int \frac{u}{\sqrt{k}}\left(\frac{dk}{2u}\right) = \int 3v\ dv$$

$$\frac{1}{2}\int \frac{1}{\sqrt{k}}\ dk = \int 3v\ dv$$

$$\frac{1}{2}\int k^{-\frac{1}{2}}\ dk = \int 3v\ dv$$

$$\frac{1}{2}\left(\frac{2}{1}\right)k^{\frac{1}{2}} = \frac{3}{2}v^2 + C$$

$$k^{\frac{1}{2}} = \frac{3}{2}v^2 + C$$

$$\left(1 + u^2\right)^{\frac{1}{2}} = \frac{3}{2}v^2 + C$$

$$\sqrt{1 + u^2} = \frac{3}{2}v^2 + C$$

Now we'll solve for u in order to find the solution to the separable differential equation.

$$1 + u^2 = \left(\frac{3}{2}v^2 + C\right)^2$$

$$u^2 = \left(\frac{3}{2}v^2 + C\right)^2 - 1$$

$$u = \sqrt{\left(\frac{3}{2}v^2 + C\right)^2 - 1}$$