

## Direct proof

### Conditional statement

If P is true, then Q is true. This can be abbreviated as  $P(\text{hypothesis}) \Rightarrow Q(\text{conclusion})$  and which is read ' $P$  implies  $Q$ '

$$P(\text{"if it is chicken"}) \Rightarrow Q(\text{"then it is poultry"})$$

$$P(\text{"if P is true"}) \Rightarrow Q(\text{"then Q is true"})$$

$$\text{then } P \Rightarrow Q$$

Not all conditional statements will be true. If we switch the hypothesis and conclusion, then it will be false.

$$P(\text{"if it is poultry"}) \Rightarrow Q(\text{"then it is chicken"})$$

## Direct proof

To give a direct proof of  $P \Rightarrow Q$ , we assume that the hypothesis  $P$  is true and then show that the conclusion  $Q$  is also true.

Exercise 1) Prove the following statements:

- a) If  $a$  is odd and  $b$  is even, then  $a + b$  is odd.
- b) If  $a$  is odd and  $b$  is odd, then  $ab$  is odd.

Exercise 2) Let  $p, q \in \mathbb{Z}$  such that  $p$  is divisible by 5 and  $q$  is divisible by 3. Prove that  $pq$  is divisible by 15.

Exercise 3) Let  $x$  and  $y$  be positive real numbers. Prove that if  $x > y$ , then  $x^2 > y^2$ .

Exercise 4) Let  $x$  and  $y$  be any two positive real numbers. Prove that  $\frac{x+y}{2} \geq \sqrt{xy}$